Magnetic quantization of electronic states in *d*-wave superconductors

N. B. Kopnin[1,2] and V. M. Vinokur[3]
[1]*Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 2200, FIN-02015 HUT, Finland*
[2]*L. D. Landau Institute for Theoretical Physics, 117940 Moscow, Russia*
[3]*Argonne National Laboratory, Argonne, Illinois 60439*
(Received 6 March 2000)

We derive a general quasiclassical approach for long-range magnetic-field quantization effects in superconductors. The method is applied to superclean *d*-wave superconductors in the mixed state for delocalized states with energies $\epsilon \gg \Delta_0 \sqrt{H/H_{c2}}$. We find that the energy spectrum consists of narrow energy bands whose centers are located at the Landau levels calculated in the absence of the vortex potential. We show that transitions between the states belonging to the different Landau levels can be observed experimentally due to resonances in the ac vortex friction.

I. INTRODUCTION

The unusual behavior of the thermodynamic and transport properties of *d*-wave superconductors as functions of magnetic field has been the subject of extensive experimental and theoretical studies. This behavior is attributed to nontrivial energy dependence of the electronic density of states[1–4] and to specific kinetic processes which are very sensitive to the fine details of electronic states brought about by the presence of vortices.[5–8] There exists, however, a conceptual controversy about the structure of the electronic spectrum in *d*-wave superconductors in the mixed state. One of the views is that the states below the maximum gap Δ_0 have a discrete spectrum due to Andreev reflections; some states are localized within vortex cores[8,9] while others are quantized at longer distances[10–12] as a particle which moves along a curved trajectory in a magnetic field hits the gap for a current momentum direction where the energy becomes equal to $|\Delta_{\mathbf{p}}|$. Other authors propose that instead of the magnetic quantization, energy bands appear in a periodic vortex potential due to the vortex lattice.[13–16]

In the present paper we develop a general quasiclassical approach for calculating the long-range magnetic-field quantization effects in superconductors in the regime where the electron wavelength is much shorter than the coherence length $p_F\xi \gg 1$. The proposed method is applied to superclean *d*-wave superconductors in the mixed state in the low-field limit $H \ll H_{c2}$. We show in Secs. II–IV that the influence of a magnetic field on delocalized excitations in a superconductor is not reduced to simply the action of an effective vortex lattice potential. The effect of magnetic field is rather twofold: (i) It creates vortices and thus provides a periodic potential for electronic excitations. (ii) It also affects the long-range motion of quasiparticles in a manner similar to that in the normal state. The long-range effects are less pronounced for low-energy excitations. On the contrary, the spectrum of excitations with energies $\epsilon \gtrsim \Delta_0 \sqrt{H/H_{c2}}$ is determined mostly by long-distance motion and exhibits magnetic quantization. We study the delocalized states with energies $\epsilon \gg \Delta_0 \sqrt{H/H_{c2}}$ and calculate their energy spectrum. We find that the spectrum consists of energy bands as one would indeed expect to be the case for a periodic potential. However, these bands in the quasiclassical limit are rather narrow; their centers are located at the Landau levels calculated in Refs. 10–12. We thus demonstrate that the picture of the energy spectrum is in fact a compromise between the two above-mentioned extremes. We emphasize that both the quasiclassical assumption $p_F\xi \gg 1$ and the high-energy condition $\epsilon \gg \Delta_0 \sqrt{H/H_{c2}}$ are crucial for our results to hold. Because of an increasing role of the periodic vortex potential, the states with lower energies deviate strongly from the Landau-level picture and resemble more the band structure of a solid obtained within the tight-binding approximation. We note also that the results of numerical solutions of the Bogoliubov–de Gennes equations of Refs. 15 and 16 cannot be directly compared with our analytical results because the calculations in these works were done for conditions where at least one of our basic assumptions is not fulfilled.

In this paper we restrict ourselves to a more qualitative analysis and concentrate on situations where the exact band structure of the electronic states is not essential, leaving the detailed numerical solution of our equations for a forthcoming publication. In Sec. V, we demonstrate that the obtained Landau-level structure of electronic states is important for understanding dynamic and transport properties of *d*-wave superconductors in a wide temperature range $T_c\sqrt{H/H_{c2}} \ll T \ll T_c$. We consider effects of the energy spectrum on the vortex dynamics which can be accessed by magneto-optical experiments in the far-infrared region (compare with Ref. 17). We show that the vortex friction for oscillating vortices displays resonances at transitions between the states belonging to different Landau levels.

II. LONG-RANGE EFFECTS OF THE MAGNETIC FIELD

We start with the standard Bogoliubov–de Gennes equations

$$\left[\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2 - p_F^2\right]u + 2m\Delta_{\hat{\mathbf{p}}}v = 2m\epsilon u,$$

$$\left[\left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A}\right)^2 - p_F^2\right]v - 2m\Delta^*_{\hat{\mathbf{p}}}u = -2m\epsilon v, \qquad (1)$$

where $\hat{\mathbf{p}}=-i\nabla$ is the canonical momentum operator. Equations (1) have particle-hole symmetry such that $u\to v^*, v\to -u^*$ under complex conjugation and $\epsilon\to-\epsilon$. For a vortex array, the order parameter phase is a multiple-valued function defined through

$$\mathrm{curl}\nabla\chi=\sum_i 2\pi\delta(\mathbf{r}-\mathbf{r}_i). \tag{2}$$

As a result

$$\nabla\chi=\sum_i \frac{\mathbf{z}\times(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|^2} \tag{3}$$

such that, on average, $\nabla\chi\approx eHr/c$ for large r.

Consider a quasiparticle in a magnetic field in the presence of a vortex lattice for energies above the gap at infinity. If the particle mean free path is longer than the Larmor radius, i.e., $\omega_c\tau\gg1$ where ω_c is the cyclotron frequency, such a particle can travel away from each vortex up to distances of the order of the Larmor radius $r_L=v_F/\omega_c$. This brings new features to Eqs. (1). Assume for a moment that $\Delta=0$. Then the wave function u describes a particle with kinetic momentum $\mathbf{P}_+=\mathbf{p}-(e/c)\mathbf{A}$ and energy $\epsilon=\mathbf{P}_+^2/2m-E_F$ while v describes a hole with kinetic momentum $\mathbf{P}_-=\mathbf{p}+(e/c)\mathbf{A}$ and energy $\epsilon=E_F-\mathbf{P}_+^2/2m$. A particle and a hole which start propagating from the same point will then move in different directions and along different trajectories which transform one into another under the transformation $\mathbf{H}\to-\mathbf{H}$. For a finite order parameter the wave function is a linear combination of a particle and a hole. It is not convenient, however, to use such a combination at distances where the trajectories of the particle and the hole go far apart, i.e., when the vector potential is no longer small compared to the Fermi momentum p_F.

Equation (1) shows that the phase of u differs from that of v by the order parameter phase χ. To construct a proper basis, one needs to bring the phases of u and v in correspondence with each other. We note that the usual transformation

$$\begin{pmatrix} u \\ v \end{pmatrix}=\begin{pmatrix} e^{i\chi/2}\tilde{u} \\ e^{-i\chi/2}\tilde{v} \end{pmatrix}, \quad \check{\psi}=\begin{pmatrix} \tilde{u} \\ \tilde{v} \end{pmatrix}, \tag{4}$$

which leads to a substitution of $\mathbf{A}$ with $-(mc/e)\mathbf{v}_s=\mathbf{A}-(c/2e)\nabla\chi$ in Eq. (1), is not convenient when considering a particle which can move at distances much larger than the size of one unit cell. Though it accounts correctly for the phase difference between u and v, it introduces an extra overall phase $\pm\chi/2$ into the new wave function $\check{\psi}$ as compared to the initial particle (or hole) basis; see the discussion later in this section. This overall phase increases with distance and is equivalent to a gauge transformation to a ''rotating frame'' where the magnetic field drops out of $\mathbf{v}_s$ because curl $\mathbf{v}_s$ vanishes on average but a Coriolis (i.e., the Lorentz) force appears instead[18] (see also Sec. V). Mathematically it follows from the fact that the transformation $e^{i\chi/2}$ is singular, $(\nabla_i\nabla_k-\nabla_k\nabla_i)\chi\neq0$, i.e., from Eq. (2). It means that the momentum in the new frame is not an integral of motion even in absence of the vortex potential associated with the superconducting velocity and spatial variations of the order parameter magnitude. The transformation of Eq. (4), however, is not dangerous if particles are bound to distances of the order of few unit cells because the phase gradient is limited $|\nabla\chi|\ll p_F$; in other words, the Lorentz force does not affect the trajectory considerably. However, for a vortex array, the phase gradient can reach values comparable with p_F.

To avoid these complications we use here another transformation which also removes the coordinate dependence of the order parameter phase. The results, of course, should be independent of the choice of transformation due to the gauge invariance. Following Refs. 11 and 14 we put in Eq. (1)

$$u=u', \quad v=\exp(-i\chi)v'. \tag{5}$$

This is a single-valued transformation. We obtain

$$[\hat{\mathbf{P}}_+^2-p_F^2]u'+2me^{-i\chi}\Delta_{\hat{\mathbf{P}}'_+}v'=2m\epsilon u', \tag{6}$$

$$[(\hat{\mathbf{P}}_+-2m\mathbf{v}_s)^2-p_F^2]v'-2me^{i\chi}\Delta^*_{\hat{\mathbf{P}}'_+}u'=-2m\epsilon v', \tag{7}$$

where $\hat{\mathbf{P}}_+=\hat{\mathbf{p}}-(e/c)\mathbf{A}$ is the operator of the particle kinetic momentum, and

$$\hat{\mathbf{P}}'_+=\hat{\mathbf{p}}-\nabla\chi/2=\hat{\mathbf{P}}_+-m\mathbf{v}_s.$$

The superconducting velocity is

$$2m\mathbf{v}_s=\boldsymbol{\nabla}\chi-\frac{2e}{c}\mathbf{A}.$$

In Eqs. (6) and (7) we use that, for a general pairing symmetry, $\Delta_{\hat{\mathbf{p}}'}\propto uv^*$ depends actually on $\hat{\mathbf{p}}'=(\hat{\mathbf{p}}_u+\hat{\mathbf{p}}_v)/2$ where $\hat{\mathbf{p}}_{u,v}$ are the canonical momentum operators which act on the Bogoliubov wave functions u and v, respectively. The term $-\nabla\chi/2$ appears in the order parameter together with the canonical momentum $\mathbf{p}$ because only one half of the momentum operator in $\Delta_{\hat{\mathbf{p}}'}$ acts on each of the wave functions u or v.

The transformation of Eq. (5) is ''u like;'' it brings the phase of v in correspondence with the phase of u. Equation (5) defines the particlelike basis; within it, Eq. (7) describes the motion of a hole as it is seen by a particle. Note that, as distinct from Eq. (1), a particle and a hole determined by Eqs. (6) and (7) for $\Delta=0$ move along the same trajectory though, of course, in different directions.

The resulting equations are not symmetric with respect to u and v: the term $\mathbf{v}_s$ is present in the second equation together with $\hat{\mathbf{P}}$ while it does not appear in the first equation. Let us perform one more transformation

$$\begin{pmatrix} u' \\ v' \end{pmatrix}=\begin{pmatrix} U_+ \\ V_+ \end{pmatrix}e^{i\chi_v/2}, \tag{8}$$

where $\boldsymbol{\nabla}\chi_v=2m\mathbf{v}_s$ such that

$$\mathrm{curl}\boldsymbol{\nabla}\chi_v=\sum_i 2\pi\delta(\mathbf{r}-\mathbf{r}_i)-\frac{2e}{c}\mathbf{H}$$

and $\chi_v=\chi-\chi_A$ where

$$\nabla\chi_A=\frac{2e}{c}\mathbf{A}. \tag{9}$$

The ''phase'' χ_v is not single valued within each unit cell; it depends on the particular path of integration. However, it is single valued on average, i.e., on a scale much larger than the intervortex distance since

$$\int \mathrm{curl}\boldsymbol{\nabla}\chi_v d^2r=0.$$

It also implies that $\nabla\chi_v$ does not have large terms increasing with distance. The transformation, Eq. (8), is thus not dangerous. The total transformation, Eqs. (5) and (8), has the form

$$u=\exp(i\chi/2-i\chi_A/2)U_+ ,$$
$$v=\exp(-i\chi/2-i\chi_A/2)V_+ . \tag{10}$$

With this transformation we finally obtain

$$[(\hat{\mathbf{P}}_+ +m\mathbf{v}_s)^2-p_F^2]U_+ +2m\tilde{\Delta}_{\hat{\mathbf{P}}_+}V_+=2m\epsilon U_+ , \tag{11}$$

$$[(\hat{\mathbf{P}}_+ -m\mathbf{v}_s)^2-p_F^2]V_+ -2m\tilde{\Delta}_{\hat{\mathbf{P}}_+}U_+=-2m\epsilon V_+ , \tag{12}$$

where

$$\tilde{\Delta}_{\hat{\mathbf{P}}_+}=e^{-i\chi}\Delta_{\hat{\mathbf{p}}-(e/c)\mathbf{A}}=e^{i\chi}\Delta^*_{\hat{\mathbf{p}}-(e/c)\mathbf{A}}.$$

Another equation can be obtained using the transformation

$$u=e^{i\chi}e^{-i\chi_v/2}U_-=\exp(i\chi/2+i\chi_A/2)U_- ,$$
$$v=e^{-i\chi_v/2}V_-=\exp(-i\chi/2+i\chi_A/2)V_- . \tag{13}$$

We get

$$[(\hat{\mathbf{P}}_- +m\mathbf{v}_s)^2-p_F^2]U_- +2m\tilde{\Delta}_{\hat{\mathbf{P}}_-}V_-=2m\epsilon U_- , \tag{14}$$

$$[(\hat{\mathbf{P}}_- -m\mathbf{v}_s)^2-p_F^2]V_- -2m\tilde{\Delta}_{\hat{\mathbf{P}}_-}U_-=-2m\epsilon V_- , \tag{15}$$

where $\hat{\mathbf{P}}_-=\hat{\mathbf{p}}+(e/c)\mathbf{A}$ is the ''hole'' kinetic momentum. The transformation, Eq. (13), is ''v-like;'' it brings the phase of u in correspondence with that of v. Equation (13) defines the holelike basis such that Eq. (14) describes motion of a particle as seen by a hole. Again, both particles and holes with $\Delta=0$ move along the same trajectory.

Note that the vector wave function

$$\check{\Psi}_+=\begin{pmatrix} U_+ \\ V_+ \end{pmatrix}$$

defined by Eq. (10) differs from $\check{\psi}$ in Eq. (4) by an additional overall phase, $\check{\psi}=e^{-i\chi_A/2}\check{\Psi}_+$. It is exactly of the same origin as the extra phase present in $\check{\psi}$ as compared to the initial particlelike basis. One can say that the transformation (4) ''removes'' the magnetic field while the phase $-\chi_A/2$ ''restores'' it. Similarly, the phase $+\chi_A/2$ in Eq. (13) restores the magnetic field in the holelike basis.

One can transform Eqs. (11) and (12) further by putting

$$\check{\Psi}_+=\exp\left(i\int \mathbf{p}\cdot d\mathbf{r}\right)\check{\phi},\quad \check{\phi}=\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}, \tag{16}$$

where

$$\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)^2=p_F^2, \tag{17}$$

and $\check{\phi}$ is a slow function which varies over distances of the order of ξ. This approximation works if $p_F\xi\gg 1$. We shall call it the *first-level* quasiclassical approximation. It is expected to be valid for most superconductors. Of course, its accuracy is not very good for those high-T_c materials which have $p_F\xi$ not considerably larger than unity.

If $\mathrm{div}\,\mathbf{A}=0$, we have

$$\mathbf{P}_+(-i\nabla+m\mathbf{v}_s)\phi_1+m\tilde{\Delta}_{\mathbf{P}_+}\phi_2=m\epsilon\phi_1 ,$$

$$\mathbf{P}_+(-i\nabla-m\mathbf{v}_s)\phi_2-m\tilde{\Delta}_{\mathbf{P}_+}\phi_1=-m\epsilon\phi_2 . \tag{18}$$

Using Eq. (16) we can transform Eqs. (14) and (15) to their first-level quasiclassical version which is Eq. (18) where $\mathbf{P}_+$ is substituted by $\mathbf{P}_-$ under the condition $|\mathbf{P}_-|^2=p_F^2$. Equation (18) and its v-like analog possess particle-hole symmetry. Under the transformation

$$\mathbf{p}\to-\mathbf{p},\epsilon\to-\epsilon;\quad \phi_1\to\phi_2^*,\phi_2\to-\phi_1^*,$$

they go one into another. Moreover, each set of equations has particle-hole symmetry separately for a given position on the trajectory if the kinetic momenta $\mathbf{P}_\pm=\mathbf{p}\mp(e/c)\mathbf{A}$ are reversed for a fixed position of the particle. Due to Eq. (17), $\mathbf{p}-(e/c)\mathbf{A}=(q\cos\alpha,q\sin\alpha)$, where α is the local direction of the momentum. The reversal corresponds to $\alpha\to\pi+\alpha$.

We take the z axis along the magnetic field. To solve Eq. (18) we define the quasiclassical particlelike trajectory by

$$\frac{dx}{dy}=\frac{P_{+x}}{P_{+y}}=\frac{p_x-(e/c)A_x}{p_y-(e/c)A_y}. \tag{19}$$

When the magnetic-flied penetration length is much longer than the distance between vortices, $\lambda_L\gg a_0$, the magnetic field can be considered homogeneous. With $\mathbf{A}$ taken in the Landau gauge,

$$\mathbf{A}=(-Hy,0,0), \tag{20}$$

the trajectory is a circle:

$$(x-x_0)^2+(y+cp_x/eH)^2=(p_\perp c/eH)^2, \tag{21}$$

where $p_\perp^2=p_F^2-p_z^2$. The local direction of the kinetic momentum is $p_x+eHy/c=p_\perp\sin\alpha$, $p_y=p_\perp\cos\alpha$. The distance along the trajectory is $ds=r_Ld\alpha$ where the Larmor radius is $r_L=p_\perp/m\omega_c$. Equation (18) can now be written in terms of the quasiclassical trajectory Eq. (19). We have

$$v_\perp\left(-i\frac{\partial}{\partial s}+mv_t\right)\phi_1+\tilde{\Delta}(\alpha)\phi_2=\epsilon\phi_1 ,$$

$$v_\perp\left(-i\frac{\partial}{\partial s}-mv_t\right)\phi_2-\tilde{\Delta}(\alpha)\phi_1=-\epsilon\phi_2 . \tag{22}$$

Here $v_\perp=p_\perp/m$, and v_t is the projection of $\mathbf{v}_s$ on the local direction of the trajectory. $\tilde{\Delta}(\alpha)$ and v_t are functions of coordinates $x(s)$, $y(s)$ and of the angle $\alpha(s)$ taken at the trajectory. Equations (22) look exactly as the quasiclassical Bogoliubov–de Gennes equations obtained using the transformation, Eq. (4), with the important difference that the trajectory is now a circle rather than a straight line.

III. ELECTRONIC STATES IN ZERO LATTICE POTENTIAL

We take the order parameter in d-wave superconductors in the form $\tilde{\Delta}_{\mathbf{p}}=\Delta_0(2p_xp_y)/(p_x^2+p_y^2)$ so that $\tilde{\Delta}_{\mathbf{p}-(e/c)\mathbf{A}}=\Delta_0\sin(2\alpha)$. Consider first the limit $v_s=0$ and $\Delta_0=\text{const}$. Equations (22) become

$$-i\omega_c\frac{\partial\phi_1}{\partial\alpha}+\Delta_0\sin(2\alpha)\phi_2=\epsilon\phi_1,$$

$$i\omega_c\frac{\partial\phi_2}{\partial\alpha}+\Delta_0\sin(2\alpha)\phi_1=\epsilon\phi_2.$$

These equations can be solved with a *second-level* quasiclassical ansatz

$$\check{\phi}=\check{C}\exp[if(\alpha)].$$

We obtain

$$f(\alpha)=\pm\int\frac{d\alpha}{\omega_c}\sqrt{\epsilon^2-\Delta_0^2\sin^2(2\alpha)}.$$

The quantization rule also includes the integral over the momentum $\mathbf{p}$ defined by Eqs. (16) and (17). We have

$$\oint\mathbf{p}\,d\mathbf{r}\pm\oint\frac{d\alpha}{\omega_c}\sqrt{\epsilon^2-\Delta_0^2\sin^2(2\alpha)}=2\pi n. \tag{23}$$

This second-level quasiclassical approximation is less general as compared to the first-level approximation: In addition to the condition $p_F\xi\gg1$, it also requires that the quantum numbers $n\gg1$ be large.[19] The $\pm$ signs distinguish between particles and holes. As was already mentioned, a particle [with the plus sign in Eq. (23)] and a hole (with the minus sign) move along the same trajectory, Eq. (21), but in the opposite directions. The phase χ_v which was introduced in Eqs. (8) and (10) gives a contribution to the action of the order of 2π because it is limited from above by an increment of the order of circulation around one vortex unit cell; it can thus be neglected for large n.

A. Subgap states

In the range $|\epsilon|<\Delta_0$, the turning points correspond to a vanishing of the square root at $\alpha=\pm\alpha_\epsilon$ where $\sin(2\alpha_\epsilon)=|\epsilon|/\Delta_0$. We have

$$\frac{4}{\omega_c}\int_0^{\alpha_\epsilon}d\alpha\sqrt{\epsilon^2-\Delta_0^2\sin^2(2\alpha)}=2\pi n, \tag{24}$$

where $n>0$. The first integral in Eq. (23) disappears because the turning points of the momentum $\mathbf{p}$ are not reached: the particle cannot go far along the trajectory, Eq. (19), and remains localized on a given trajectory at distances $s\sim r_L(\epsilon/\Delta_0)$ smaller than the Larmor radius r_L. Note also that the contribution from χ_v vanishes identically because the particle after being Andreev reflected transforms into a hole which returns to the starting point along the same trajectory. Using the substitution $\sin x=(\Delta_0/\epsilon)\sin(2\alpha)$ we find

$$\int_0^{\alpha_\epsilon}d\alpha\sqrt{\epsilon^2-\Delta_0^2\sin^2(2\alpha)}=\frac{\Delta_0}{2}\left[E\left(\frac{\epsilon}{\Delta_0}\right)-\left(1-\frac{\epsilon^2}{\Delta_0^2}\right)K\left(\frac{\epsilon}{\Delta_0}\right)\right],$$

where $K(k)$ and $E(k)$ are the full elliptic integrals of the first and second kinds, respectively. Applying the Bohr-Sommerfeld quantization rule, Eq. (23), we obtain

$$\frac{2\Delta_0}{\omega_c}\left[E\left(\frac{\epsilon_n}{\Delta_0}\right)-\left(1-\frac{\epsilon_n^2}{\Delta_0^2}\right)K\left(\frac{\epsilon_n}{\Delta_0}\right)\right]=2\pi n. \tag{25}$$

These states are degenerate with the same degree as in the normal state: for each n, there are $\Phi/2\Phi_0=N_v/2$ states for particles and $N_v/2$ states for holes, where Φ is the total magnetic flux through the superconductor, and N_v is the total number of vortices.

Consider $\epsilon\ll\Delta_0$. Expanding in small k

$$E(k)=\frac{\pi}{2}\left(1-\frac{k^2}{4}\right),\quad K(k)=\frac{\pi}{2}\left(1+\frac{k^2}{4}\right),$$

we find from Eq. (25)

$$\epsilon_n=\pm\sqrt{4\Delta_0\omega_c n}. \tag{26}$$

Equation (26) agrees with the result of Refs. 10 and 11.

B. Extended states

If $|\epsilon|>\Delta_0$, we get for the Landau gauge, Eq. (20), $p_x=\text{const}$ and

$$\oint\mathbf{p}\,d\mathbf{r}=\oint p_y\,dy=2\int_{y_1}^{y_2}\sqrt{p_\perp^2-(p_x+eHy/c)^2}dy=\pi c p_\perp^2/eH.$$

The turning points $y_{1,2}$ correspond to the values of Larmor radius where $p_x+eHy_{1,2}/c=\pm p_\perp$. The corresponding trajectory is a closed circle where α varies by 2π. The second integral in Eq. (23) gives

$$\int_0^{2\pi}\frac{d\alpha}{\omega_c}\sqrt{\epsilon^2-\Delta_0^2\sin^2(2\alpha)}=\frac{4\epsilon}{\omega_c}E\left(\frac{\Delta_0}{\epsilon}\right). \tag{27}$$

The quantization rule (23) yields

$$\pm\frac{2\epsilon_n}{\pi}E\left(\frac{\Delta_0}{\epsilon_n}\right)=\omega_c n+\frac{p_z^2}{2m}-E_F. \tag{28}$$

For an s-wave superconductor we get, in particular,

$$\pm\sqrt{\epsilon_n^2-\Delta_0^2}=\omega_c n+\frac{p_z^2}{2m}-E_F. \tag{29}$$

IV. EFFECTS OF THE PERIODIC POTENTIAL

A. Bloch functions

At low magnetic fields $H \ll H_{c2}$, one can consider that the particle trajectory always passes far from cores. The oscillating part of the vortex potential comes mostly from the superconducting velocity. The corresponding Doppler energy $\zeta = p_\perp v_t$ is of the order of $\Delta_0 \sqrt{H/H_{c2}}$. This periodic potential transforms the discrete energy spectrum into energy bands. Equations (12) and (15) or the quasiclassical version, Eq. (18), are invariant under the magnetic translations by periods of the regular vortex lattice. Consider the particlelike equations (12) or (18). The particlelike operator of magnetic translations in a homogeneous field is[20]

$$\hat{T}(\mathbf{R}_l) = \exp\left[-i\mathbf{R}_l \left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A} \right) \right], \tag{30}$$

where $\hat{\mathbf{p}} = -i\nabla$ is the canonical momentum and $\mathbf{R}_l$ is a vector of the vortex lattice. Its zero-field version corresponds to a shift

$$\hat{T}_0(\mathbf{R}_l) f(\mathbf{r}) = \exp[-i\mathbf{R}_l \hat{\mathbf{p}}] f(\mathbf{r}) = f(\mathbf{r} - \mathbf{R}_l).$$

The operator $\hat{T}(\mathbf{R}_l)$ commutes with the Hamiltonian because $\mathbf{v}_s$ and Δ are periodic in the vortex lattice and the commutator:

$$\left[\left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A} \right)_i , \left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A} \right)_j \right] = 0.$$

Since $\mathbf{P}_+$ does not change under the action of the operator, Eq. (30), magnetic translations for functions $\check{\phi}$ in Eq. (18) are equivalent to the usual translations $\hat{T}_0(\mathbf{R}_l)$ in space for a fixed kinetic momentum of the particle.

It is more convenient to consider magnetic translations in the symmetric gauge $\mathbf{A} = \mathbf{H} \times \mathbf{r}/2$. In this case,

$$\hat{T}(\mathbf{R}_l) f(\mathbf{r}) = \exp\left(-\frac{ie}{2c} \mathbf{R}_l [\mathbf{H} \times \mathbf{r}] \right) f(\mathbf{r} - \mathbf{R}_l).$$

For this gauge, the wave functions, Eq. (16), can be more conveniently written in a slightly different form

$$\check{\Psi}(p_x;\mathbf{r}) = \exp\left[ieHxy/2c + ip_x x + i\int_{y_1}^{y} p_y dy' \right] \check{\phi}. \tag{31}$$

The extra phase factor $\exp[ieHxy/2c]$ is associated with our choice of the vector potential and allows us to reduce the problem to the Landau gauge. The particle trajectory takes the form of Eq. (21) with $p_y = \sqrt{p_\perp^2 - (p_x + eHy/c)^2}$. The function $\check{\phi}$ satisfies Eq. (22).

If a_0 and b_0 are the unit cell vectors along x and y, respectively, the magnetic translation operators for functions of Eq. (31) are

$$\hat{T}_x(la_0) \check{\Psi}_n(p_x;x,y) = e^{-ip_x l a_0} \check{\Psi}_n(p_x;x,y), \tag{32}$$

$$\hat{T}_y(lb_0) \check{\Psi}_n(p_x;x,y) = \check{\Psi}_n\left(p_x - \frac{eHlb_0}{c};x,y \right). \tag{33}$$

When deriving these expressions we have used the periodicity of $\mathbf{v}_s$ and the fact that the trajectory depends on y only through $y + cp_x/eH$. The turning point y_1 is thus shifted by lb_0 when p_x is shifted by $-eHlb_0/c$.

The functions, Eq. (31), can be used to construct two independent basis functions

$$\check{\Phi}_n^+(k_x,k_y;x,y) = \sum_l e^{ik_y 2lb_0} \hat{T}_y(2lb_0) \check{\Psi}_n(k_x;x,y), \tag{34}$$

$$\check{\Phi}_n^-(k_x,k_y;x,y) = \sum_l e^{ik_y(2l+1)b_0} \hat{T}_y[(2l+1)b_0] \times \check{\Psi}_n(k_x;x,y), \tag{35}$$

with even and odd translations, respectively. Starting from Eq. (34) we replace p_x with k_x. The functions $\check{\Phi}^\pm$ belong to the same energy. The wave vector k_y has an arbitrary value at this stage; we shall establish it later. The generic translation is $2b_0$ which is the size of the magnetic unit cell along the y axis. The magnetic unit cell contains two vortices because the superconducting magnetic flux quantum correspond to one-half of the 2π phase circulation of a single-particle wave function. The functions $\check{\Phi}^\pm$ transform into each other under odd translations

$$\hat{T}_y[(2m+1)b_0] \check{\Phi}^\pm(k_x,k_y) = e^{-ik_y(2m+1)b_0} \check{\Phi}^\mp(k_x,k_y). \tag{36}$$

The functions, Eqs. (34) and (35), have the Bloch form

$$\hat{T}_x(la_0) \check{\Phi}^\pm(k_x,k_y) = (\pm 1)^l e^{-ik_x la_0} \check{\Phi}^\pm(k_x,k_y), \tag{37}$$

$$\hat{T}_y(2mb_0) \check{\Phi}^\pm(k_x,k_y) = e^{-ik_y 2mb_0} \check{\Phi}^\pm(k_x,k_y). \tag{38}$$

We omit the coordinates x,y in the arguments of $\check{\Phi}^\pm$ for brevity.

Since the magnetic translation $\hat{T}_y(lb_0)$ commutes with the Hamiltonian, the energy is degenerate with respect to k_y. This degeneracy is spurious, however. To see this, consider the transformations, Eqs. (37) and (38). For $l=1$, the transformed function in Eq. (37) is periodic in k_x with the period $2\pi/a_0$. This period corresponds to the shift of the center of orbit $y_0 = ck_x/eH$ by one size of the magnetic unit cell $2b_0$. Obviously, the transformation, Eq. (38), should also have the same symmetry. For one magnetic unit cell, a shift by $2b_0$ (i.e., for $m=1$) along the y axis should combine with one period along the x axis. The period in k_y is π/b_0; it should thus correspond to the shift of the coordinate x_0 by a_0. We thus put

$$k_y = eHx_0/c. \tag{39}$$

The energy depends on the position of the trajectory within the vortex unit cell through the Doppler energy ζ. The energy $\epsilon(k_x,k_y)$ has a band structure due to periodicity of ζ; it is periodic with the periods $eHb_0/c = \pi/a_0$ and $eHa_0/c = \pi/b_0$ in k_x and k_y, respectively, which correspond to shifts of the center of orbit by one vortex unit cell vector.

B. Spectrum

Consider energies $\epsilon \gg \Delta_0 \sqrt{H/H_{c2}}$. Applying the second-level quasiclassical approximation to Eq. (22) we find

$$\check{\phi} = \check{C} \exp[\pm i A(s)], \tag{40}$$

where the action is

$$A(s) = \int_{s_1}^{s} \sqrt{(\epsilon - \zeta)^2 - \Delta_0^2 \sin^2(2\alpha)} \, \frac{ds}{v_\perp}. \tag{41}$$

This quasiclassical approximation is justified because the wave vector $\partial A/\partial s \sim \epsilon/v_F$ is much larger than the inverse characteristic scale $1/a_0$ of variation of the vortex potential ζ for $\epsilon \gg \Delta_0 \sqrt{H/H_{c2}}$. The function $\zeta = p_\perp v_t$ is taken at the trajectory which is a part of a circle specified by the coordinates of its center x_0 and $y_0 = -c p_x / eH$; they determine the position of the trajectory within the vortex unit cell.

For energies $\Delta_0 \sqrt{H/H_{c2}} \ll \epsilon < \Delta_0$, quasiparticle trajectory is extended over distances of the order of $r_L(\epsilon/\Delta_0)$. The quantization rule defines the energy

$$\int_{s_1}^{s_2} \sqrt{(\epsilon - \zeta)^2 - \Delta_0^2 \sin^2(2\alpha)} \, \frac{ds}{v_\perp} = \pi n. \tag{42}$$

Here s_1 and s_2 are the turning points. Expanding in small $\zeta \ll \epsilon$ we find

$$m \int_{y_1}^{y_2} \sqrt{\epsilon^2 - \Delta_0^2 \sin^2(2\alpha)} \, \frac{dy}{p_y} - m \int_{y_1}^{y_2} \frac{\zeta(x,y)\epsilon}{\sqrt{\epsilon^2 - \Delta_0^2 \sin^2(2\alpha)}} \frac{dy}{p_y} = \pi n. \tag{43}$$

Here $\zeta(x,y) = (k_x + eHy/c) v_{sx} + p_y v_{sy}$ while y_1 and y_2 are the turning points which correspond to a vanishing of the square root: $k_x + eHy_{1,2}/c = p_\perp \sin(2\alpha_\epsilon)$. The energy ϵ_n is a function of k_x and x_0 which determine the location of the particle trajectory with respect to vortices. The energy is thus periodic in k_x with the period eHb_0/c and in x_0 with a period a_0 when the center is shifted by one period of the vortex lattice.

The ζ term under the second integral in Eq. (43) oscillates rapidly over the range of integration and mostly averages out. The remaining contribution determines the variations of energy with k_x and x_0 and can be estimated as follows. Variation of action for $\epsilon \ll \Delta_0$ due to a change in energy $\delta\epsilon$ is

$$\delta A \sim (\delta\epsilon/v_F)(\epsilon/\Delta_0) r_L \sim (\epsilon \delta\epsilon)/(\Delta_0 \omega_c).$$

Variation of action due to a shift of the center of orbit by a distance of the order of the lattice period is $\delta A \sim (a_0/v_F)\zeta \sim 1$. The corresponding energy variation is thus $\delta\epsilon \sim \Delta_0 \omega_c / \epsilon$. Since x_0 is coupled to k_y through Eq. (39), the energy can be written as

$$\epsilon_n(k_x, k_y) = \sqrt{4 \Delta_0 \omega_c [n + \zeta_0(k_x, k_y)]}, \tag{44}$$

where $\zeta_0 \sim 1$. The energy, Eq. (44), has a band structure; the bandwidth is of the order of the distance between the Landau levels. It is small as compared to the energy itself. It is clear that the spectrum for energies $\epsilon \gtrsim \Delta_0$ can also be obtained from Eqs. (25), (28), and (29) through the substitution $n \to n + \zeta_0(k_x, k_y)$.

Equation (44) needs a discussion. First, we show that the vortex potential indeed does not destroy the $\sqrt{n}$ dependence of the particle energy for fixed quasimomenta k_x and k_y, Eq. (26), if $\epsilon \gg \Delta_0 \sqrt{H/H_{c2}}$. We note in this connection that the $\sqrt{n}$ behavior of the levels in Eq. (26) is preserved if the contribution to the action from the oscillating potential changes by an amount much less than unity for transitions between the neighboring levels with $n \to n \pm 1$. For an energy ϵ, the distance between the neighboring levels is $\delta\epsilon \sim \Delta_0 \omega_c / \epsilon$. This corresponds to a change in the length of the trajectory by

$$\delta s_\epsilon \sim \frac{v_F}{\omega_c} \frac{\delta\epsilon}{\Delta_0} \sim \frac{v_F}{\epsilon}.$$

The variation in the length is much smaller than the intervortex distance $a_0 \sim \xi \sqrt{H_{c2}/H}$ if $\epsilon \gg \Delta_0 \sqrt{H/H_{c2}}$, and the action changes by a quantity much less than 1. It shows that the distance between the levels with different n as given by Eq. (26) is not affected by the vortex potential. Finally, we demonstrate that small regions on a trajectory where the expression under the square root in Eq. (42) is negative do not affect the spectrum if $\epsilon \gg \Delta_0 \sqrt{H/H_{c2}}$. Let s_0 be the size of the region where $(\epsilon - \zeta)^2 < \Delta^2(\alpha)$. The estimate shows that $s_0 \sim (\zeta/\epsilon) a_0$. One can write

$$(\epsilon - \zeta)^2 - \Delta^2(\alpha) \sim s \epsilon \zeta / s_0.$$

The imaginary part of the action becomes

$$\operatorname{Im} A \sim \frac{s^{3/2}}{v_F} \sqrt{\frac{\zeta\epsilon}{s_0}}.$$

The decay length λ of the wave function is $\lambda \sim s_0 (\epsilon/\zeta)^{1/3}$. We see that it is much longer than the length of the classically inaccessible region s_0: The wave function does not feel the inaccessible regions and the trajectory is not destroyed.

The situation changes drastically for smaller energies $\epsilon \lesssim \Delta_0 \sqrt{H/H_{c2}}$: The centers of bands will deviate strongly from the positions determined by Eq. (26) due to a considerable contribution from the periodic vortex potential to the turning points in Eq. (42). Moreover, the applicability of the quasiclassical approximation, Eq. (42), itself is violated; the potential ζ is strong enough to break the particle trajectory into separate pieces[14] and to cause large deformations of the energy spectrum. Some states can even become effectively localized near the vortex cores.[9] We conclude that the condition $\epsilon \gg \Delta_0 \sqrt{H/H_{c2}}$ is vital for existence of the Landau quantization.

In the present paper we do not calculate the band structure of the spectrum exactly. The corresponding numerical analysis will be published elsewhere. In the following sections, we rather consider a situation where the particular band structure is not essential while the Landau-level quantizations are of a crucial importance.

V. INDUCED TRANSITIONS BETWEEN THE LANDAU LEVELS

In this section, we discuss how the Landau quantization affects transport properties of superconductors. We show that, by studying some transport characteristics, one can experimentally observe the Landau-level structure of the energy spectrum. It is known that the vortex motion induces transitions between the quasiparticle states. The transitions between low-energy core states with $\epsilon \ll \Delta_0 \sqrt{H/H_{c2}}$ were considered in Ref. 9. It was shown that the vortex core states determine the vortex response to dc and ac electric fields. For temperatures $T_c \sqrt{H/H_{c2}} \ll T$ extended states dominate. It was found in Ref. 9 that the vortex response is determined by what was called ''collective modes'' which are associated with the electron states outside the vortex cores. In this section we demonstrate that these collective modes are nothing but transitions between the electronic states, Eq. (44), specified by the same quasimomentum but by different principal quantum numbers n. We start with noting that the transition matrix elements are proportional to[21] $\langle \check{\Phi}_n(\mathbf{k}_i) \nabla \check{H}_1 \check{\Phi}_m(\mathbf{k}_j) \rangle$ where the Hamiltonian $\check{H}_1$ is composed of $\Delta_{\mathbf{p}}$ and ζ, while $\mathbf{k}$ is the quasimomentum. $\check{H}_1$ is periodic with the period of the vortex lattice; thus transitions are possible between the quasimomenta which differ by vectors of the reciprocal lattice. Since the band energy is periodic in the quasimomenta with the periods of the reciprocal lattice, the energy difference for these transitions corresponds to the energy difference for states with the same quasimomentum but with different quantum numbers n. For $\zeta_0 \ll n$ the transition energy is just the distance between the Landau levels: $\delta\epsilon_n(k_x, k_y) = \delta\epsilon_n$ determined by Eqs. (25), (28), or (29). For low energies in a d-wave superconductor, one has $\delta\epsilon(k_x, k_y) = 2\Delta_0 \omega_c / \epsilon_n$ in accordance with Eq. (26).

Consider the vortex-induced transitions between the levels in more detail. We use the microscopic kinetic-equation approach which has been applied earlier for s-wave superconductors in Ref. 22. The kinetic equations for the distribution functions f_1 and f_2 have the form[18]

$$\left[e(\mathbf{v}_F \cdot \mathbf{E}) g_- + \frac{1}{2}\left(f_- \frac{\hat{\partial}\Delta^*_{\mathbf{p}}}{\partial t} + f^\dagger_- \frac{\hat{\partial}\Delta_{\mathbf{p}}}{\partial t} \right) \right] \frac{\partial f^{(0)}}{\partial \epsilon} + (\mathbf{v}_F \cdot \nabla) \times (g_- f_2) + g_- \frac{\partial f_1}{\partial t} + \left[\frac{e}{c}[\mathbf{v}_F \times \mathbf{H}] g_- - \frac{1}{2}(f_- \hat{\nabla}\Delta^*_{\mathbf{p}} + f^\dagger_- \hat{\nabla}\Delta_{\mathbf{p}}) \right] \cdot \frac{\partial f_1}{\partial \mathbf{p}} + \frac{1}{2}\left(f_- \frac{\partial \Delta^*_{\mathbf{p}}}{\partial \mathbf{p}} + f^\dagger_- \frac{\partial \Delta_{\mathbf{p}}}{\partial \mathbf{p}} \right) \cdot \nabla f_1 = J \tag{45}$$

and

$$g_-(\mathbf{v}_F \cdot \nabla) f_1 = 0. \tag{46}$$

Here J is the collision integral,

$$\check{g}^{R(A)} = \begin{pmatrix} g^{R(A)} & f^{R(A)} \\ -f^{\dagger R(A)} & -g^{R(A)} \end{pmatrix}$$

are the retarded (advanced) quasiclassical Green functions, and $\check{g}_- = (\check{g}^R - \check{g}^A)/2$. In Eq. (45) we encounter the Lorentz force which has appeared due to the transfromation $u = e^{i\chi_A/2}\tilde{u}$, $v = e^{-i\chi_A/2}\tilde{v}$ used for derivation of this equation.[18] This is exactly the point which we discussed in Sec. I in connection with the transformation of Eq. (4).

For an extended state with an energy $\epsilon > \Delta_{\mathbf{p}}$, the particle trajectory crosses many vortex unit cells at various distances from vortices. Since the distribution function f_1 is constant along the trajectory according to Eq. (46), it should be also independent of the impact parameter (i.e., of the distance from the trajectory to the vortex). We thus look for a distribution function f_1 which is independent of coordinates. One can then omit the last term in the left-hand side (LHS) of Eq. (45). Let us average Eq. (45) over an area which contains many vortex unit cells but has a size small compared with the Larmor radius, $a_0 \ll r \ll r_L$. Since $r \ll r_L$, the momentum $\mathbf{p}$ is still an integral of motion. We have (compare with Ref. 18)

$$\int_{S_0} g_- \frac{\partial f_1}{\partial t} d^2 r - \frac{1}{2}\mathrm{Tr}\int_{S_0} d^2 r\, \check{g}_-(\nabla \check{H}) \cdot \frac{\partial f_1}{\partial \mathbf{p}} - \int_{S_0} J\, d^2 r = \frac{1}{2}\mathrm{Tr}\int_{S_0} d^2 r\, \check{g}_-(\mathbf{v}_L \cdot \nabla \check{H}) \frac{\partial f^{(0)}}{\partial \epsilon}.$$

Here Tr is the trace in the Nambu space; $S_0 = \Phi_0/B$ is the area of the vortex unit cell. The collision integral has the form[23]

$$J = -\frac{1}{\tau}[(f_1\langle g_-\rangle - \langle f_1 g_-\rangle) g_- - (f_1\langle f^\dagger_-\rangle - \langle f_1 f^\dagger_-\rangle) f_- + (f_1\langle f_-\rangle - \langle f_1 f_-\rangle) f^\dagger_-],$$

where $\langle \cdots \rangle$ is an average over the Fermi surface. Using the identity

$$\frac{1}{2}\mathrm{Tr}\int_{S_0} d^2 r [(\nabla \check{H})\check{g}_-] = \pi[\mathbf{z} \times \mathbf{v}_\perp]$$

derived in Ref. 22 we find

$$-\pi[\mathbf{z} \times \mathbf{v}_\perp] \cdot \frac{\partial f_1}{\partial \mathbf{p}} + \frac{\partial f_1}{\partial t}\int_{S_0} g_- d^2 r - \int_{S_0} J\, d^2 r = \pi(\mathbf{v}_L \cdot [\mathbf{z} \times \mathbf{v}_\perp]) \frac{\partial f^{(0)}}{\partial \epsilon}. \tag{47}$$

We shall concentrate on energies $\epsilon \gg \Delta\sqrt{H/H_{c2}}$. In the leading approximation

$$g_- = \frac{\epsilon}{\sqrt{\epsilon^2 - \Delta^2(\alpha)}} \Theta[\epsilon^2 - \Delta^2(\alpha)],$$

$$f_- = \frac{\Delta(\alpha)}{\sqrt{\epsilon^2 - \Delta^2(\alpha)}} \Theta[\epsilon^2 - \Delta^2(\alpha)].$$

We have

$$\langle f_1 \rangle = \langle f_1 g_- \rangle = 0, \quad \langle f_1 f_- \rangle = \langle f_1 f^\dagger_- \rangle = 0.$$

For a d-wave superconductor also $\langle f_- \rangle = \langle f^\dagger_- \rangle = 0$.

In the collision integral, the main contribution for $\epsilon \gg \Delta\sqrt{H/H_{c2}}$ comes from the delocalized states. Indeed, including contributions from the states in the core[22] with energies $E_n(b)$ we would have

$$\int_{S_0} J\, d^2 r \approx -S_0 \left[\sum_n \frac{p_\perp \omega_c}{\tau_n} \int \delta(\epsilon - E_n) db + \frac{\langle g_- \rangle g_-}{\tau} \right] f_1 ,$$

where b is the impact parameter. The first term in square brackets comes from the core states. Since $\tau_n \sim \tau$ and $b \sim \xi\sqrt{H_{c2}/H}$, the core contribution is of the order of $\tau^{-1}\sqrt{H/H_{c2}}$. The delocalized states, however, give $(\epsilon/\Delta_0)\tau^{-1}$ which is much larger than the first term. Neglecting the core contribution we find

$$J = -\frac{1}{\tau}\langle g_- \rangle g_- f_1 .$$

Let us put

$$f_1 = -\frac{\partial f^{(0)}}{\partial \epsilon} \{([\mathbf{u} \times \mathbf{p}_\perp] \cdot \hat{\mathbf{z}})\gamma_{\mathrm{O}} + (\mathbf{u} \cdot \mathbf{p}_\perp)\gamma_{\mathrm{H}}\}. \tag{48}$$

The functions $\gamma_{\mathrm{O,H}}$ satisfy the following set of equations:

$$\frac{\partial \gamma_{\mathrm{O}}}{\partial \alpha} - \gamma_{\mathrm{H}} - V(\alpha)\gamma_{\mathrm{O}} + 1 = 0,$$

$$\frac{\partial \gamma_{\mathrm{H}}}{\partial \alpha} + \gamma_{\mathrm{O}} - V(\alpha)\gamma_{\mathrm{H}} = 0, \tag{49}$$

which is derived from Eq. (47). Here

$$V(\alpha) = \frac{(-i\omega + \langle g_- \rangle/\tau) g_-}{\omega_c} \tag{50}$$

and ω is the frequency of the applied field.

The general solution of Eqs. (49) can be obtained[8] by putting $W_\pm = \gamma_{\mathrm{H}} \pm i\gamma_{\mathrm{O}}$. We have

$$\frac{\partial W_\pm}{\partial \alpha} \mp i W_\pm - V(\alpha) W_\pm \pm i = 0,$$

whence

$$W_\pm = \left[C_\pm \mp i \int_0^\alpha e^{\mp i\alpha' - F(\alpha')} d\alpha' \right] e^{\pm i\alpha + F(\alpha)}, \tag{51}$$

where

$$F(\alpha) = \int_0^\alpha V(\alpha') d\alpha' .$$

The constant $C_\pm$ is found from the condition of periodicity $W(\alpha) = W(\alpha + \pi/2)$:

$$C_\pm = \frac{\exp[F(\pi/2)] \int_0^{\pi/2} \exp[\mp i\alpha - F(\alpha)] d\alpha}{1 - \exp[\pm i\pi/2 + F(\pi/2)]}. \tag{52}$$

In the limit $\tau \to \infty$, the functions

$$\gamma_{\mathrm{H}} = (W_+ + W_-)/2; \quad \gamma_{\mathrm{O}} = (W_+ - W_-)/2i$$

have poles when

$$F(\pi/2) = \frac{\pi i}{2}(1 + 2M), \tag{53}$$

where M is an integer.

A. High energies

Excitations with high energies, $\epsilon > \Delta_0$, have resonances at

$$\frac{\omega}{\omega_c} \int_0^{2\pi} \frac{\epsilon}{\sqrt{\epsilon^2 - \Delta^2(\alpha)}} d\alpha = 2\pi(1 + 2M).$$

The lowest frequency $M = 0$ exactly corresponds to the condition

$$\omega = (d\epsilon_n/dn),$$

where $d\epsilon_n/dn$ is the distance between the Landau levels determined by Eq. (23). The resonant frequencies are in the range $\omega \lesssim \omega_c$ and appoach the cyclotron frequency for $\epsilon \gg \Delta_0$. For illustration, consider an s-wave superconductor. Equations (49) have the form

$$\gamma_{\mathrm{H}} + V\gamma_{\mathrm{O}} = 1,$$

$$\gamma_{\mathrm{O}} - V\gamma_{\mathrm{H}} = 0, \tag{54}$$

where

$$V(\alpha) = \left[\frac{-i\omega}{\omega_c} \frac{\epsilon}{\sqrt{\epsilon^2 - \Delta_0^2}} + \frac{1}{\omega_c \tau} \right] \Theta[\epsilon^2 - \Delta_0^2]$$

since

$$J = -\frac{1}{\tau}(f_1 - \langle f_1 \rangle)\Theta[\epsilon^2 - \Delta_0^2].$$

One has from Eq. (54)

$$\gamma_{\mathrm{H}} = \frac{1}{1 + V^2}, \quad \gamma_{\mathrm{O}} = \frac{V}{1 + V^2}.$$

The resonances appear when $\omega_c \tau \gg 1$; the poles correspond to $V = \pm i$ so that

$$\omega = \omega_c \frac{\sqrt{\epsilon^2 - \Delta_0^2}}{\epsilon} = \frac{d\epsilon_n}{dn}, \tag{55}$$

where ϵ_n is determined by Eq. (29). For not very low temperatures $T \sim T_c$, the resonances are practically not distinguishable from the cyclotron resonance. However, the situation changes for lower temperatures $T \ll T_c$, where the low-energy states dominate.

B. Low energies

For energies $\epsilon \ll \Delta_0$, the resonant frequencies are essentially above the cyclotron resonance; this could be anticipated from the fact that $d\epsilon_n/dn \gg \omega_c$ as follows from Eq. (25) with $\epsilon \ll \Delta_0$. We start our discussion with the observation that the condition, Eq. (53), is not simply the distance between the Landau levels determined by Eq. (25). Indeed, one has from Eq. (24)

$$\frac{d\epsilon_n}{dn}\int_{-\alpha_\epsilon}^{\alpha_\epsilon}\frac{\epsilon}{\sqrt{\epsilon^2-\Delta^2(\alpha)}}d\alpha=\pi\omega_c,$$

where $\Delta(\alpha_\epsilon)=\epsilon$. At the same time, Eq. (53) gives the lowest resonant frequency

$$\frac{\omega}{\omega_c}N\int_{-\alpha_\epsilon}^{\alpha_\epsilon}\frac{\epsilon}{\sqrt{\epsilon^2-\Delta^2(\alpha)}}d\alpha=2\pi,$$

where N is the number of gap nodes ($N=4$ for a d-wave superconductor). We see that the resonance occurs at

$$N\omega=2\frac{d\epsilon_n}{dn}. \tag{56}$$

When the vortex oscillates, all N nodes participate in exciting quasiparticles which accounts for the factor N on the LHS of Eq. (56). This is similar to the process of multiphoton absorption. The factor of 2 on the RHS is explained by noting that states with momentum directions α and $\alpha+\pi$ are simultaneously excited.

Consider now the dynamic vortex response for energies $\Delta_0\sqrt{H/H_{c2}}\ll\epsilon\ll\Delta_0$ when the states in the gap nodes far from vortex cores dominate over the contribution from the core states.[9] Solution of Eqs. (49) and (50) for this energy range was obtained in Ref. 9. We recall it for completeness. In the main region of angles, $|\alpha|>\alpha_\epsilon=\epsilon/2\Delta_0$, one has according to Eqs. (51) and (52)

$$\gamma_{\rm O}=A\cos\alpha+B\sin\alpha,$$

$$\gamma_{\rm H}=1-A\sin\alpha+B\cos\alpha, \tag{57}$$

with

$$A=\frac{e^{\lambda}\sinh\lambda}{2\sinh^2\lambda+1},\quad B=\frac{e^{-\lambda}\sinh\lambda}{2\sinh^2\lambda+1}. \tag{58}$$

Here

$$\lambda=F(\alpha_\epsilon),\quad F(\pi/2)=2\lambda$$

and we use $F(\pi/2-\alpha)=2\lambda-F(\alpha)$. The expression for λ is easily obtained from Eq. (50):

$$\lambda=\frac{\pi}{4}\langle g_-\rangle\frac{-i\omega+\langle g_-\rangle/\tau}{\omega_c}. \tag{59}$$

We see that the effective relaxation rate is $1/\tau_{eff}=|\epsilon|/\Delta_0\tau$ since $\langle g_-\rangle=|\epsilon|/\Delta_0$. Note that a τ approximation was used in Ref. 9 for the collision integral. To get the present expression for λ from that obtained in Ref. 9 one has to replace $1/\tau$ with $1/\tau_{eff}$.

In the superclean limit $\tau\to\infty$, the response, Eqs. (57) and (58), has poles at $i\lambda=(2M+1)\pi/4$, i.e., for

$$\omega=(2M+1)E_0(\epsilon),\quad E_0(\epsilon)=\Delta_0\omega_c/|\epsilon|. \tag{60}$$

We have for $M=0$

$$\omega=\frac{1}{2}\frac{d\epsilon_n}{dn},$$

where ϵ_n is determined by Eq. (26). This condition agrees with Eq. (56). The resonance frequencies are above the cyclotron resonance $\omega\gg\omega_c\sqrt{H_{c2}/H}$ for excitations with energies $\Delta_0\sqrt{H/H_{c2}}\ll\epsilon\ll\Delta_0$.

These resonances were first predicted in Ref. 9. Note the different numerical factor in Eq. (60) as compared to Ref. 9; this is because a simplified version of $V(\alpha)$ has been used in Ref. 9. The main effect of resonances is that vortices experience a considerable friction force, Eq. (61), even in a superclean case $\omega\tau_{eff}\gg1$.

C. Vortex friction

A vortex moving with a velocity $\mathbf{v}_L$ experiences a force from the environment which is usually parametrized as (see, for example, Ref. 18)

$$\mathbf{F}_{\rm env}=-\eta\mathbf{v}_L-\eta'[\mathbf{v}_L\times\mathbf{z}]. \tag{61}$$

According to Ref. 22, the delocalized states contribute to the constants as follows:

$$\eta_{\rm del}=\pi N\left\langle\int_{\rm del}\gamma_{\rm O}\frac{df^{(0)}}{d\epsilon}\frac{d\epsilon}{2}\right\rangle_\alpha, \tag{62}$$

where $\langle\cdots\rangle_\alpha$ is an average over $d\alpha$. The factor η' is determined by the same expression where $\gamma_{\rm O}$ is replaced with $\gamma_{\rm H}$.

The presence of resonances makes the dissipative constant $\eta_{\rm del}$ finite even in the superclean limit $\omega_c\tau\to\infty$. As we know, excitations both below and above Δ_0 can participate. For s-wave superconductors, the contribution of the core states with $\epsilon<\Delta_0$ has been considered in Ref. 24. These resonances occur at $\omega=\omega_0$ above the cyclotron resonance, $\omega_0\sim E_F/\Delta_0^2$ being the distance between the Caroli–de Gennes–Matricon bound states in the vortex core.[25] On the contrary, the high-energy states for an s-wave case give

$$\gamma_{\rm O}=\frac{\pi E}{2}[\delta(\omega-E)+\delta(\omega+E)],$$

where $E=\omega_c\sqrt{1-\Delta_0^2/\epsilon^2}$, as follows from Eq. (55). It requires $\omega<\omega_c$, of course. The friction constant due to high-energy states becomes

$$\eta_{\rm del}=\pi^2N\Delta_0\frac{\omega^2/\omega_c^2}{(1-\omega^2/\omega_c^2)^{3/2}}\frac{df^{(0)}(\epsilon_0)}{d\epsilon}, \tag{63}$$

where $\epsilon_0=\Delta_0/\sqrt{1-\omega^2/\omega_c^2}$.

A detailed discussion of the resonant vortex friction for a d–wave superconductor in the frequency range $\omega>\omega_c$ at low temperatures can be found in Ref. 9. These resonances can be, in principle, observed in the far-infrared region in magneto-optical experiments. Indeed, for a magnetic field of 8 T used in Ref. 17 the cyclotron frequency was of order of few kelvins which provides quite reasonable temperature range for detecting the predicted resonances.

VI. CONCLUSIONS

We discussed and analyzed the ''Landau-level'' vs ''energy-band'' opposition in the description of the structure of the excitation spectrum in the mixed state of superconductors and, in particular, *d*-wave superconductors. We find that the actual picture of quantization is an interplay between the two limiting images of the energy spectrum. Our analysis shows that the influence of the magnetic field on delocalized excitations in a superconductor is not reduced to a mere action of the effective vortex lattice potential. In fact, the magnetic field has a twofold effect: On the one hand, it creates vortices and thus provides a periodic potential for excitations; on the other hand, it also affects the long-range motion of quasiparticles in a manner similar to that in normal metals. For low-energy excitations, the long-range effects are less pronounced. However, excitations with energies $\epsilon > \Delta_0\sqrt{H/H_{c2}}$ mostly show long-range quantization. The energy spectrum consists of ''Landau levels'' which are split into bands by the periodic vortex potential. In the quasiclassical approximation $p_F\xi \gg 1$, the bandwidth is of the order of the distance between the Landau levels; it is small compared to the energy itself.

An ac electric field induces transitions between the states belonging to different Landau levels. Using the microscopic kinetic equations we demonstrate that these transitions can be seen as an increase in the vortex friction due to a resonant absorption at frequencies corresponding to the energy differences between the Landau levels.

ACKNOWLEDGMENTS

We are grateful to G. Blatter, A. Mel'nikov, Z. Tešanović, and G. Volovik for illuminating discussions. This research was supported by the US DOE BES-Materials Sciences under Contract No. W-31-109-ENG-38, by NSF, STCS No. DMR91-20000, and by the Russian Foundation for Basic Research, Grant No. 99-02-16043.

[1] G. E. Volovik, Pis'ma Zh. Éksp. Teor. Fiz. **58**, 457 (1993) [JETP Lett. **58**, 469 (1993)].
[2] N. B. Kopnin and G. E. Volovik, Pis'ma Zh. Éksp. Teor. Fiz. **64**, 641 (1996) [JETP Lett. **64**, 690 (1996)].
[3] S. H. Simon and P. A. Lee, Phys. Rev. Lett. **78**, 1548 (1997).
[4] I. Maggio-Aprile, Ch. Renner, A. Erb, E. Walker, and Ø. Fisher, Phys. Rev. Lett. **75**, 2754 (1995); Ch. Renner, B. Revaz, K. Kadowaki, I. Maggio-Aprile, and Ø. Fisher, Phys. Rev. Lett. **80**, 3606 (1998).
[5] P. A. Lee, Phys. Rev. Lett. **71**, 1887 (1993).
[6] K. Krishana, J. M. Harris, and N. P. Ong, Phys. Rev. Lett. **75**, 3529 (1995).
[7] L. Taillefer, B. Lussier, R. Cagnon, K. Behnia, and H. Aubin, Phys. Rev. Lett. **79**, 483 (1997).
[8] N. B. Kopnin and G. E. Volovik, Phys. Rev. Lett. **79**, 1377 (1997).
[9] N. B. Kopnin, Phys. Rev. B **57**, 11 775 (1998).
[10] L. P. Gor'kov and J. R. Schrieffer, Phys. Rev. Lett. **80**, 3360 (1998).
[11] P. W. Anderson, cond-mat/9812063 (unpublished).
[12] B. Jankó, Phys. Rev. Lett. **82**, 4703 (1999).
[13] M. Nielsen and P. Hedegård, Phys. Rev. B **51**, 7679 (1995).
[14] A. S. Mel'nikov, J. Phys.: Condens. Matter **11**, 4219 (1999).
[15] M. Franz and Z. Tešanović, Phys. Rev. Lett. **84**, 554 (2000).
[16] T. Kita, J. Phys. Soc. Jpn. **67**, 2075 (1998); K. Yasui and T. Kita, Phys. Rev. Lett. **83**, 4168 (1999).
[17] J. Černe, M. Grayson, D. C. Schmadel, G. S. Jenkins, H. D. Drew, R. Huges, A. Dabkowski, J. S. Preston, and P.-J. Kung, Phys. Rev. Lett. **84**, 3418 (2000).
[18] G. Blatter, V. B. Geshkenbein, and N. B. Kopnin, Phys. Rev. B **59**, 14 663 (1999).
[19] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon Press, New York, 1977).
[20] E. Brown, Phys. Rev. **133**, 1038 (1964).
[21] N. B. Kopnin and M. M. Salomaa, Phys. Rev. B **44**, 9667 (1991).
[22] N. B. Kopnin and A. V. Lopatin, Phys. Rev. B **51**, 15 291 (1995).
[23] A. I. Larkin and Yu. N. Ovchinnikov, in *Nonequilibrium Superconductivity*, edited by D. N. Langenberg and A. I. Larkin (Elsevier Science, New York, 1986), p. 493.
[24] N. B. Kopnin, Pis'ma Zh. Éksp. Teor Fiz. **27**, 417 (1978) [JETP Lett. **27**, 390 (1978)].
[25] C. Caroli, P. G. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).