Quantum Algorithms for Simulating Quantum Field Theories

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joint work with:

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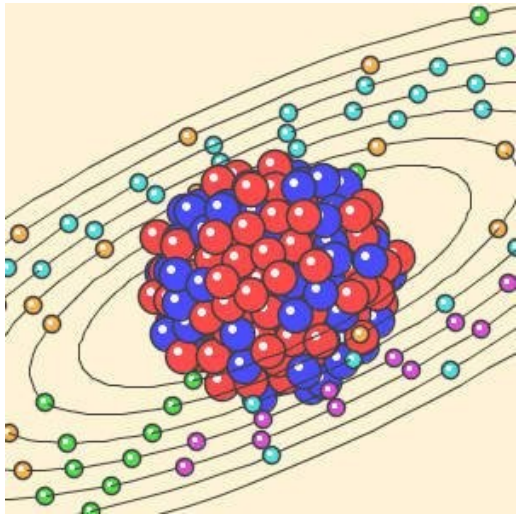
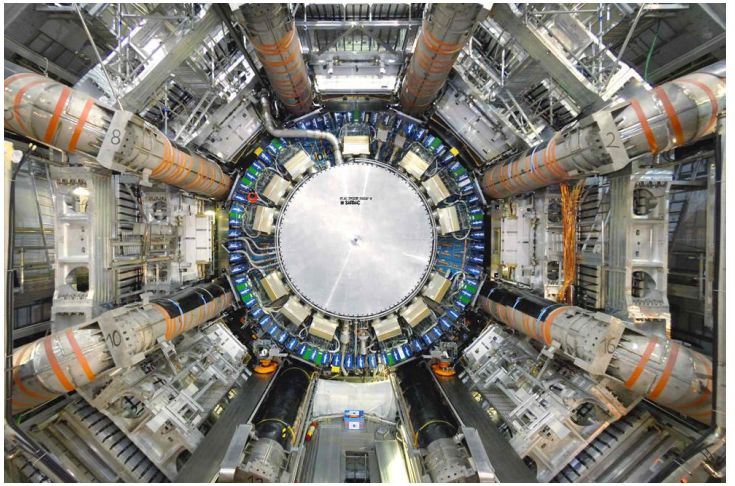
NIST

[ArXiv:1111.5306 and 1112.4833]

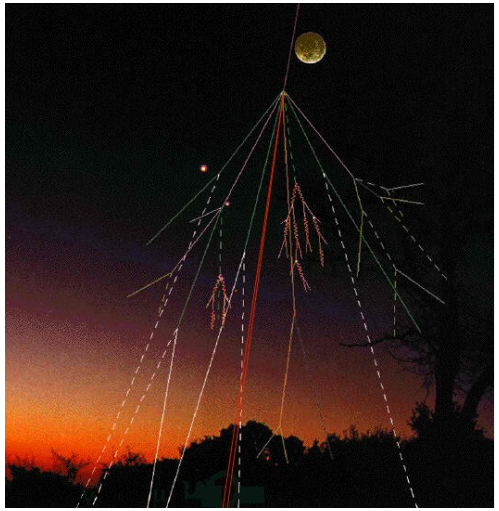
When do we need QFT?



Nuclear Physics



Accelerator Experiments

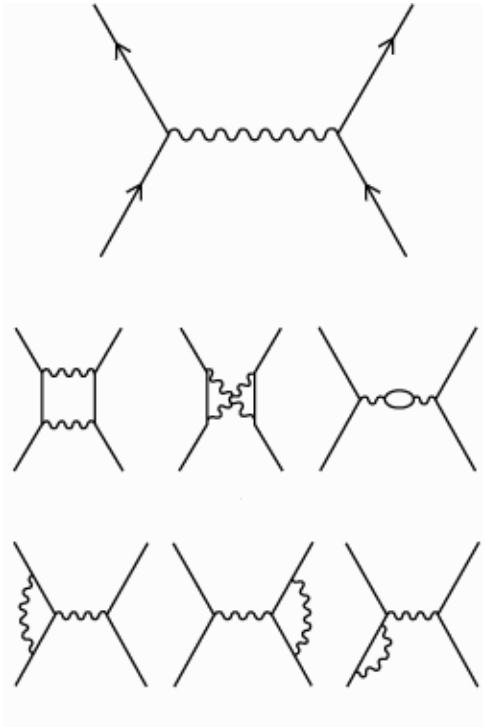


Cosmic Rays

➔Whenever quantum mechanical and relativistic effects are both significant.

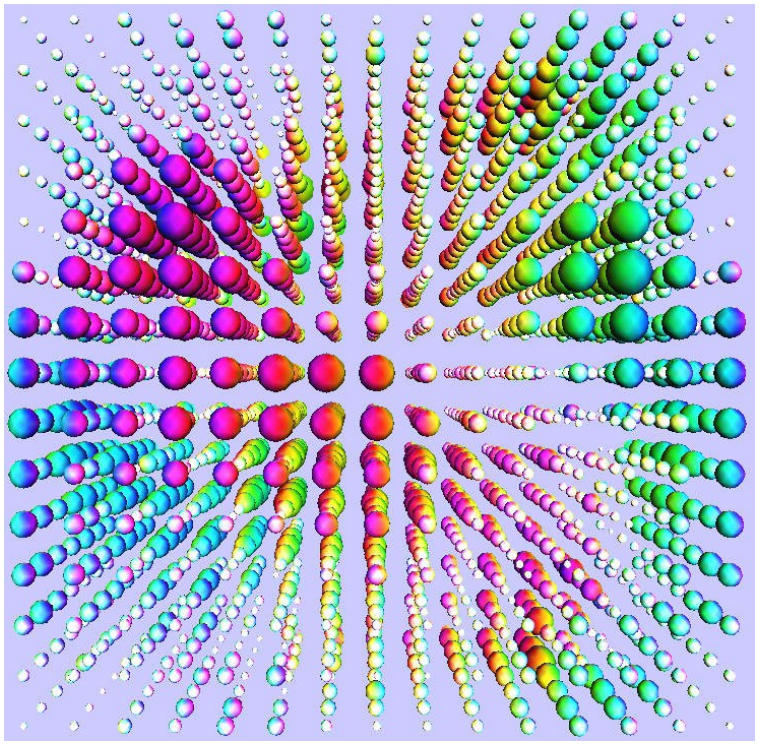
Classical Algorithms

Feynman Diagrams



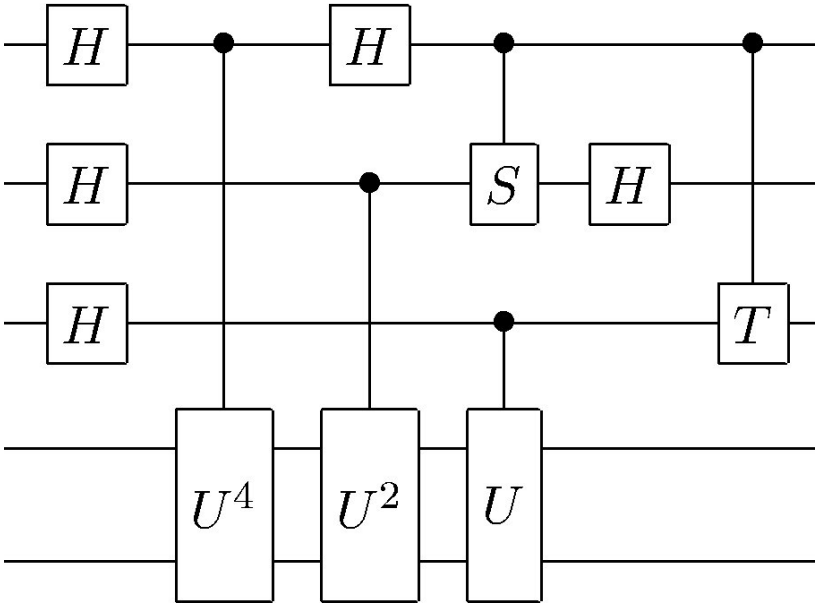
break down at strong coupling or high precision

Lattice Methods

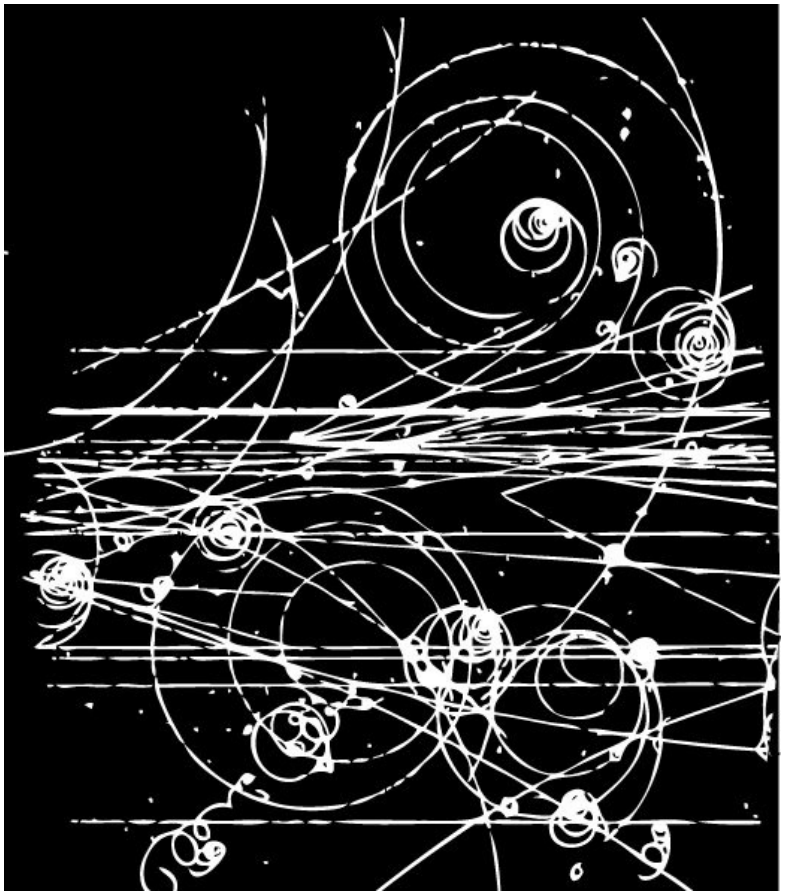


cannot calculate dynamical quantities

What is the computational power of our universe?



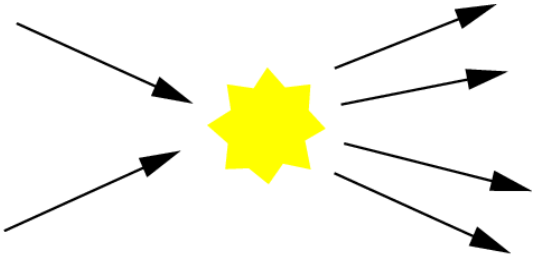
simulate

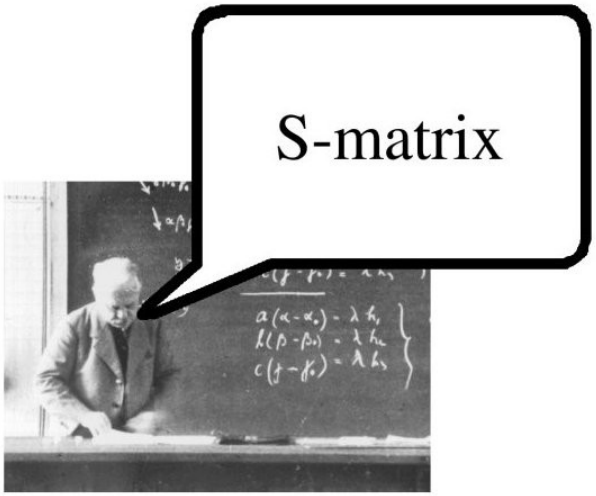


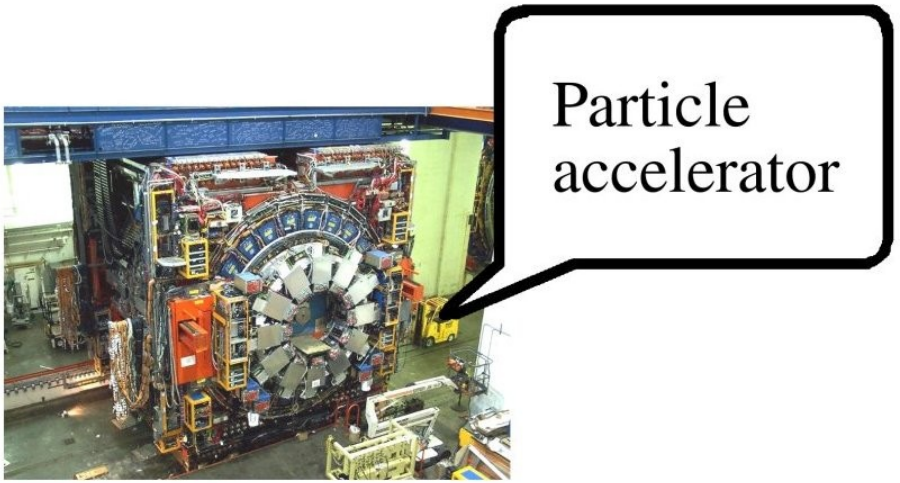
A QFT Computational Problem

input: a list of momenta of incoming particles

output: a list of momenta of outgoing particles time *t* later







I will present a polynomial-time quantum algorithm to compute scattering amplitudes in ϕ^4 -theory with nonzero mass.

ϕ^4-theory is a simple toy model, but it illustrates some of the main difficulties of simulating QFTs

Discretizing spacetime

Preparing initial states

Measuring observables

Time evolution is the easy part (use Trotter)

ϕ^4-theory

Lagrangian density:

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

For quantum simulation, we prefer Hamiltonian formulation (equivalent):

$$H = \int d^d x \left[\pi^2 + (\nabla\phi)^2 + m^2\phi^2 + \lambda\phi^4\right]$$

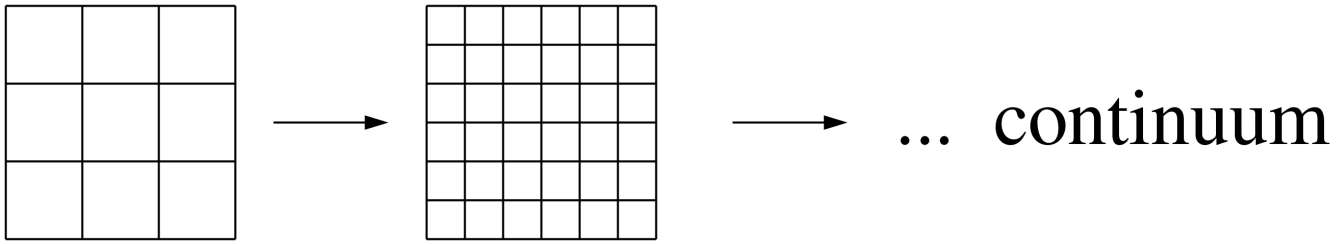
$$[\phi(x), \pi(y)] = i\delta^{(d)}(x-y)$$

Lattice Cutoff

To get finite answers one needs a cutoff. We use a spatial lattice:

$$H(a) = \frac{1}{2} \sum_{x \in \Omega} a^d \left[\pi^2 + (\nabla \phi)^2 + m^2 \phi^2 + \lambda \phi^4 \right]$$

Continuum QFT = limit of a sequence of theories on successively finer lattices:



m and λ are functions of *a* !

After imposing a spatial lattice we have a many-body quantum system with a local Hamiltonian

Simulating the time evolution in polynomial time is a solved problem

Standard methods scale as N^2 We can do N .)

Questions:

Convergence as $a \to 0$

How to prepare initial wavepackets

How to measure particle momenta

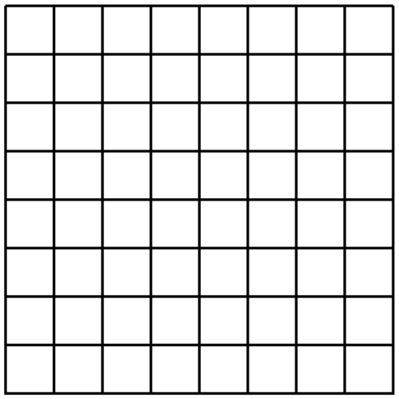
Convergence as $a \to 0$

Particles of energy E have associated

lengthscale $\frac{hc}{E}$

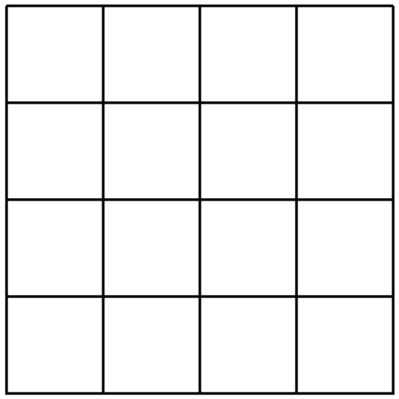
Intuition: if $a \ll \frac{hc}{E}$ discretization errors should be small

We can formalize this by analyzing a sequence of effective Hamiltonians (RG flow)

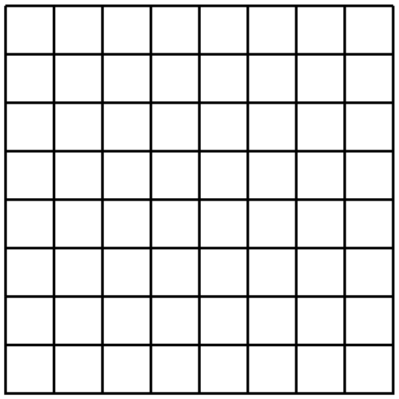


$$H = \frac{1}{2} \sum_{x \in \Omega} a^d \left[\pi^2 + (\nabla \phi)^2 + m^2 \phi^2 + \lambda \phi^4 \right]$$

Coarse grain



$$H_{\text{eff}} = \frac{1}{2} \sum_{x \in \Omega'} (2a)^d \left[\pi^2 + (\nabla' \phi)^2 + m_{\text{eff}}^2 \phi^2 + \lambda_{\text{eff}} \phi^4 + g \phi^6 + \dots \right]$$



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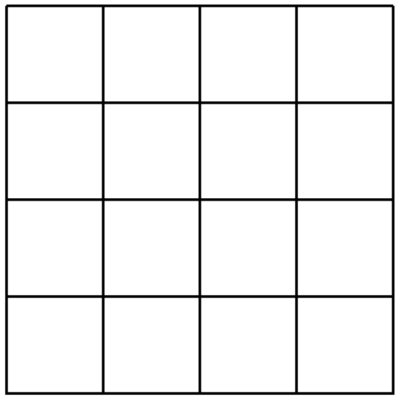

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How does simulation convergence as $a \to 0$?

Answer: as a^2

TODO:

Efficiently prepare initial wavepackets

Efficiently measure particle momenta

Creating a particle

$a_p^\dagger$ creates a particle of momentum *p* and

energy $\omega_p = \sqrt{p^2 + m^2}$

We can define single-particle states in the interacting theory by adiabatic continuation from the free theory.

Quantum State-Preparation Algorithm:

1. Build the free vacuum (Gaussian)
2. Excite wavepackets
3. Adiabatically turn on interaction

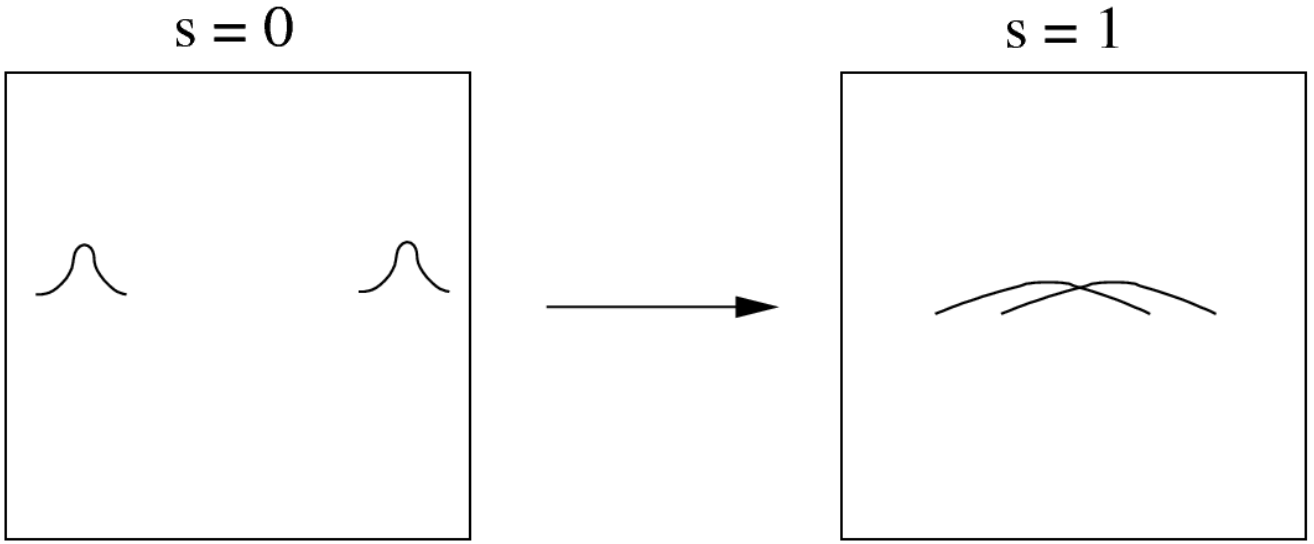
Adiabatically Turn on Interaction

$$H(s) = \sum_{x \in \Omega} \left[\pi^2 + (\nabla \phi)^2 + m^2 \phi^2 + s\lambda \phi^4 \right]$$

Use standard Trotter technique to simulate *H*(*s*) with *s* slowly varying from 0 to 1

This almost works...

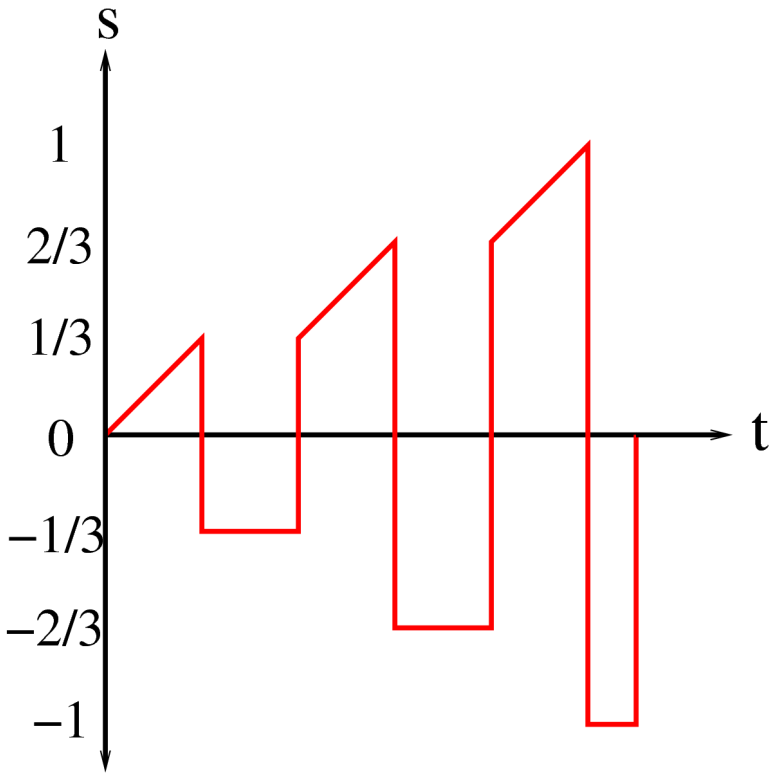
During all this slow time-evolution



the wavepackets propagate and broaden!

Solution: intersperse backwards time-evolutions with time-independent Hamiltonians

This winds back the dynamical phase on each eigenstate, without undoing the adiabatic change of eigenbasis.



Adiabaticity

Q. How slowly must we vary *s*?

A. By the adiabatic theorem, this is determined by the energy gap against excitations. There are two important excitation processes:

particle creation from vacuum: $\gamma = m_{\rm phys}$

one particle splits into three: $\gamma \simeq \dfrac{m_{\rm phys}^2}{p}$

Measurements

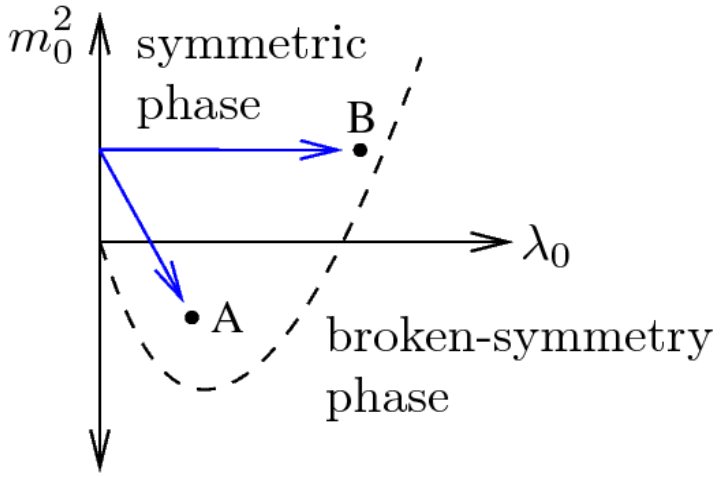
In the free theory, use the number operators: $a_p^\dagger a_p$

Defining particle number is hard in the interacting theory!

Adiabatically go back to the free theory and measure the number operators using Kitaev's phase estimation technique.

Strong Coupling

ϕ^4-theory in 1+1 and 2+1 dimensions has phase transition in which $\phi \to -\phi$ symmetry is spontaneously broken



Near phase transition perturbation theory breaks down and mass gap vanishes:

$$m_{\rm phys} \sim (\lambda_c - \lambda)^\nu \qquad \nu = \begin{cases} 1 & d = 1 \\ 0.63\ldots & d = 2 \end{cases}$$

Strong Coupling

The only part of the quantum algorithm affected by strong coupling is adiabatic state preparation.

Splitting: $$\gamma \simeq \frac{m_{\rm phys}^2}{p} \sim \frac{(\lambda_c - \lambda)^{2\nu}}{p}$$

Creation from vacuum: $$\gamma = m_{\rm phys} \sim (\lambda_c - \lambda)^{\nu}$$

Complexity is polynomial in $(\lambda_c - \lambda)$ and *p*

Eventual goal:

Simulate the Standard model in BQP

In progress:

Fermions

Open problems:

Massless particles

Gauge symmetries

Phases inaccessible from free theory

Conclusion

Quantum computers can efficiently calculate scattering amplitudes for ϕ^4 theory in 3 or fewer spatial dimensions.

Work remains to be done regarding more complicated theories.

I thank my collaborators:

Thank you for your attention.