Ramsey Numbers and Adiabatic Quantum Computing

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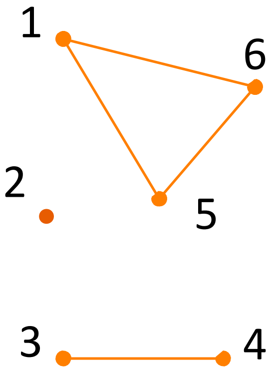
Experiment
Z. Bian, F. Chudak, & W. G. Macready;
D-Wave Systems, Inc.

References:

(1)) F. Gaitan and L. Clark , Phys. Rev. LeV. **108**, 010501 (2012)
(2) Z. Bian, F. Chudak, W. G. Macready, L. Clark, & F. Gaitan, arXiv.org:1201.1842

Ramsey Numbers
Ramsey Numbers

- Ramsey theory (1928): emergence of order in large disordered structures



Party problem: N ≧ 6 ⟹ 3 mutual friends or 3 mutual strangers

All graphs N ≧ R(m,n) vertices ⟹ m-clique or n-independent set

- R(m,n) = two-color Ramsey number
- Very hard to calculate — for m,n ≧ 3, <u>only 9 are known</u>!

Mar v in the M rtian Erdős y a e

<u>Alien Demand</u>:

In one year — R(5,5)

Okay

<u>Alien Demand</u>:

In one year — R(6,6)

First strike !!!

Ramsey Numbers via Optimization

[Gaitan & Clark, PRL (2012)]

Ramsey Numbers via Optimization
[Gaitan & Clark, PRL (2012)]

- Optimization Problem:

Inputs: N, m, n

Cost function: $h(G)$ = (# of m-cliques) + (# of n-independent sets)

SOLUTION: graph(s) G_* = global min of $h(G)$;

$N < R(m,n) \Rightarrow h(G_*) = 0$
$N \geq R(m,n) \Rightarrow h(G_*) \geq 1$

- N-vertex graph G $\Leftrightarrow$ Adjacency matrix $A(G) = (a_{ij})$

1. $g(G) = a_{2,1} \cdots a_{N,1}\, a_{3,2} \cdots a_{N,2} \cdots a_{N-1,N}$

2. $g(G) \Leftrightarrow |g(G)\rangle \Longrightarrow a_{ij} \Leftrightarrow |a_{ij}\rangle \Longrightarrow L_N = N(N-1)/2$ qubits

Ramsey Numbers Quantum Algorithm

[Gaitan & Clark, PRL (2012)]

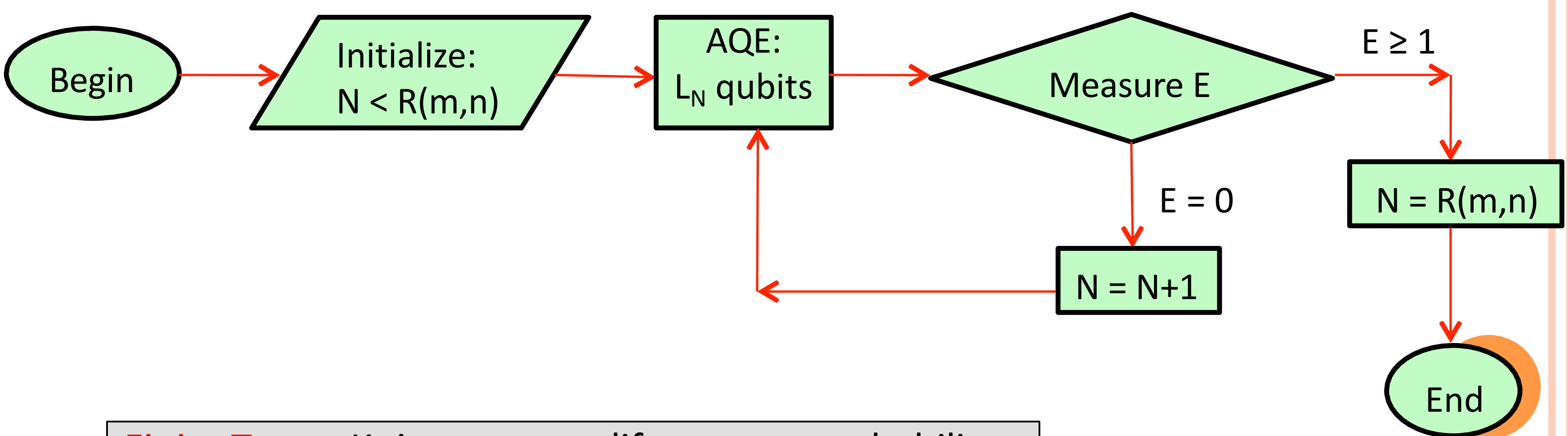
- Adiabatic quantum evolution (AQE) [Farhi et al., arXiv.org:quant--ph/0001106]

$$H(t) = (1 - t/T)\, H_i + (t/T)\, H_P \quad \Longrightarrow \quad H_P\, |g(G)\rangle = h(G)\, |g(G)\rangle$$

$$\Downarrow$$

$$(0 \leq t \leq T)$$

- ALGORITHM:



Finite T: run K times to amplify success probability

NUMERICAL SIMULATIONS

[GAITAN & CLARK, PRL (2012)]

- Numerically simulated adiabatic quantum computation of R(m,n):

$$R(s,2) = s \text{ and } R(3,3) = 6$$

	R(5,2)		R(6,2)		R(7,2)		R(3,3)
N	**(E_{gs} , D)**	**N**	**(E_{gs} , D)**	**N**	**(E_{gs} , D)**	**N**	**(E_{gs} , D)**
3	(0.0, 1)	4	(0.0, 1)	5	(0.0, 1)	4	(0.0,18)
4	(0.0, 1)	5	(0.0, 1)	6	(0.0, 1)	5	(0.0, 12)
5	(1.0, 11)	6	(1.0, 16)	7	(1.0, 22)	6	(2.0, 1760)

Experimental Realization – D-Wave Device

[Bian, Chudak, Macready, Clark, Gaitan; arXiv.org:1201.1842]

Experimental Realization – D-Wave Device
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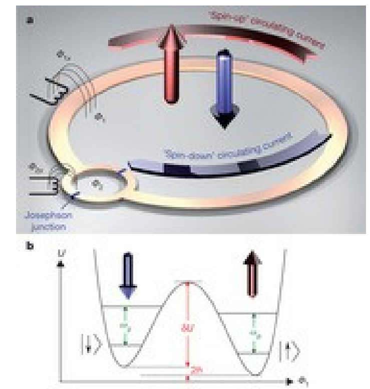
- Device implements quantum annealing; solves QUBO problems



- QUBO = Quadratic Unconstrained Binary Optimization

State:

$y = (y_1, \ldots, y_N)$; $y_i = \pm 1$;

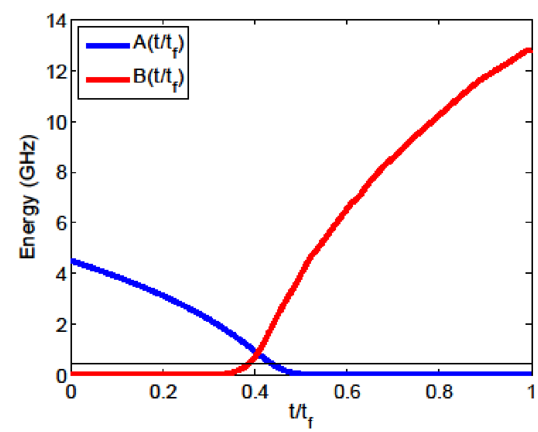
Cost function:

$\mathbb{C}(y) = \sum_i h_i y_i + \sum_{i,j} K_{ij} y_i y_j$

PROBLEM: Given $\{h_i, K_{ij}\}$ – find global minima y_* of $\mathbb{C}(y)$

- Quantum annealing

$$H(t) = A(t) H_i + B(t) H_P$$



1. Adiabatic evolution;
2. Ground-state of H_i $\Longrightarrow$ ground-state of H_P (prob. → 1)

EXPERIMENTAL REALIZATION – EMBEDDING RAMSEY INTO SC CHIP

[BIAN ET AL.; ARXIV.ORG:1201.1842]

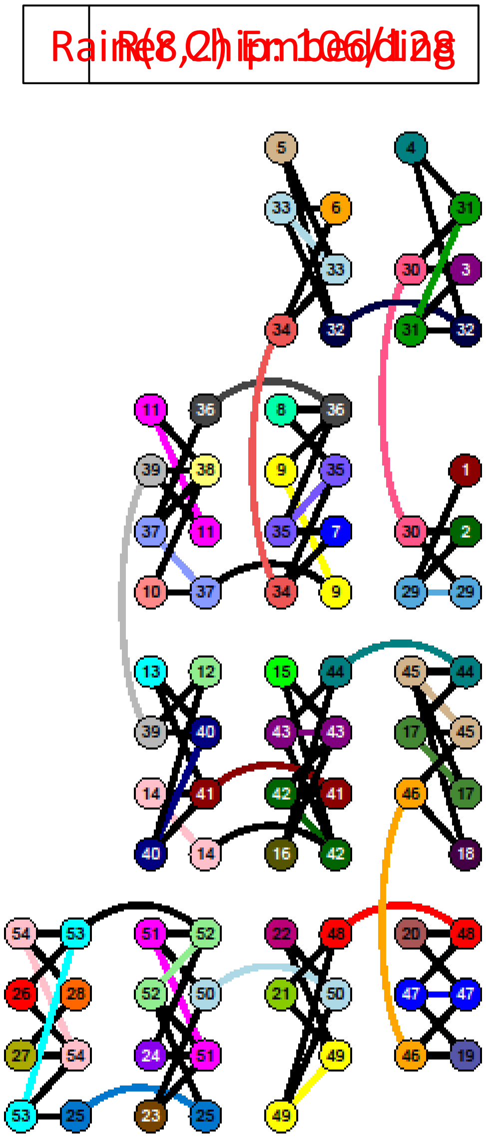
- Ramsey cost function:

$$h(g) = \sum_{\alpha=1}^{C(N,m)} \prod_{e_{ij} \in \xi_\alpha} a_{i,j} + \sum_{\alpha=1}^{C(N,n)} \prod_{e_{ij} \in \xi_\alpha} \bar{a}_{i,j} \;;$$

$$g = a_{2,1} \dots a_{N,1}\, a_{3,2} \dots a_{N,2} \dots a_{N,N-1}\;; \qquad C(N,m) = N!/m!(N-m)!$$

1. Operator expression: $a_{i,j}$ ⟹ projection operators
2. k-qubit interactions: $k = \max\{ C(m,2), C(n,2) \}$
3. k-qubit ⟹ 2-qubit interactions: ancilla qubits
4. 2-qubit coupling: Ramsey ⟹ Chip

ancilla qubits - strong ferromagnetic coupling



EXPERIMENTAL RESULTS: R(3,3) = 6

[BIAN ET AL.; ARXIV.ORG:1201.1842]

N = 6

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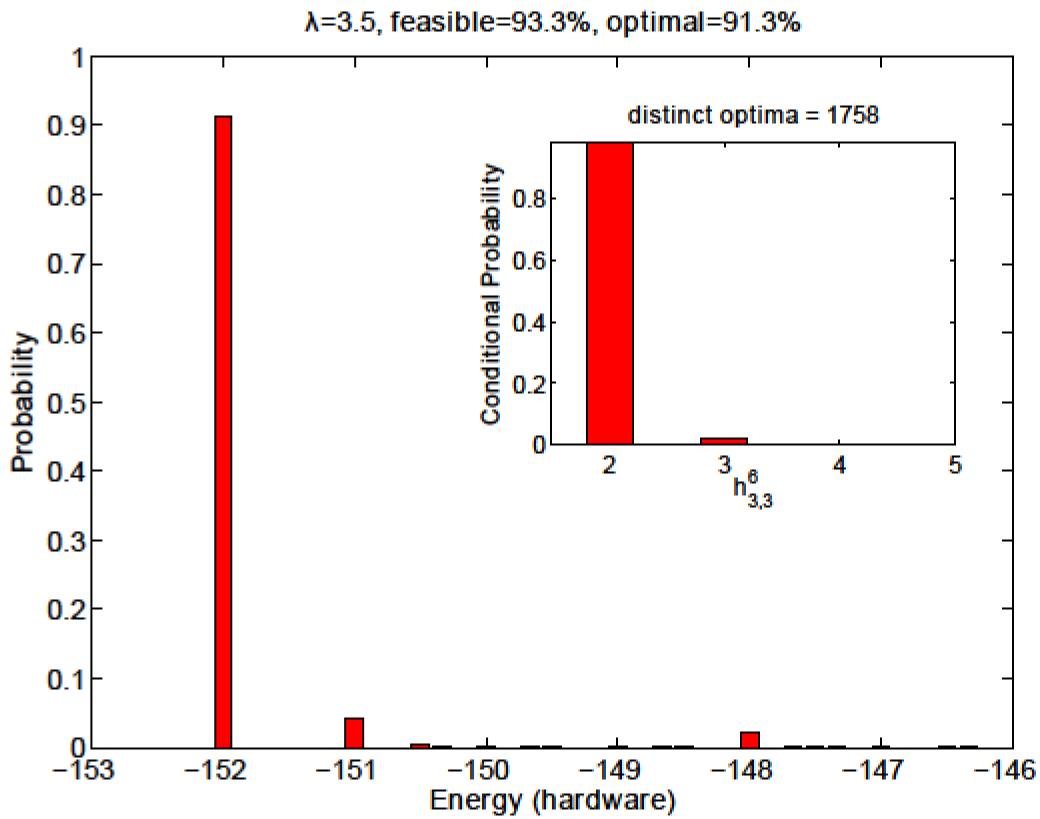


N	E_{gs}	D
4	0 (0)	18 (18)
5	0 (0)	12 (12)
6	2 (2)	1758 (1760)

(1) 100,000 QA runs
(2) 1 ms/QA run

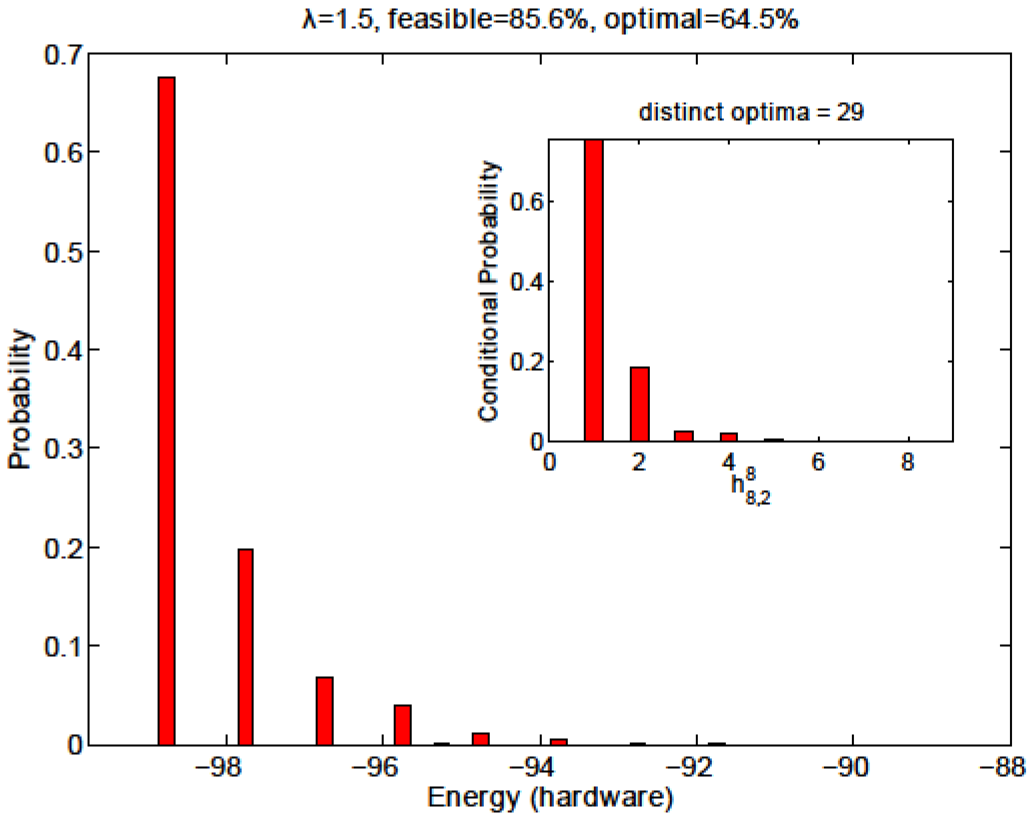
EXPERIMENTAL RESULTS: $R(s,2) = s$

[BIAN ET AL.; ARXIV.ORG:1201.1842]

- Measured $R(s,2)$ for $4 \leq s \leq 8$; show data for $R(8,2)$

N = 8

N	E_{gs}	D
7	0 (0)	1 (1)
8	1 (1)	29 (29)



(1) 100,000 QA runs
(2) 1 ms/QA run

EXPERIMENTAL RESULTS - SUMMARY

N = Graph order; Egs = final ground-state energy; D = ground-state degeneracy

R(s,2) = s

	R(4,2)			R(5,2)			R(6,2)			R(7,2)			R(8,2)	
N	Egs	D	N	Egs	D	N	Egs	D	N	Egs	D	N	Egs	D
3	0 (0)	1 (1)	4	0 (0)	1 (1)	5	0 (0)	1 (1)	6	0 (0)	1 (1)	7	0 (0)	1 (1)
4	1 (1)	7 (7)	5	1 (1)	11 (11)	6	1 (1)	16 (16)	7	1 (1)	22 (22)	8	1 (1)	29 (29)

R(3,3) = 6

	R(3,3)	
N	E_{gs}	D
4	0 (0)	18 (18)
5	0 (0)	12 (12)
6	2 (2)	1758 (1760)

SUMMARY

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- Introduced quantum algorithm to calculate two-color Ramsey numbers R(m,n)

- Ramsey quantum algorithm implemented numerically and experimentally:

1. Numerically -correctly determined R(3,3) and R(s,2) for $5 \leq s \leq 7$

2. Experimentally -correctly determined R(3,3) and R(s,2) for $4 \leq s \leq 8$

3. Experimental implementation of R(8,2) computation:

    (a) used 28 computational qubits & 84 total qubits;

    (b) largest experimental implementation of scientifically meaningful quantum algorithm.

References: (1) Gaitan and Clark, PRL **108**, 010501 (2012);

(2) Bian, Chudak, Macready, Clark, Gaitan; arXiv.org:1201.1842.

Two-color Ramsey numbers R(m,n)

Two-color Ramsey numbers R(m,n)

$m\backslash n$	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2		2	3	4	5	6	7	8	9	10
3			6	9	14	18	23	28	36	40-43
4				18	25	35-41	49-61	56-84	73-115	92-149
5					43-49	58-87	80-143	101-216	125-316	143-442
6						102-165	113-298	127-495	169-780	179-1171
7							205-540	216-1031	233-1713	289-2826
8								282-1870	317-3583	317-6090
9									565-6588	580-12677
10										798-23556

Table 1: Bounds on Ramsey numbers. The lower triangular part of the table is filled in using the symmetry $R(m,n) = R(n,m)$.