REDUCING OHMIC HEATING IN THE LITHIUM LENS BY SMOOTHING THE CURRENT PULSE

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6-Oct-93

I. Summary.

The purpose of this report is to demonstrate the extent to which smoothing the shape of the current pulse to the lithium lens reduces unnecessary ohmic heating in the lens. This investigation is important because of the continuing interest in increasing the lens gradient to improve pbar yield. In particular, lens heating can be reduced 7-10% by judicious use of a saturating inductor in the power supply feed. Required modifications to the power supply are described. The emphasis is on the collection lens, but similar results can be expected for the proton lens.

II. Calculation of Ohmic Heating.

In a lithium lens, a pulsed power supply delivers a half-sine wave current to the lens described by $I(t) = I_0 \cdot \exp(-\alpha t) \cdot \sin(\omega t)$, where I_0 is the peak current without damping, α is the damping constant, and ω is the angular frequency determined by the capacitance and inductance of the pulser circuit. The current and magnetic field diffuse into the lithium core on a time scale that depends on the ratio of the skin depth to the lens radius δ/r_0. For a current waveform given by I(t), Ref. 1 provides a solution in terms of Bessel functions to the magnetic diffusion equation $\nabla^2 \mathbf{H} = \sigma\mu(\partial \mathbf{H} / \partial t)$ in cylindrical geometry. The radial distribution of ohmic heating in the lithium core is then determined by integrating the ohmic power $\rho J_z^2(r)$ where ρ is the electrical resistivity of the lithium, and the current density J_z comes from $\mathbf{J} = \nabla \times \mathbf{H}$. Previous solutions of the lens problem did not take into account eddy currents after the current pulse is switched off, *i.e.* $\omega t > \pi$. It is straightforward to include the eddy current contribution, by using the principle of superposition to add an additional current $I'(t) = I(t - \pi/\omega) \cdot \exp(-\alpha t)$ to the waveform I(t) such as to force the current to be zero for $\omega t > \pi$. The magnetic field **H** is then the sum of the fields calculated from the two currents. Figure 1 shows the resulting current density waveforms at 0, 2, 4, 6, 8, and 10 mm radius. The expression for total ohmic heating is

$$q_0(r) = \int_0^{\infty} \rho J_z^2(r) \cdot dt \qquad (1)$$

where J_z is determined from **H**. The temperature of the lithium as a function of time in a case in which the field reaches a gradient of 1000T/m at the nominal phase $\omega t = 0.7\pi$ (lens voltage = 2587

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[1] A. J. Lennox, pbar note #269 (Jan. 1983).

V, I_0 = 861 kA), is shown in Figure 2. Fits[2] for the electrical conductivity and specific heat at constant pressure were used. For an assumed initial temperature of 76 C, the curves show a final temperature of 183 C at the outer edge of the lithium, implying that the lithium is melting. The large edge temperature is due to the large current density at the edge, compounded by the thermal runaway effect caused by increase in resistivity of the lithium with temperature. The residual eddy-current heating is concentrated at the center of the lithium core, and is relatively small (less than 10% averaged over the cross section of the lens). But the technique of superposition can be used to calculate ohmic heating in the lens for alternate waveforms.

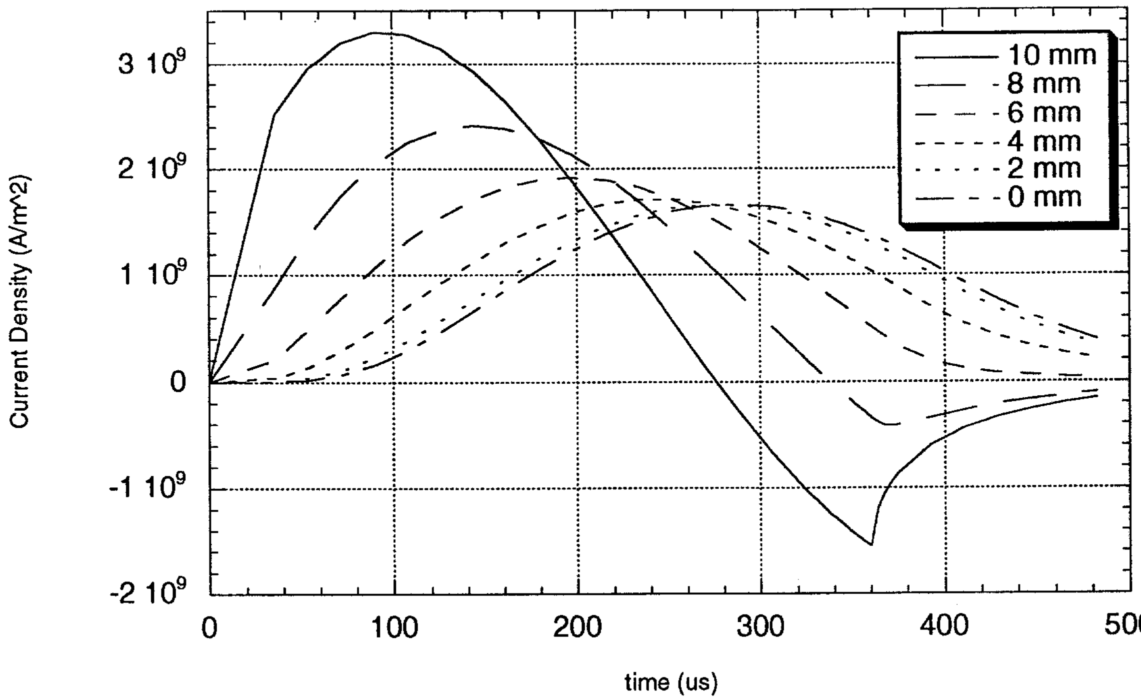

Figure 1. Current density in collection lens for V=2584 V (1000 T/m). The current pulse ends at t=360 μs.

III. Effect of Smoothing the Waveform.

The simple damped sine-wave current pulse is relatively inefficient in penetrating to the center of the lithium core because of the sharp turn-on and turn-off of the current. A waveform which is easily modelled and which has a smoother shape is the current pulse

$$I(t) = \frac{I_0}{1 + a/3}\left(\sin(\omega_0 t) - \frac{a}{3}\sin(3\omega_0 t)\right) \tag{2}$$

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[2]R. K. Williams, G.L. Coleman, D.W. Yarbrough, ORNL-TM-10622 (Mar. 1988).

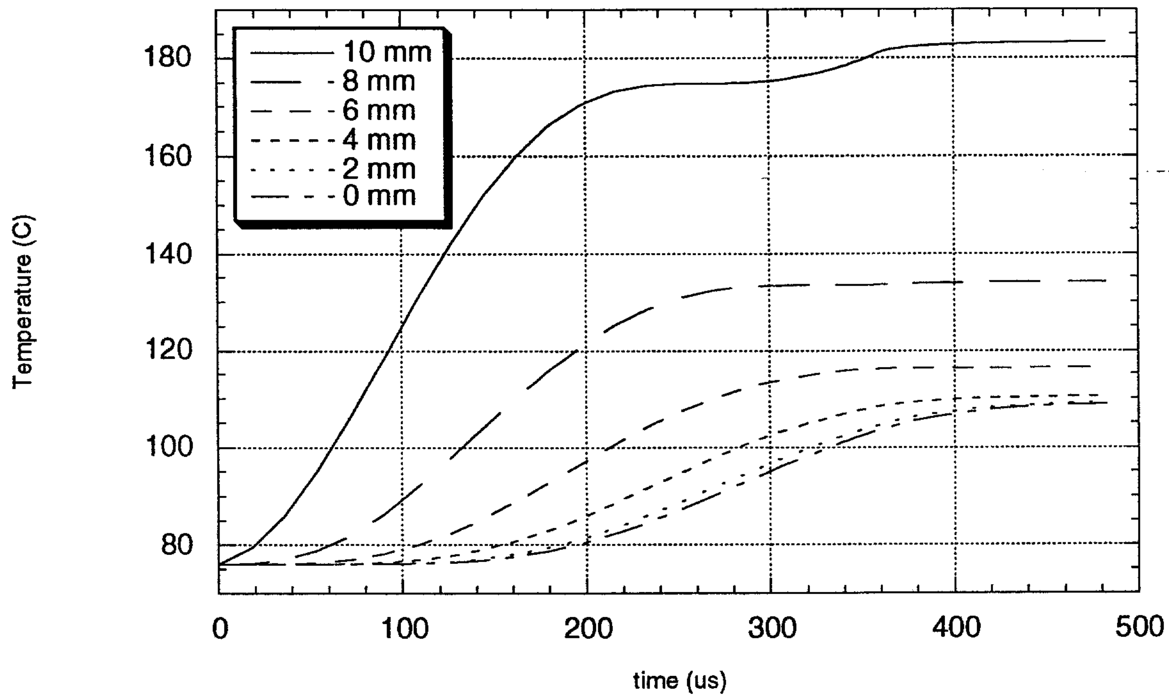

Figure 2. Temperature distribution in the collection lens as a function of time for V=2584V. Initial temperature is assumed to be at a steady-state operating temperature of 76 C.

where the shape factor a is a measure of the slope of the current at t=0 (a=0 nominal pulse, a=1 no initial slope), and the current pulse is normalized to peak current I_0. In order to maintain constant the total charge passing through the lens, the pulse frequency ω_0 must scale as

$$\frac{\omega_0}{\omega_{00}} = \frac{1-a/9}{1+a/3} \tag{3}$$

where ω_{00} is the frequency at a=0. The ohmic heating $Q=\int I^2 R\,dt$ then scales with a as

$$\left(\frac{Q}{Q_0}\right) = \left(\frac{1+(a/3)^2}{(1+a/3)(1-a/9)}\right) \tag{4}$$

which is plotted in Figure 3. The function has a minimum of 0.929 near a=0.6. Therefore wasted heat is minimized at this point. Calculating the Fourier transform of the waveform I(t)

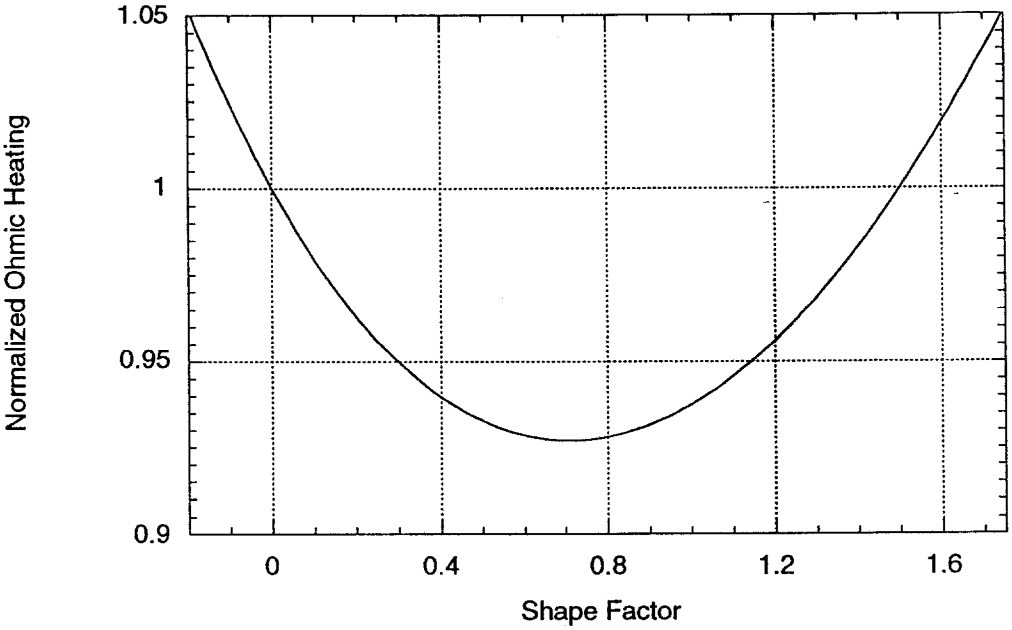

Figure 3. Normalized ohmic heat deposition as a function of shape factor a, from Eq. 4.

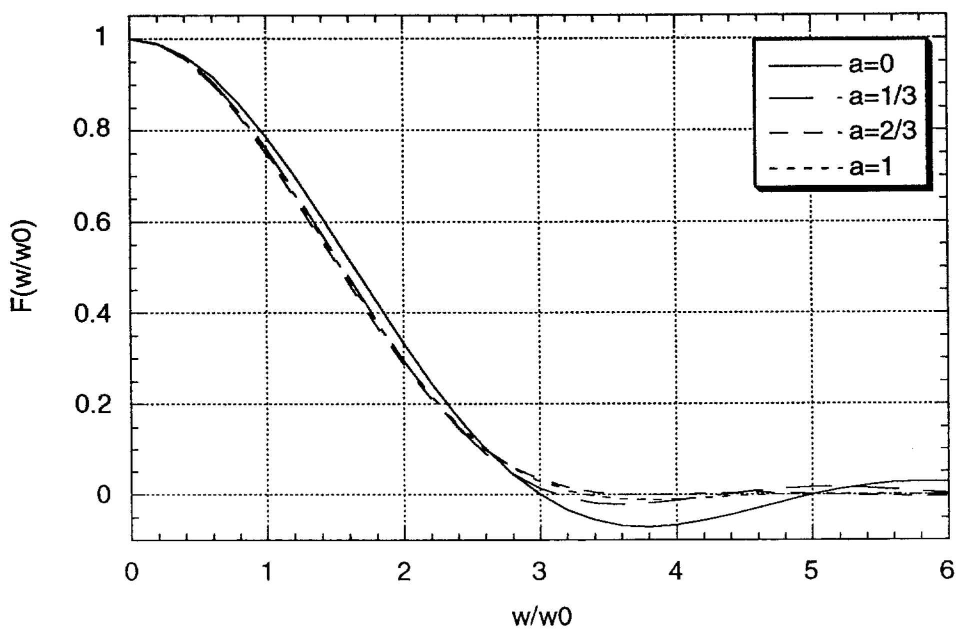

Figure 4. Fourier transforms of the current pulse shapes (Eq. 5) for several values of shape factor a.

$$F(\omega)=\left(\frac{1}{1+a/3}\right)\left(\frac{1}{\omega}\cos\frac{\pi\omega}{2\omega_0}\right)\left[\left(\frac{1}{1-\omega^2/\omega_0^2}\right)-\left(\frac{a}{9-\omega^2/\omega_0^2}\right)\right] \tag{5}$$

shows clearly the high frequency components that are dissipated as heat at the outer edge of the lens. The Fourier transform of the current pulse $F(\omega)$ is plotted in Figure 4 for a=0, 1/3, 2/3, and 1. The high-frequency ripple in the curve for a=0 is not present in the smoothed curves.

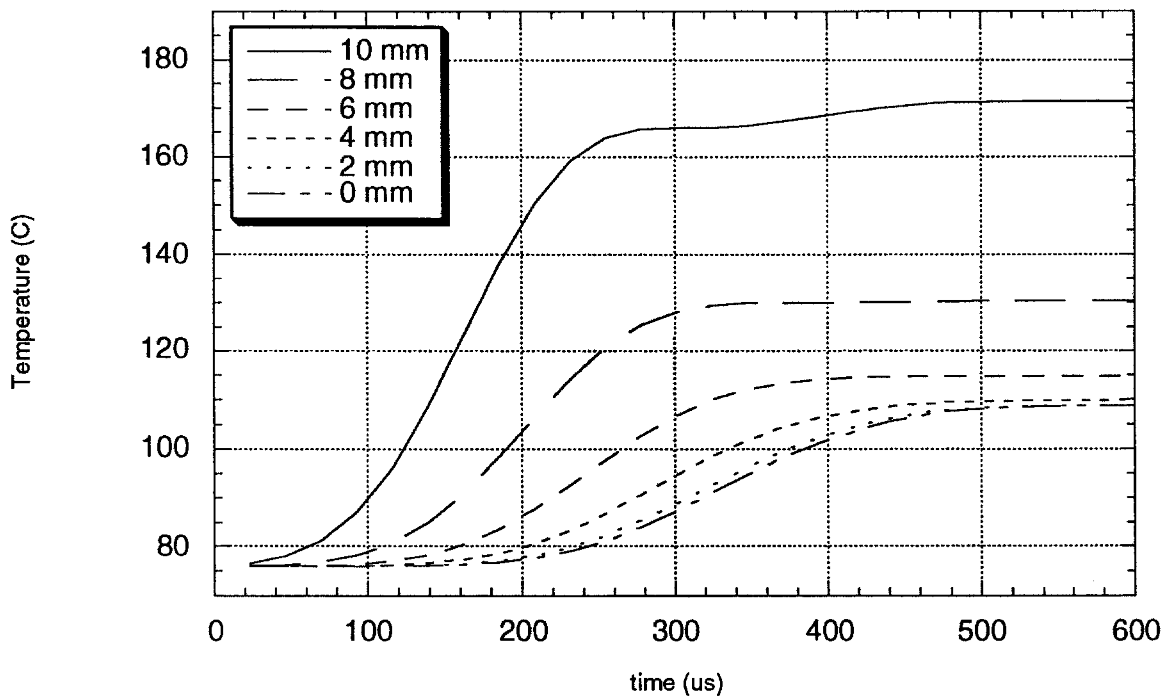

Figure 5. Temperature distribution as a function of time in the collection lens as a function of time for a=0.6. Pulse time and amplitude are scaled to maintain the same focusing field in the core as that in Fig. 2. Assumed initial temperature remains at 76 C. The current pulse ends at t=463 μs.

A calculation of the ohmic heating for a smoothed pulse is shown in Figure 5. In this case, I_0 = 956 kA, and the pulse length has been lengthened. The decay constant remains the same α=1800 sec^{-1}. The final temperature of the lithium at the outer edge is 171 C, which is reduced by 12 C with respect to the nominal pulse. Of course, the reduced overall energy deposition will reduce the steady-state operating temperature from 76 C for the same rep rate and external cooling, further lowering the final temperature. Figures 6 and 7 show the two current pulses, and Monte Carlo calculations of the pbar yield for the two current pulses. The curves show that peak current and yield are essentially the same for the two cases.

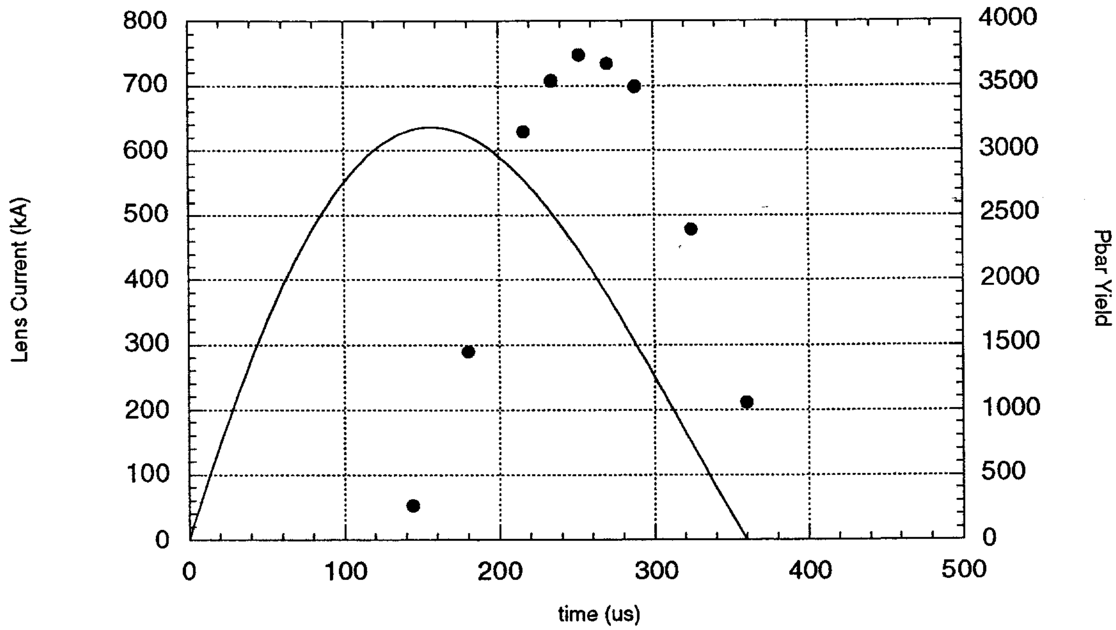

Figure 6. Current pulse and calculated yield for the nominal pulse of Figs. 1 and 2.

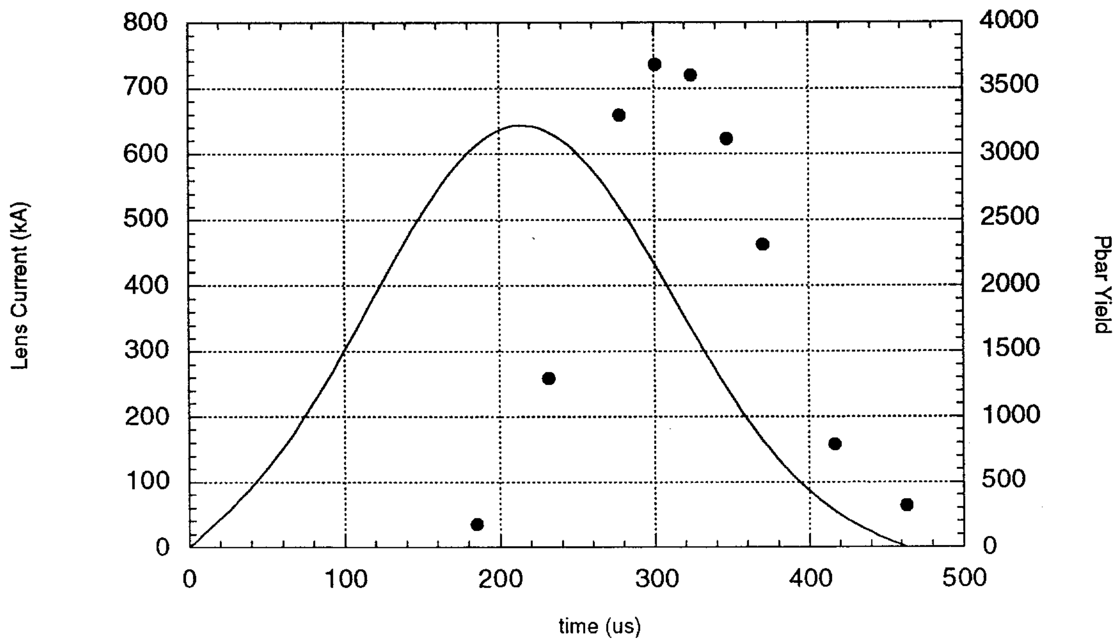

Figure 7. Current pulse and calculated yield for the smoothed pulse of Fig. 5.

IV. Implementing the Smoothing.

The most straightforward way to implement the smoothing is by a saturating inductor. A simple calculation, based on a 100 mV-sec series inductor (4 μH on, 0 μH off) leads to the following results.

Parameter	**Nominal Current Pulse**	**100 mV-sec inductor**
Capacitance	4500 μF	4000 μF
Voltage	2584 V	2820 V
Integrated Current (Coul)	17.74	17.89
Ohmic Heating (J)	10872	10425
Peak Current (A)	78311	78229

A 4% reduction in ohmic heating is realized in this example (figure 8). It would be necessary to increase the operating voltage of the pulser, and reduce the capacitance, in addition to introducing the large 100 mV-sec inductor. More specific calculations can be made, with realistic B-H loop data, should this modification be deemed feasible.

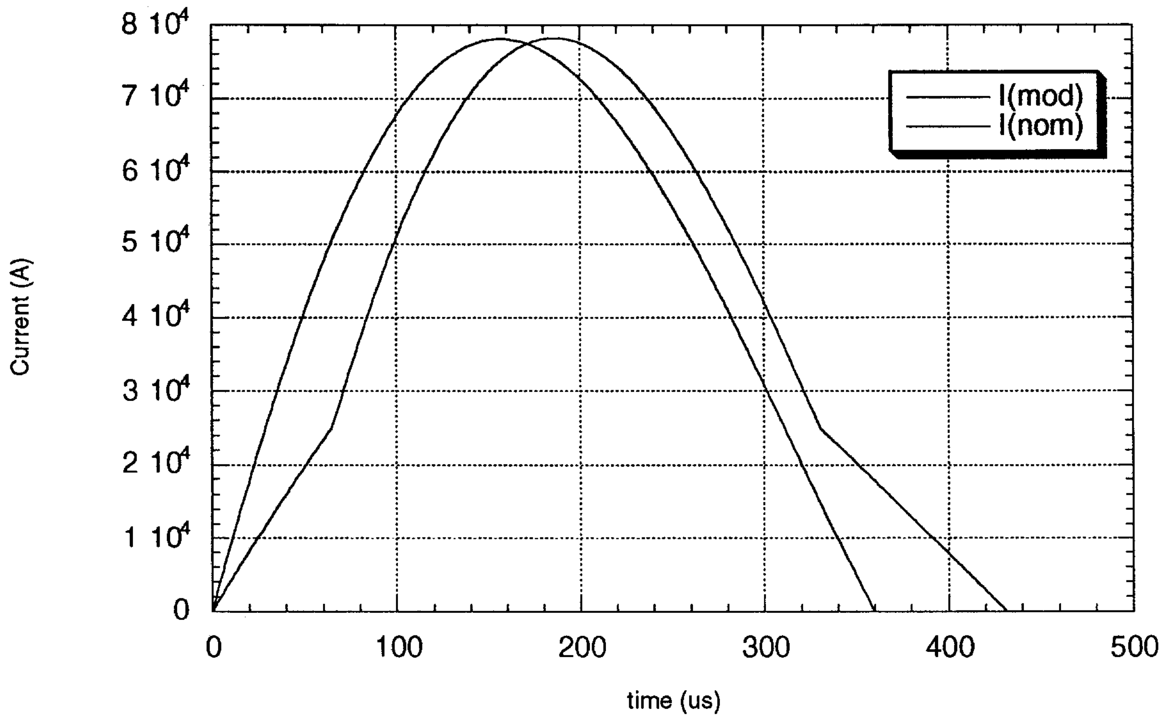

Figure 8. Current pulse for nominal pulser, I(nom), and for a crude saturating inductor model, I(mod).