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Vortex plasma and transport in superconducting films with magnetic dots

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Abstract. – We consider a superconducting film supplied with a magnetic dot array. The dot magnetization is random and oriented perpendicular to the film. The concentration of the superconducting vortices bound to the dots is a step-like function of the dot magnetic flux. The concentration of the unbound vortices is an oscillating function of the same variable. The resistivity is determined by the thermal drift of the vortices through the corner points of a checker-board formed by positive and negative unbound vortices.

The emerging heterogeneous magneto-superconducting systems open a new class of physical phenomena related to formation of the spatially modulated vortex plasma, analogous by its structure to the electrone-hole liquid in overcompensated semiconductors. The current interest is motivated not only by an important technological promise, but also by the wealth of physical effects produced by the interplay of inhomogeneous magnetic field acting on superconducting film and the potential relief due to induced superconducting vortices. In this letter we focus on the vortex plasma which consists of spatially separated vortex nano-droplets with alternating sign pinned on the nanoscale. Such a plasma appear in the extensively investigated arrays of magnetic dots deposited on thin (about 100 nm) superconducting films [1]. It was predicted [2] that the magnetic dots with moments normal to the film induce randomly oriented vortices upon zero-cooling below superconducting transition temperature T_c. These vortices are localized near their parental dots and, in their turn, create a potential relief favoring thermal generation of free vortex pairs. Secondary vortices are not bound by dots. Therefore, the pairs will dissolve and vortices will separate according to their signs giving rise to a peculiar random checker-board structure (see fig. 1). A typical size of the cell in such a checker-board is the effective penetration length in the film $\lambda_{\mathrm{eff}} = \lambda^2/d$, where λ is the London

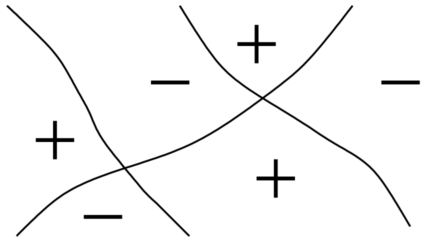

Fig. 1 – The checker-board average structure of the vortex plasma.

penetration length and d is the sample thickness [3]. The unbound vortices were suggested to be in a resistive state [2].

We study the thermodynamics and transport of such checker-board vortex plasma. We find the dependence of the concentrations of bound and unbound vortices on the magnetic moment of a single magnetic dot and temperature. Having established thermodynamic properties of vortex plasma, we calculate the resistivity of this system.

The model. – The random potential favoring formation of the vortex plasma is the superposition of slow (logarithmically) varying single-bound-vortex potentials. For the sake of simplicity we replace this slowly varying potential $V(\boldsymbol{r})$ by a potential having a constant value within the single cell: $V_0 = 2\epsilon_0$ at the distance $r < \lambda_{\rm eff}$ and zero at $r > \lambda_{\rm eff}$, where $\epsilon_0 = \Phi_0^2/(16\pi^2\lambda_{\rm eff})$, Φ_0 is the magnetic flux quantum. Considering the film as a set of almost unbound cells of the linear size $\lambda_{\rm eff}$ we arrive at the following Hamiltonian for such a cell:

$$H = -U\sum_i \sigma_i n_i + \epsilon\sum_i n_i^2 + 2\epsilon_0\sum_{i>j} n_i n_j, \tag{1}$$

where n_i is integer vorticity on either a dot or a site of the dual lattice (between the dots) which we conventionally associate with location of unbound vortices. $\sigma_i = \pm 1$, where the subscript i relates to the dot, describes the random sign of the dot magnetic moments. $\sigma_i = 0$ on the sites of the dual lattice. The first term of the Hamiltonian (1) describes the binding energy of the vortex at the magnetic dot and $U \approx \epsilon_0\Phi_d/\Phi_0$, with Φ_d being the magnetic flux through a single dot. The second term in the Hamiltonian is the sum of single vortex energies, $\epsilon = \epsilon_0 \ln(\lambda_{\rm eff}/a)$, where a is the period of the dot array, ξ is the superconducting coherence length. The third term mimics the intervortex interaction. Redefining the constant ϵ, one can replace the last term of eq. (1) by $\epsilon_0(\sum n_i)^2$. The sign of the vorticity on a dot follows two possible ("up-" and "down-") orientations of its magnetization. The vortices located between the dots (n_i on the dual lattice) are correlated on the scales of order $\lambda_{\rm eff}$ and form the above-mentioned irregular checker-board potential relief.

Thermodynamics. – We consider a cell with a large number of dots of each sign $\sim (\lambda_{\rm eff}/a)^2 \gg 1$. The energy (1) is minimal when the "neutrality" condition $Q \equiv \sum n_i = 0$ is satisfied. Indeed, if $Q \neq 0$ the interaction energy grows as Q^2, whereas the first term of the Hamiltonian behaves as $|Q|$ and cannot compensate for the last one unless $Q \sim 1$. The neutrality constraint means that the unbound vortices screen almost completely the "charge" of those bound by dots, that is $K \sim (N_+ - N_-) \sim \sqrt{N_\pm} \sim \lambda_{\rm eff}/a$, where K is the difference between the numbers of positive and negative dots and $N_\pm$ are the numbers of positive and negative vortices, respectively. Neglecting the total charge $|Q|$ as compared with $\lambda_{\rm eff}/a$, we minimize the energy (1) accounting for the neutrality constraint. At $Q = 0$ the Hamiltonian

(1) can be written as the sum of one-vortex energies:

$$H = \sum H_i; \qquad H_i = -U\sigma_i n_i + \epsilon n_i^2. \tag{2}$$

The minima for any H_i is achieved by choosing $n_i = n_i^0$, an integer closest to the magnitude $\sigma_i\kappa = \sigma_i U/(2\epsilon)$. The global minimum consistent with the neutrality is realized by values of n_i that differ from the local minima values n_i^0 not more than over ±1. Indeed, in the configuration with $n_i = n_i^0$, the total charge $|\sum n_i^0| \sim \kappa|\sum \sigma_i| = \kappa K$. Hence, if $\kappa \ll \lambda_{\rm eff}/a$, then the change of the vorticity at a small part of sites by ±1 restores neutrality. To be more specific, let us consider $K > 0$. Let $\bar{n}$ be the integer closest to κ, and consider the case $\kappa < \bar{n}$. Then the minimal energy corresponds to a configuration with vorticity $n_i = -\bar{n}$ at each negative dot and with vorticity $\bar{n}$ or $\bar{n} - 1$ at positive dots. The neutrality constraint implies that the number of positive dots with vorticity $\bar{n} - 1$ is $M = K\bar{n}$. In the opposite case $\kappa > \bar{n}$ the occupancies of all the positive dots are $\bar{n}$, whereas the occupancies of the negative dots are either $\bar{n}$ or $\bar{n}+1$. Note that in our model the unbound vortices are absent in the ground state unless κ is an integer. Indeed, the transfer of a vortex from a dot with the occupancy n to a dual site changes the energy by $\Delta E = 2\epsilon(\kappa - n + 2)$. Hence, the energy transfer is zero if and only if κ is an integer, otherwise the energy change upon the vortex transfer is positive. At integer κ, the number of the unbound vortices can vary from 0 to $K\bar{n}$ without change of energy. The ground state is degenerate at any non-integer κ since, while the total number of dots with different vorticities is fixed, the vortex exchange between two dots with vorticities n and $n \pm 1$ does not change the total energy. Thus, our model predicts a step-like dependence of dot occupancies on κ at the zero temperature and peaks in the concentration of unbound vortices as shown in fig. 2. The thermal behavior of the model is governed by its partition function

$$Z = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}k \sum_{n_i} \exp\left[-\frac{\tilde{H}}{T}\right], \tag{3}$$

where $\tilde{H}$ is the truncated Hamiltonian which includes the two first terms of eq. (1) but no intervortex interactions; the integration over k ensures the neutrality. After the summation over n_i the integral can be calculated by the saddle-point method. The result is

$$Z = \Theta^{\frac{N+K}{2}}(x, ik_0 + y)\Theta^{\frac{N-K}{2}}(x, ik_0 - y)\Theta^N(x, ik_0), \tag{4}$$

where $x = \epsilon/T$, $y = U/T$, k_0 is the saddle-point, and

$$\Theta(x, y) = \sum_{n=-\infty}^{n=+\infty} \exp[-xn^2 + yn]. \tag{5}$$

The function $\Theta(x, y)$ can be expressed in terms of the elliptic theta-functions [4]. Since $\gamma = K/N$ is small, the saddle-point value k_0 is also small and assumes the form

$$k_0 = \frac{2i\gamma F(x, y)}{G(x, y) + G(x, 0)}, \tag{6}$$

where $F(x, y) = \partial/\partial y[\ln\Theta(x, y)]$, $G(x, y) = -\partial/\partial x[\ln\Theta(x, y)] - (F(x, y))^2$. Let $\tilde{N}_\pm$ denote the absolute values of the vortex charge on all positive (negative) dots, $n_\pm = \tilde{N}_\pm/N$. Then

$$n \equiv n_+ + n_- = \frac{1}{N}\frac{\partial}{\partial y}[\ln Z(x, y)] = F(x, y), \tag{7}$$

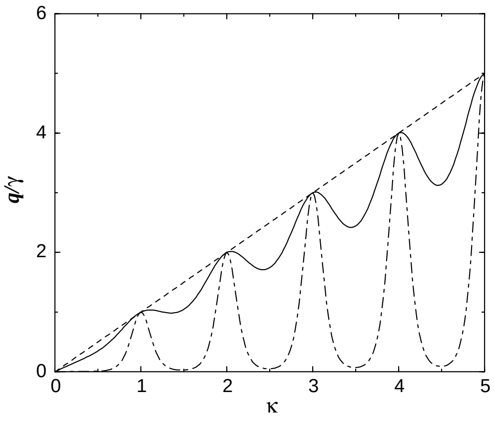

Fig. 2

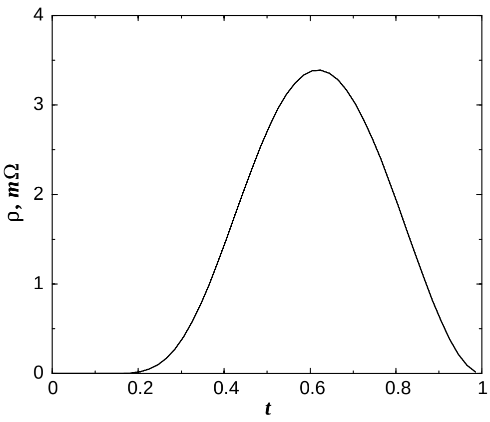

Fig. 3

Fig. 2 – The average number of the unbound vortices in the cell of size a via the parameter κ proportional to the dot magnetic moment. The dot-dashed line corresponds to $T/\epsilon_0 = 0.15$, the solid line corresponds to $T/\epsilon_0 = 0.4$, the dashed line corresponds to $T/\epsilon_0 = 2$.

Fig. 3 – The static resistance ρ of the film *vs.* dimensionless temperature $t = T/T_c$ at typical values of parameters.

$$q \equiv n_+ - n_- = \frac{2\gamma F(x,y)G(x,0)}{G(x,y) + G(x,0)} \, . \tag{8}$$

The values (7) and (8) can be treated as the concentrations of the bound and unbound vortices, respectively. The dependencies of the unbound vortex concentration on $\kappa = y/(2x)$ for several values of $x = \epsilon/T$ are shown in fig. 2. Oscillations are well pronounced for $x \gg 1$ and are suppressed at small x (large temperatures). At low temperatures, $x \gg 1$, the half-widths of the peaks in the density of the unbound vortices are $\Delta\kappa \approx 1/x$ and the heights of peaks are $\approx \gamma n$.

Incomplete screening of the intervortex interactions also smears the peaks. Since the relative amplitude of the fluctuations of interaction energies is $a/\lambda_{\rm eff}$, the half-widths of the peaks due to these fluctuations is of the same order of magnitude. In the presence of the external magnetic field B the neutrality constraint is to be replaced by the condition $Q = B\lambda_{\rm eff}^2/\Phi_0$ (Φ_0 is the magnetic flux associated with a vortex), and the above calculations can be carried over to the finite field case. The density of the bound vortices does not depend on the field up to $B \sim \Phi_0/a^2$, while the density of unbound vortices q changes substantially even at small fields $B \sim \gamma\Phi_0/a^2$:

$$q = \frac{(b + 2\gamma F(x,y))G(x,0)}{G(x,y) + G(x,0)} \, , \tag{9}$$

where $b = Ba^2/\Phi_0$. At $b > 2\gamma\kappa$ the irregular checker-board vanishes and vortices of one sign determined by the magnetic field prevail. The oscillations of q *vs.* κ vanish at $b \sim \gamma$ or $x \sim 1$. At $b \ll \gamma$ the only qualitative change in the κ-dependence of the density is the appearance of an additional maximum at $\kappa = 0$.

Vortex transport. – At moderate external currents j the vortex transport and dissipation are controlled by percolation of unbound vortices through the corners of the checker-board cells where the saddle-points in the potential relief for the vortices are located.

The typical energy barrier associated with the corner ridge is ϵ_0, this barrier indeed exceeds the bulk single vortex barriers due to point defect pinning [5], thus dominating the transport. The unbound vortex density is $m \sim a^{-2}\gamma \sim (a\lambda_{\rm eff})^{-1}$ and oscillates with κ as was shown above. The average distance between the unbound vortices is $l \sim \sqrt{a\lambda_{\rm eff}}$, which is also an average distance between the corner saddle-point and the nearest unbound vortex. The transport current exerts the Magnus (Lorentz) force $F_{\rm M} = j\Phi_0/c$ acting on a vortex. Since the condiditon $T \ll \epsilon_0$ is satisfied in the vortex state everywhere except for the regions too close to $T_{\rm c}$, the percolation through the coner occurs via the thermally activated jumps with the rate

$$\nu = \nu_0 \exp[-\epsilon_0/T] = (\mu j\Phi_0/cl)\exp[-\epsilon_0/T], \tag{10}$$

where $\mu = (\xi^2\sigma_n)/(4\pi e^2)$ is the Bardeen-Stephen vortex mobility [6]. The induced electric field near the corner is, accordingly,

$$E_{\rm c} = l\dot{B}/c = m\Phi_0\nu l/c\,, \tag{11}$$

The Ohmic losses per corner are $W_{\rm c} = jE_{\rm c}\lambda_{\rm eff}a = j\Phi_0\nu l/c$ giving rise to the dc resistivity as

$$\rho_{\rm dc} = \frac{W_c}{j^2\lambda_{\rm eff}^2} = \frac{\mu\Phi_0^2}{c^2\lambda_{\rm eff}^2}\exp[-\epsilon_0(T)/T]\,. \tag{12}$$

The energy barriers at the corner are random and fluctuate around ϵ_0; this average value of the corner barrier corresponds to the percolation level through the checker-board.

For typical values $\epsilon_0|_{T=0}/T_{\rm c} \sim 2$, $\lambda_{\rm eff} = 10^{-3}$ cm, $\mu \sim 2\times 10^{15}$ CGS the dependence of $\rho_{\rm dc}$ on T is shown in fig. 3. Interestingly, the static resistivity does not depend on the dot density, but is proportional to the corner density $\lambda_{\rm eff}^{-2}$. Note the non-monotonic dependence of $\rho_{\rm dc}$ on temperature T. The low fequency, $\omega \ll \Delta$, ac resistivity is governed by two competing processes, corresponding to two kinds of motion each unbound vortex can participate in. The first process is the bulk energy dissipation due to oscillations of unbound vortices within the cells of the checker-board [7]. The second process is the activation of the vortices through the corners of the irregular checker-board considered above. Summing up the ac contributions from different channels similarly to [7] we arrive at

$$\rho_{\rm ac} = \rho_{\rm dc}(1 + i\omega/\nu) + \frac{iq\omega\Phi_0^2}{c^2(-\alpha_{\rm L} + i\omega\eta)q^2}\,, \tag{13}$$

where $\alpha_{\rm L} \simeq \epsilon_0 \ln(\lambda_{\rm eff}/a)/\lambda_{\rm eff}^4$ is the rigidity of a random potential well associated with the bulk pinning, and η is the vortex viscosity. The predicted oscillations of q *vs.* κ can be observed in the high-frequency limit of $\Re\rho_{\rm ac}$. In conclusion, we have found the density of the unbound vortices in the superconducting film supplied with the periodic array of ferromagnetic randomly magnetized dots. We have found that this density is an oscillating function of the flux through a dot. The resistivity of such a system is determined by thermally activated jumps of vortices through the corners of the irregular checker-board formed by the positive or negative unbound vortices and oscillates with $\Phi_{\rm d}$. These oscillations can be observed by additional deposition (or removal) of the magnetic material to the dots.

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