Domain-wall superconductivity in hybrid superconductor-ferromagnet structures

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On the basis of a phenomenological Ginzburg-Landau approach we investigate the problem of order parameter nucleation in hybrid superconductor-ferromagnet systems with a domain structure in an applied external magnetic field. Both isolated domain boundaries and periodic domain structures in ferromagnetic layers are considered. We study the interplay between superconductivity localized at the domain walls and far from the walls and show that such an interplay determines a peculiar field dependence of the critical temperature T_c. For a periodic domain structure the behavior of the upper critical field of superconductivity nucleation near T_c is strongly influenced by the overlapping of the superconducting nuclei localized over different domains.

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I. INTRODUCTION

The problem of the coexistence of superconducting and magnetic orderings has been studied for several decades (see, e.g., Refs. 1 and 2 for reviews). One can separate two basic mechanisms responsible for the interaction of superconducting order parameter with magnetic moments in the ferromagnetic state: (i) the electromagnetic mechanism (interaction of Cooper pairs with magnetic field induced by magnetic moments), which was first discussed by Ginzburg[3] in 1956, and (ii) the exchange interaction of magnetic moments with electrons in Cooper pairs. The revival of interest in the fundamental questions of magnetism and superconductivity coexistence has been stimulated, in particular, by recent investigations of hybrid superconductor-ferromagnet (S/F) systems. Such thin-film structures consist of a ferromagnetic insulator film and superconducting film deposited on it. A similar situation can be obtained with a metallic ferromagnet when a superconducting film is evaporated on the buffer oxide layer in order to avoid a proximity effect. The superconducting properties of such structures have attracted a growing interest due to the large potential for applications. In particular, such hybrid S/F systems have been intensively investigated in connection with the problem of controlled flux pinning. Enhancement of the depinning critical current density j_c has been observed experimentally for superconducting films with arrays of submicron magnetic dots,[4–6] antidots,[7] and for S/F bilayers with domain structure in ferromagnetic films.[8] A theory of vortex structures and pinning in S/F systems at rather low magnetic fields (in the London approximation) has been developed in Refs. 9–18.

A nonhomogeneous magnetic field distribution induced by the domain structure in a ferromagnetic layer influences strongly the conditions of the superconducting order parameter nucleation, and, as a consequence, hybrid S/F systems reveal a nontrivial phase diagram in an external applied magnetic field **H** (see, e.g., Refs. 19–21). In this paper we focus on a theoretical study of this phase diagram on the basis of the phenomenological Ginzburg-Landau (GL) model. We assume that the electromagnetic mechanism mentioned above plays a dominant role and neglect the exchange interaction which is obviously suppressed provided superconducting and ferromagnetic layers are well separated by an insulating barrier. We also assume that the domain walls are well pinned and do not take account of changes in the domain structure with an increase in H. It should be noted that we consider ferromagnetic films with high coercivity (e.g., perpendicular magnetic materials for magneto-optical recording with coercivity about 1 kOe [e.g., multilayered systems Co/Pt (Ref. 22)]). The magnetic field induced in the superconductor is weakened by the insulating layer and can be less than the coercivity field of the ferromagnet that allows us to neglect the magnetic reversal effect in domains. Still the magnitude of the field can be rather large to observe the effects discussed in the paper.

The distribution of the magnetic field induced by the domain structure is determined by the ratio of two length scales: the thickness of the ferromagnetic film, D, and the distance between the domain walls, w. Hereafter we neglect a finite width of the domain wall—i.e., consider this width to be much less than both the nucleus localization length (which is of the order of the superconducting coherence length) and the ferromagnetic film thickness. Provided the ferromagnetic film is rather thick ($D \gg w$), the magnetic field in a thin superconducting film is almost homogeneous over the domain and suppresses the critical temperature of superconductivity nucleation. In this case with the decrease in the temperature the superconductivity must first appear just above the domain wall (see Refs. 23 and 24) due to a mechanism analogous to the one responsible for the surface superconductivity below H_{c3} (see Ref. 25). Thus, in this limit the domain walls stimulate the nucleation of the superconducting order parameter. Note that the same effect should appear for two-dimensional magnetic field distributions induced, e.g., by magnetic dots and results in the dependence of the upper critical field on the angular momentum of the superconducting nucleus wave function (see Refs. 26–28).

For a thin ferromagnetic film ($D \ll w$) the magnetic field decays with an increase in the distance from the domain wall and almost vanishes inside the domain. In the absence of an external field such a domain wall should locally weaken superconductivity as was discussed in Ref. 9. The supercon-

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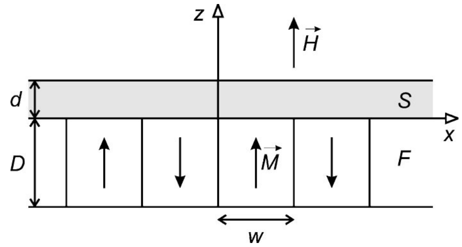

FIG. 1. Superconductor-ferromagnet (S/F) bilayer.

ducting nucleus in this case should appear far from the domain wall. As we switch on an external magnetic field, we can control the position of the superconducting nucleus, suppressing the order parameter inside the domains. Thus, the phase diagram of the S/F bilayer is generally determined by the interplay between the superconductivity nucleated at the domain walls and in between these walls. For small-period domain structures (when w is comparable with the nucleus size) this simple physical picture based on consideration of isolated superconducting nuclei should be modified, taking account of the interaction between the superconducting nuclei localized above different domain walls.

Our further consideration is based on the linearized GL equation for the order parameter Ψ:

$$-\left(\nabla+\frac{2\pi i}{\Phi_0}\mathbf{A}\right)^2\Psi=\frac{1}{\xi^2(T)}\Psi. \quad (1)$$

Here $\mathbf{A}(\mathbf{r})$ is the vector potential, $\mathbf{B}(\mathbf{r})=\nabla\times\mathbf{A}(\mathbf{r})$, Φ_0 is the flux quantum, $\xi(T)=\xi_0/\sqrt{1-T/T_{c0}}$ is the coherence length, and T_{c0} is the critical temperature of the bulk superconductor at $B=0$. Note that in Eq. (1) we neglect the corrections to the vector potential, caused by the supercurrents (which would result in terms nonlinear in the order parameter amplitude). For superconducting films with thickness d much smaller than the coherence length the role of the parallel component of the magnetic field is negligibly small. Thus, we can take account only of the magnetic field component B_z perpendicular to the film surface and also neglect the dependence of the order parameter on z. For the sake of simplicity we restrict ourselves to consideration of the one-dimensional case $B_z(x)=H+b(x)$, where H is a uniform external magnetic field and $b(x)$ is the z component of the field induced by the magnetization $\mathbf{M}=M(x)\mathbf{z}_0$ (see Fig. 1).

Choosing the gauge $\mathbf{A}=A(x)\mathbf{y}_0$, one can easily see that the momentum along the y axis is conserved; hence we can find the solution of the Schrödinger-like equation (1) in the form $\Psi(\mathbf{r})=f_k(x)\exp(-iky)$, where function $f_k(x)$ should be determined from a solution of the one-dimensional problem:

$$-\frac{d^2f_k}{dx^2}+\left(\frac{2\pi}{\Phi_0}A(x)-k\right)^2 f_k=\frac{1}{\xi^2(T)}f_k. \quad (2)$$

Nontrivial solutions of Eq. (2) exist only for a discrete set of temperatures $T_n(k)$. The superconducting critical temperature T_c should be defined as the highest value $\max\{T_n(k)\}$, corresponding to the lowest "energy level" $1/\xi^2(T)$ of the Schrödinger-like equation (2). Note that a similar problem of the energy spectrum $E(k)$ of two-dimensional electronic gas in periodic magnetic field profiles has been analyzed for zero external magnetic field $H=0$ in Ref. 29 for extremely large H values[30] and for a steplike magnetic profile.[31]

II. SUPERCONDUCTIVITY NUCLEATION AT A DOMAIN WALL: AN ISOLATED ORDER PARAMETER NUCLEUS

Let us start from consideration of a superconducting nucleus at a single domain wall taking the magnetization $\mathbf{M}$ near the wall in the form $\mathbf{M}=M\,\mathrm{sgn}(x)\mathbf{z}_0$ (we assume that the domain wall width is much less than the superconducting coherence length).

A. Domain wall in a thick ferromagnetic film: Steplike magnetic field profile

As was mentioned above, for a rather thick ferromagnetic film ($D\gg w$) the expression for the distribution of magnetic field near the surface reads $B_z=4\pi M\,\mathrm{sgn}(x)+H$, where H is an external applied magnetic field. We choose the gauge in the form $\mathbf{A}=(4\pi M|x|+Hx)\mathbf{y}_0$. At high temperatures the superconductivity far from the domain wall can be completely suppressed due to the orbital effect. On the contrary, near the boundary the superconducting nucleus can be still energetically favorable due to a mechanism analogous to the one responsible for the existence of the H_{c3} critical field for a superconducting nucleus near the superconductor-insulator interface (see, e.g., Ref. 25). Thus, a change of the magnetization direction which occurs at a domain boundary is responsible for a partial decrease of the orbital effect which provides conditions for the formation of localized superconducting nuclei at the domain walls at high temperatures (above the critical temperature far from the walls). Such a localized nucleus can appear only if we take account of the proximity effect—i.e., consider Cooper pairs to exist on both sides of the domain boundary. Such systems can reveal interesting behavior in an external magnetic field. An external magnetic field applied to the sample results in a partial compensation of the field above one of the domains. As a result, the critical temperature of the superconductor can depend nonmonotoneously on the applied magnetic field. Both the critical temperature of superconductivity nucleation far from the domain wall and the critical temperature of the formation of localized superconductivity at the wall should increase up to an external field value equal to the magnetic induction induced by the ferromagnetic moment.

It is convenient to rewrite Eq. (2) in the following dimensionless form:

$$-\frac{\partial^2 f_k}{\partial t^2}+(|t|+ht-t_0)^2 f_k=Ef_k, \quad (3)$$

where $t=x/L$, $t_0=kL$, $L^2=\Phi_0/(2\pi B_0)$, $h=H/B_0$, $E=(T_{c0}-T)/\Delta T_c^{orb}$, the value $\Delta T_c^{orb}=T_{c0}\xi_0^2/L^2$ characterizes the shift of critical temperature due to the orbital mechanism, and B_0 is the maximum absolute value of the field b (in this subsection $B_0=4\pi M$).

For the case $|t_0|\to\infty$ a superconducting nucleus will appear far from the domain boundary at a certain T_c^∞. In this limit the lowest eigenvalue $E=|1-|h||$ of Eq. (3) and, hence, the critical temperature is not disturbed by the presence of the domain boundary. On the contrary, for finite t_0 values the superconducting nuclei to the left and to the right from the domain wall cannot be considered separately due to the proximity effect. Provided the lowest energy level in the resulting potential well in Eq. (3) is minimal for a certain finite t_0 coordinate, we get a superconducting nucleus localized at the domain boundary for temperatures above T_c^∞. The mechanism resulting in the appearance of such a localized nucleus is analogous to the one responsible for the existence of surface superconductivity at the superconductor-insulator boundary for magnetic fields $H_{c2}<H<H_{c3}$. Indeed, for $h=0$ the potential well $V(t)$ in Schrödinger equation (3) is symmetric $[V(t)=V(-t)]$ and the eigenvalue problem (3) can be considered only for $t>0$ with the boundary condition $f_k'(t=0)=0$. For this particular case the energy minimum corresponds to $t_0^2=E_{min}=0.59010$ (Ref. 25). An increase in the h value will obviously result in an increasing asymmetry of the well $V(t)$ and, thus, in the suppression of superconductivity localized at the domain wall. Equation (3) can be solved exactly in terms of Weber functions (see Refs. 25 and 32):

$$f_k=C_1W\left(\sqrt{1+h}t-\frac{t_0}{\sqrt{1+h}},\frac{E}{1+h}\right),\quad t>0, \tag{4}$$

$$f_k=C_2W\left(-\sqrt{1-h}t-\frac{t_0}{\sqrt{1-h}},\frac{E}{1-h}\right),\quad t<0. \tag{5}$$

Here C_1 and C_2 are constants, and the Weber function $W(s,\varepsilon)$ is the solution of the following equation:

$$-\frac{\partial^2 W}{\partial s^2}+s^2W=\varepsilon W, \tag{6}$$

with the boundary condition $W(s\to+\infty,\varepsilon)\to 0$. Matching these solutions at $t=0$ we obtain

$$\frac{\sqrt{1+h}W_s'\left(-\frac{t_0}{\sqrt{1+h}},\frac{E}{1+h}\right)}{W\left(-\frac{t_0}{\sqrt{1+h}},\frac{E}{1+h}\right)}=-\frac{\sqrt{1-h}W_s'\left(-\frac{t_0}{\sqrt{1-h}},\frac{E}{1-h}\right)}{W\left(-\frac{t_0}{\sqrt{1-h}},\frac{E}{1-h}\right)}. \tag{7}$$

This equation can be solved numerically which allows us to obtain the function $E(t_0,h)$. The resulting dependence of the critical temperature of superconductivity nucleation on parameter h is shown in Fig. 2. One can see that the external field suppresses the localized superconducting nuclei and the superconductivity localized at the domain wall exists only at a relatively weak applied field. As we increase an external field the superconducting nucleus shifts away from the domain wall towards the region where the absolute value of the total magnetic field is minimal. For $0\leqslant|h|\leqslant 1$ the curve $E(h)$ calculated numerically can be fitted by the following simple expression:

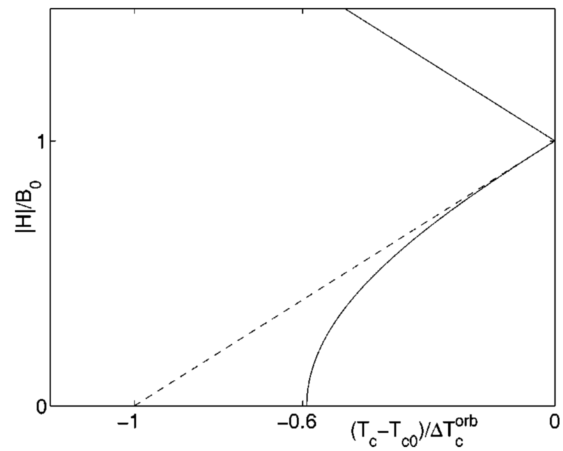

FIG. 2. The temperature dependence of the critical magnetic field for a S/F system with a thick ferromagnetic layer. The solid (dashed) line corresponds to the superconductivity nucleation at the domain boundary (far from the domain boundary).

$$E(h)\simeq\left(E_{min}-\frac{1}{2}\right)h^4+\left(\frac{1}{2}-2E_{min}\right)h^2+E_{min}. \tag{8}$$

B. Domain wall in a thin ferromagnetic film

In this subsection we proceed with consideration of another limiting case $D\ll w$ and consider the problem of superconductivity nucleation in the field of an isolated domain wall in a thin ferromagnetic film: $B_z(x,z=0)=4M\tan^{-1}(D/x)+H$. Obviously, for rather weak external magnetic fields $H<B_0$ (in this subsection the maximum value of the domain-wall field at $z=0$ is given by the expression $B_0=2\pi M$) the superconducting order parameter nucleates in the region near the point x_0 where $B_z(x_0)=0$. Provided the localization length ℓ of the superconducting nucleus is much smaller than the characteristic length scale of magnetic field distribution, we can expand vector potential as

$$A(x)\simeq A(x_0)+\frac{1}{2}B_z'(x_0)(x-x_0)^2.$$

Such a local approximation is valid if the following conditions are fulfilled:

$$\left|\frac{B_z''(x_0)}{B_z'(x_0)}\ell\right|\ll 1\quad\text{and}\quad \ell\ll x_0. \tag{9}$$

Introducing a new coordinate $t=(x-x_0)/\ell$ we obtain the dimensionless equation

$$-\frac{d^2f}{dt^2}+(t^2-Q)^2f=\epsilon f, \tag{10}$$

$$\ell=\sqrt[3]{\frac{\Phi_0}{\pi|B_z'(x_0)|}}=D\sqrt[3]{\frac{\Phi_0}{4\pi MD^2\sin^2(H/4M)}}, \tag{11}$$

$$\epsilon=\frac{\ell^2}{\xi_0^2}\left(1-\frac{T}{T_{c0}}\right), \tag{12}$$

$$Q=\sqrt[3]{\frac{\Phi_0}{\pi B_z'(x_0)}}\left(k-\frac{2\pi}{\Phi_0}A(x_0)\right). \tag{13}$$

The lowest eigenvalue of Eq. (10), $\epsilon_0\simeq 0.904$, is achieved at $Q\simeq 0.437$. For the critical temperature T_c of superconductivity nucleation we obtain

$$\frac{T_{c0}-T_c}{\Delta T_c^{orb}}=\frac{\epsilon_0}{\pi}\left(\frac{\Phi_0}{2B_0D^2}\right)^{1/3}\sin^{4/3}\left(\frac{\pi|H|}{2B_0}\right). \tag{14}$$

This expression is valid when

$$\left|\frac{\sin^{1/3}(H/4M)}{\cos(H/4M)}\right|\ll\frac{4\pi MD^2}{\Phi_0}. \tag{15}$$

Note that close to T_{c0} the upper critical field has an unusual temperature dependence: $\propto(T_{c0}-T)^{3/4}$.

As we increase an external magnetic field H the position of superconducting nucleus shifts from infinity to the domain wall at $x=0$. For rather large fields H the nucleus appears to be localized at the domain wall. Thus, the behavior of the nucleus coordinate in an external field is an opposite to the one considered in the Sec. II A. The critical temperature for high-field H limit is given by the expression

$$\frac{T_c-T_{c0}}{\Delta T_c^{orb}}=1-|H|/B_0. \tag{16}$$

The simple asymptotical formulas given above are in good agreement with our numerical simulations of Eq. (2) (see Fig. 3).

For numerical analysis of the localized states of Schrödinger-like equation (2) with an external magnetic field we approximated it on a equidistant grid and obtained the eigenfunctions $f_k(x)$ and eigenvalues $1/\xi^2(T)$ by the diagonalization method of the tridiagonal difference scheme. The typical behavior of the ground-state wave function is shown in Fig. 4.

III. NUCLEATION OF SUPERCONDUCTIVITY FOR A PERIODIC DOMAIN STRUCTURE

In this section we consider the effect of interaction of Cooper pair wave functions nucleated at different domain walls. Surely such an interaction is important only for temperatures close to T_{c0} ($\xi(T)>w$); otherwise for a rather large domain size $w\gg\xi(T)$ the overlapping of superconducting nuclei above different domain walls is exponentially small. For the sake of simplicity we consider here the case $w\ll D$ and take the steplike distribution of magnetic fields induced by the domain structure with period $a=2w$: $b(x)=B_0\mathrm{sgn}(x)$, for $|x|<w$ and $b(x+na)=b(x)$, where n is an integer. The corresponding vector potential can be chosen in the form $A(x)=B_0|x|$ for $|x|<w$ and $A(x+na)=A(x)$.

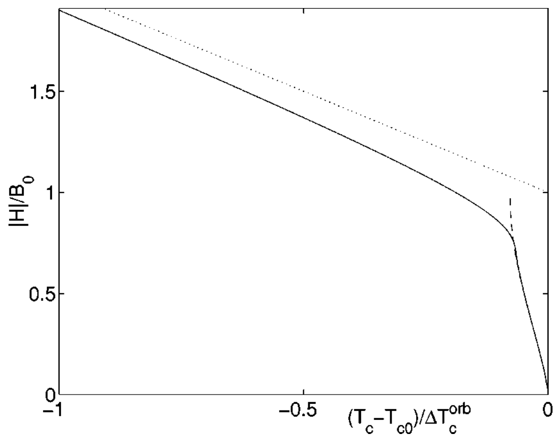

FIG. 3. The temperature dependence of the upper critical field for a domain wall in a S/F system with a thin ferromagnetic layer for $B_0D^2/\Phi_0=25$ (solid line). The dashed line corresponds to the analytical expression (14) at low fields; the dotted line corresponds to the high-field asymptotics (16).

In the absence of an external field the general solution of Eq. (2) meets the Bloch theorem:

$$f_{kq}(x+a)=f_{kq}(x)e^{iqa}, \tag{17}$$

where q is a quasimomentum. The nodeless wave function of the ground state corresponds to the value $q=0$ and is an even function of x, and thus we obtain $f_k'(0)=f_k'(w)=0$. So we conclude that the solution at zero external field is identi-

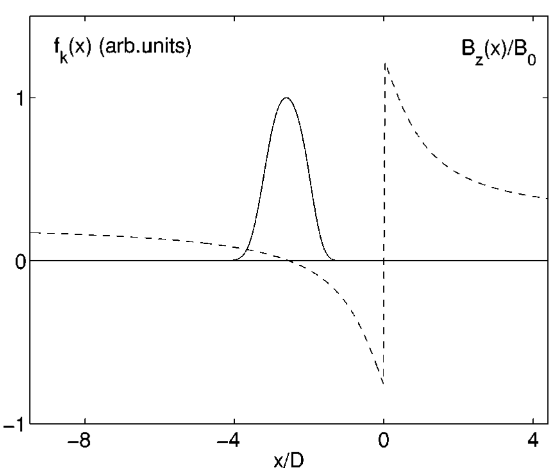

FIG. 4. The typical behavior of the ground-state wave function for a domain wall in a S/F system with a thin ferromagnetic layer (solid line). The magnetic field profile is shown by the dashed line. The parameters are $B_0D^2/\Phi_0=25$ and $H/B_0=0.24$.

cal to the one describing the superconductivity nucleation in a superconducting film of thickness w in uniform magnetic field B_0. Following Ref. 25 we can obtain the ground-state wave functions and energy $E=(T_{c0}-T_c)/\Delta T_c^{orb}$ as a function of momentum k. The behavior of the resulting dependence of $E(k)$ strongly depends on the parameter w/L. Two different regimes could be realized: for small values of $w/L<2.5$ there is only one minimum of $E(k)$ at $k=w/(2L^2)$, which corresponds to superconductivity nucleation above the domain center. For larger values of w/L one obtains two minima with equal energies E at k_1^{min} and k_2^{min} ($k_1^{min}+k_2^{min}=w/L^2$). For $w/L\gg 1$ the coordinates of these minima k_1^{min} and k_2^{min} and minimum energy E approach the values corresponding to the ones for isolated domain walls (see Section II A). Depending on the k-momentum value the superconducting nuclei appear either above the walls at $x=na$ (for k_1^{min}) or at $x=w+na$ (for k_2^{min}). Thus, the nuclei at neighboring domain walls do not interact within the linearized GL theory. The dependence of the critical temperature on the field B_0 in a periodic domain structure is described by the formula

$$1-T_c/T_{c0}=\frac{4\xi_0^2}{w^2}F\left(\frac{\pi B_0 w^2}{2\Phi_0}\right),$$

where the function $F(z)$ coincides with that for a superconducting film in the uniform magnetic field B_0 which is plotted, e.g., in Ref. 25. For a finite domain thickness w the critical temperature $T_c(B_0)$ appears to be larger than the one for a single domain wall. This difference in T_c becomes rather large for small values $z=\pi B_0 w^2/2\Phi_0$ when the nucleus is not localized near the domain boundary. For large z values one can obtain $F(z)\to z/1.69$, which corresponds to the dependence $T_c(B_0)$ for a nucleus at a single domain wall.

If we apply an external magnetic field H, the Bloch theorem is no longer valid and the solution $f_k(x)$ appears to be localized. The energy level $E(k)$ becomes a periodic function of the momentum k: $E(k+4\pi Hw/\Phi_0)=E(k)$. The behavior of the upper critical field and structure of superconducting nuclei are controlled by the parameter w/L. The results of our calculations carried out using the same numerical scheme as in Section II B are shown in Fig. 5.

For large values w/L the phase transition line is very close to the one found in the Section II A, except for the small temperature region close to $T_{c0}:\Delta T\sim 4T_{c0}\xi_0^2/w^2$. Outside this narrow temperature interval (and for $H<B_0$) the wave function is localized at the domain walls (see Fig. 6).

The coordinates of these localized nuclei shift at ma as we change the momentum at $4\pi Hwm/\Phi_0$ (m is an integer). Let us note that for rather weak magnetic fields $H<B_0$ we observed a very peculiar behavior of the order parameter for a discrete set of field values given by the condition $k_2^{min}-k_1^{min}=4\pi Hwm/\Phi_0$: the ground-state wave function $f_k(x)$ has a two-peak structure (see Fig. 7). This fact is a natural consequence of the equivalence of the momenta k and $k'=k+4\pi Hwm/\Phi_0$ and the resulting resonant interaction of nuclei localized at domain walls separated by the distance $w(2m-1)$.

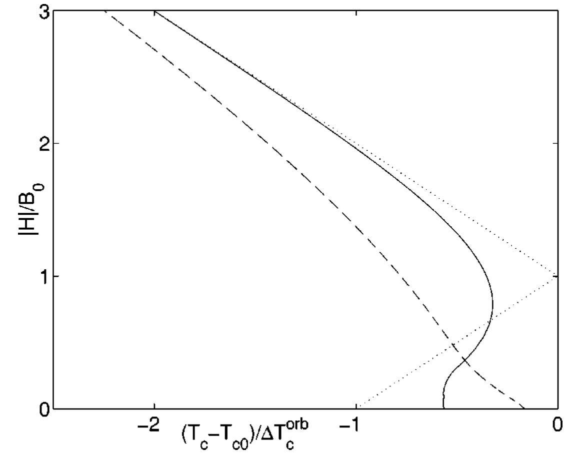

FIG. 5. The temperature dependence of the upper critical field for a periodic domain structure in a S/F system with a thick ferromagnetic layer for $\pi B_0 w^2/\Phi_0=5$ (solid line) and $\pi B_0 w^2/\Phi_0=1$ (dashed line).

For not very large values $w/L<2.0$ the critical temperature becomes a monotonic function of the external magnetic field because of the strong overlapping of wave functions corresponding to different domains. Therefore, the wave function is no longer localized in a single domain (see Fig. 8). However, even in this case we still observe a change in the slope of the phase transition line (see the dashed line in Fig. 5).

The behavior of the upper critical field discussed above is not specific for steplike field distributions. To demonstrate this fact we studied the superconductivity nucleation for the field profile $B_z(x)=B_0\cos(2\pi x/a)+H$. The phase diagram on the plane H-T appears to be qualitatively similar to the one shown in Fig. 5. The critical temperature is a monotonic function of the external magnetic field for $a/L<4.5$. For large parameters $a/L\gg 1$ and $H<B_0$ the behavior of the critical temperature can be analyzed analytically following the approach used in Section II B. The characteristic size of a

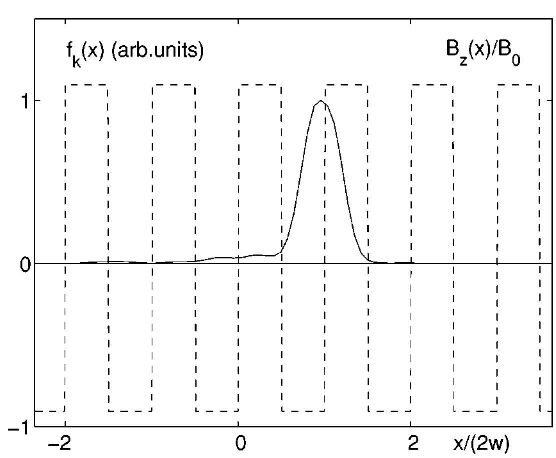

FIG. 6. The behavior of the ground-state wave function (solid line) localized at a domain wall in a periodic domain system. The magnetic field profile is shown by the dashed line. The parameters are $\pi B_0 w^2/\Phi_0=5$ and $H/B_0=0.095$.

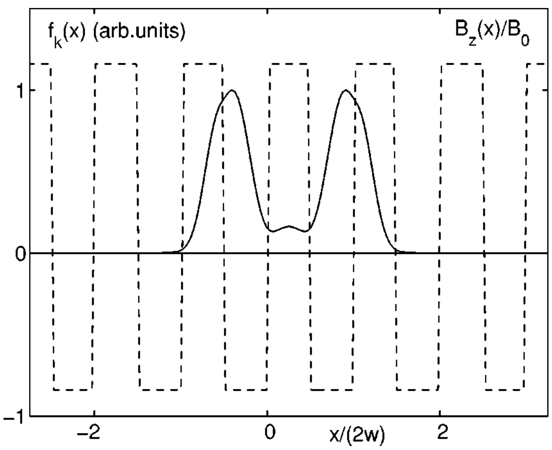

FIG. 7. The two-peak structure of the ground-state wave function (solid line) for a periodic domain system. The magnetic field profile is shown by the dashed line. The parameters are $\pi B_0 w^2/\Phi_0 = 5$ and $H/B_0 = 0.16$.

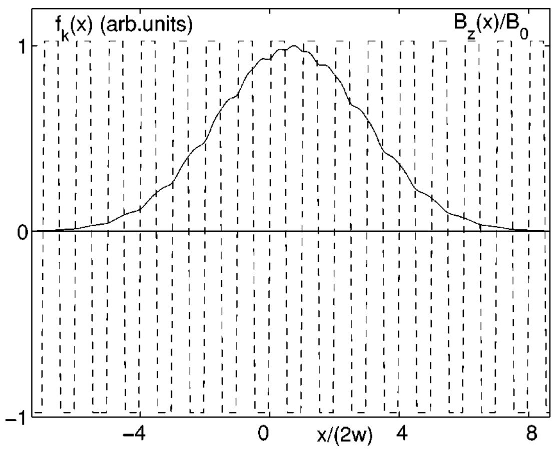

FIG. 8. The behavior of the ground-state wave function in a periodic domain structure for $\pi B_0 w^2/\Phi_0 = 1$ and $H/B_0 = 0.025$ (solid line). The magnetic field profile is shown by the dashed line.

superconducting nucleus and the critical temperature of superconductivity nucleation are given by the expressions

$$\ell = a\sqrt{[}3\frac{\Phi_0}{2\pi^2 B_0 a^2}\left(1-\frac{H^2}{B_0^2}\right)^{-1/6},$$

$$\left(1-\frac{T_c}{T_{c0}}\right) = \epsilon_0 \frac{\xi_0^2}{a^2}\sqrt[3]{\frac{2\pi^2 B_0 a^2}{\Phi_0}}\left(1-\frac{H^2}{B_0^2}\right). \qquad (18)$$

The validity range of this approximate description is defined by the conditions

$$\left(\frac{H/B_0}{1-H^2/B_0^2}\right)^{2/3} \ll \left(\frac{2\pi^2 B_0 a^2}{\Phi_0}\right)^{1/3}, \quad \left(\frac{2\pi^2 B_0 a^2}{\Phi_0}\right)^{1/3} \gg 1.$$

IV. CONCLUSIONS

To summarize, we investigated the conditions of nucleation of localized superconductivity at the domain boundaries in hybrid S/F systems. The appearance of these localized superconducting nuclei should result in a broadening of the superconducting transition probed, e.g., by the resistivity measurements. We predict different regimes for the temperature dependence of the upper critical field near T_c. The crossover between these regimes could be easily seen in experiments. In fact, the beginning of the resistivity decrease with the temperature decrease would correspond to the domain-wall superconductivity, while its complete disappearance would signal the bulk superconductivity. An external magnetic field would shrink the region of the domain wall superconductivity. Let us discuss some estimates of the physical parameters for systems where the nucleation of superconductivity at domain boundaries could be observed. We can take, for example, the parameters of Nb ($T_c \sim 9$ K and $dH_{c2}/dT \sim 0.5$ kOe/K) and typical values of magnetization for ferromagnetic insulators, $4\pi M \sim 1$–10 kOe. The resulting increase in the critical temperature above a domain wall is quite strong: $\delta T_c \sim 1$–3 K. The thickness of a superconducting film must be much smaller than the distance between domains and ideal conditions correspond to a thickness of the order of several coherence lengths. So we conclude that the effects discussed above may be easily observed and could be quite important. Note that the behavior observed in Ref. 21 for S/F bilayers with bubble domains in a ferromagnetic film is qualitatively similar to our predictions. Generally the temperature behavior of the critical field in S/F structures can be very rich (see Figs. 2, 3, and 5) and it is strongly dependent on the domain structure and method of determination of the critical field. Careful measurements of the resistive and magnetic transition (including the measurements of the transition broadening) on the samples with a controllable domain structure would be very useful for the interpretation of the phase diagram and could give important information on the domain-wall superconductivity.

Note in conclusion that the existence of localized superconducting channels near the domain walls in S/F heterostructures can provide an interesting possibility to realize a switching behavior provided we can move the ferromagnetic domain wall. The superconducting channel in this case should follow the motion of the domain wall, which provides a possibility to control the conductance between certain static leads.

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[1] *Superconductivity and Magnetism*, edited by M. B. Maple and O. Fisher (Springer-Verlag, Berlin, 1982).
[2] J. Flouquet and A. Buzdin, Phys. World **15**, 41 (2002).
[3] V.L. Ginzburg, Zh. Éksp. Teor. Phys. **31**, 202 (1956) [Sov. Phys. JETP **4**, 153 (1956)].
[4] J.I. Martin, M. Vélez, J. Nogués, and I.K. Schuller, Phys. Rev. Lett. **79**, 1929 (1997); J.I. Martin, M. Vélez, A. Hoffmann, I.K. Schuller, and J.L. Vicent, *ibid.* **83**, 1022 (1999).
[5] D.J. Morgan and J.B. Ketterson, Phys. Rev. Lett. **80**, 3614 (1998).
[6] M.J. Van Bael, K. Temst, V.V. Moshchalkov, and Y. Bruynseraede, Phys. Rev. B **59**, 14 674 (1999); M.J. Van Bael, L. Van Look, M. Lange, J. Bekaert, S.J. Bending, A.N. Grigorenko, V.V. Moshchalkov, and Y. Bruynseraede, Physica C **369**, 97 (2002).
[7] M.J. VanBael, S. Raedts, K. Temst, J. Swerts, V.V. Moshchalkov, and Y. Bruynseraede, J. Appl. Phys. **92**, 4531 (2002).
[8] A. Garsia-Santiago, F. Sánchez, V. Varela, and J. Tejada, Appl. Phys. Lett. **77**, 2900 (2000).
[9] E.B. Sonin, Pis'ma Zh. Tekh. Phys. **14**, 1640 (1988) [Sov. Tech. Phys. Lett. **14**, 714 (1988)].
[10] I.D. Tokman, Phys. Lett. A **166**, 412 (1992).
[11] I.F. Lyuksyutov and V.L. Pokrovsky, Phys. Rev. Lett. **81**, 2344 (1998).
[12] R. Šášik and T. Hwa, cond-mat/0003462 (unpublished).
[13] L.N. Bulaevsky, E.M. Chudnovsky, and M.P. Maley, Appl. Phys. Lett. **76**, 2594 (2000).
[14] Yu.I. Bespyatykh and W. Wasilevski, Fiz. Tverd. Tela (St. Petersburg) **43**, 215 (2001) [Phys. Solid State **43**, 224 (2001)]; Yu.I. Bespyatykh, W. Wasilevski, M. Gajdek, I.P. Nikitin, and S.A. Nikitov, *ibid.* **43**, 1754 (2001) [*ibid.* **43**, 1827 (2001)].
[15] S. Erdin, I.F. Lyuksyutov, V.L. Pokrovsky, and V.M. Vinokur, Phys. Rev. Lett. **88**, 017001 (2002); S. Erdin, A.F. Kayali, I.F. Lyuksyutov, and V.L. Pokrovsky, Phys. Rev. B **66**, 014414 (2002).
[16] L.E. Helseth, Phys. Rev. B **66**, 104508 (2002).
[17] M.V. Milošević, S.V. Yampolskii, and F.M. Peeters, Phys. Rev. B **66**, 174519 (2002); J. Low Temp. Phys. **130**, 321 (2003).
[18] R. Laiho, E. Lähderanta, E.B. Sonin, and K.B. Traito, Phys. Rev. B **67**, 144522 (2003).
[19] Y. Otani, B. Pannetier, J.P. Nozières, and D. Givord, J. Magn. Magn. Mater. **126**, 622 (1993).
[20] M. Lange, M.J. Van Bael, Y. Bruynseraede, and V.V. Moshchalkov, Phys. Rev. Lett. **90**, 197006 (2003).
[21] M. Lange, M. J. Van Bael, and V. V. Moshchalkov (unpublished).
[22] W.B. Zeper, F.J.A. Greidanus, P.F. Garcia, and C.R. Fisher, J. Appl. Phys. **65**, 4971 (1989).
[23] A.I. Buzdin, L.N. Bulaevskii, and S.V. Panyukov, Zh. Éksp. Teor. Phys. **87**, 299 (1984) [Sov. Phys. JETP **60**, 174 (1984)].
[24] A.I. Buzdin and A.S. Mel'nikov, Phys. Rev. B **67**, 020503 (2003).
[25] D. Saint-James, G. Sarma, and E. J. Thomas, *Type-II Superconductivity* (Pergamon Press, New York, 1969), Chap. 4.
[26] M.V. Milošević, S.V. Yampolskii, and F.M. Peeters, Phys. Rev. B **66**, 024515 (2002).
[27] S.-L. Cheng and H.A. Fertig, Phys. Rev. B **60**, 13107 (1999).
[28] I.K. Marmorkos, A. Matulis, and F.M. Peeters, Phys. Rev. B **53**, 2677 (1996).
[29] I.S. Ibrahim and F.M. Peeters, Phys. Rev. B **52**, 17321 (1995).
[30] F.M. Peeters and P. Vasilopoulos, Phys. Rev. B **47**, 1466 (1993).
[31] J. Reijniers and F.M. Peeters, J. Phys.: Condens. Matter **12**, 9771 (2000).
[32] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, Natl. Bur. Stand. Appl. Math. Ser. No. 55 (U.S. GPO, Washington, D.C., 1965).