PBAR NOTE 583
ATTENUATION OF WAVEGUIDE MODES WITH ABSORBING WALLS

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INTRODUCTION

For the 4-8 GHz Debuncher Upgrade, attenuation of the microwave modes in the beam pipe is very important. This note describes how to calculate the attenuation of rectangular waveguide modes with absorber placed on the top and side walls of the waveguide.

ABSORBER ON THE SIDE-WALLS OF THE WAVEGUIDE

The geometry of the problem is shown in Figure 1. Without the absorber, the dominant mode in the waveguide will be the Transverse Electric to Z 1,0 mode (TE^{Z}_{10}). With the introduction of the absorber, the waveguide modes can no longer be classified as transverse to Z (TE^{Z}, TM^{Z}) but can be classified as transverse to X (TE^{X}, TM^{X}).[1] Since the TE^{X}_{10} mode is the same as the TE^{Z}_{10} mode in the absence of absorber, and the incident mode on the absorbing section of waveguide will be TE^{Z}_{10} , we will consider TE^{X} modes which are even in X only. TE^{X} modes can be derived from a x-directed electric vector potential:

$$\vec{F}(x,y,z) = \hat{x}F_x(x,y)e^{-\gamma z} \tag{1}$$

The electric field is given as:

$$\vec{E} = -\frac{1}{\varepsilon}\vec{\nabla}\times\vec{F} \tag{2}$$

The magnetic field is given as:

$$\vec{H} = -\frac{1}{j\omega\mu\varepsilon}\vec{\nabla}\times\vec{E} \tag{3}$$

From Equations 1-3, electric field is:

$$\vec{E} = \left(0\hat{x} + \frac{\gamma}{\varepsilon}F_x\hat{y} + \frac{1}{\varepsilon}\frac{\partial F_x}{\partial y}\hat{z}\right)e^{-\gamma z} \tag{4}$$

The magnetic field is:

$$\vec{H} = \frac{1}{j\omega\mu\varepsilon}\left(-\left(\frac{\partial^2 F_x}{\partial y^2} + \gamma^2 F_x\right)\hat{x} + \frac{\partial^2 F_x}{\partial x \partial y}\hat{y} - \gamma\frac{\partial F_x}{\partial x}\hat{z}\right)e^{-\gamma z} \tag{5}$$

[1] Time Harmonic Electromagnetic Fields, R.F. Harrington, McGraw-Hill, Inc., 1961, pg. 158

To meet the boundary conditions in Region I of Figure 1, the electric vector potential must be of the form:

$$F_x(x, y) = F_1 \cos(f_1 x) \cos\left(\frac{n\pi}{b} y\right) \tag{7}$$

The electric field in Region I is:

$$E_x(x, y) = 0 \tag{8}$$

$$E_y(x, y) = \frac{\gamma}{\varepsilon_1} F_1 \cos(f_1 x) \cos\left(\frac{n\pi}{b} y\right) \tag{9}$$

$$E_z(x, y) = -\frac{1}{\varepsilon_1}\left(\frac{n\pi}{b}\right) F_1 \cos(f_1 x) \sin\left(\frac{n\pi}{b} y\right) \tag{10}$$

The magnetic field in Region I is:

$$H_x(x, y) = -\frac{1}{j\omega\mu_1\varepsilon_1}\left(\gamma^2 - \left(\frac{n\pi}{b}\right)^2\right) F_1 \cos(f_1 x) \cos\left(\frac{n\pi}{b} y\right) \tag{11}$$

$$H_y(x, y) = \frac{1}{j\omega\mu_1\varepsilon_1} f_1 \frac{n\pi}{b} F_1 \sin(f_1 x) \sin\left(\frac{n\pi}{b} y\right) \tag{12}$$

$$H_z(x, y) = \frac{\gamma}{j\omega\mu_1\varepsilon_1} f_1 F_1 \sin(f_1 x) \cos\left(\frac{n\pi}{b} y\right) \tag{13}$$

To meet the boundary conditions in Region II of Figure 1, the electric vector potential must be of the form:

$$F_x(x, y) = F_2 \sin\left(f_2\left(x - \frac{a}{2} - \delta\right)\right) \cos\left(\frac{n\pi}{b} y\right) \tag{14}$$

The electric field in Region II is:

$$E_x(x, y) = 0 \tag{15}$$

$$E_y(x, y) = \frac{\gamma}{\varepsilon_2} F_2 \sin\left(f_2\left(x - \frac{a}{2} - \delta\right)\right) \cos\left(\frac{n\pi}{b} y\right) \tag{16}$$

$$E_z(x, y) = -\frac{1}{\varepsilon_2}\left(\frac{n\pi}{b}\right) F_1 \sin\left(f_2\left(x - \frac{a}{2} - \delta\right)\right) \sin\left(\frac{n\pi}{b} y\right) \tag{17}$$

The magnetic field in Region II is:

$$H_x(x,y) = -\frac{1}{j\omega\mu_2\varepsilon_2}\left(\gamma^2 - \left(\frac{n\pi}{b}\right)^2\right)F_2 \sin\left(f_2\left(x - \frac{a}{2} - \delta\right)\right)\cos\left(\frac{n\pi}{b}y\right) \tag{18}$$

$$H_y(x,y) = -\frac{1}{j\omega\mu_2\varepsilon_2} f_2 \frac{n\pi}{b} F_2 \cos\left(f_2\left(x - \frac{a}{2} - \delta\right)\right)\sin\left(\frac{n\pi}{b}y\right) \tag{19}$$

$$H_z(x,y) = -\frac{\gamma}{j\omega\mu_2\varepsilon_2} f_2 F_2 \cos\left(f_2\left(x - \frac{a}{2} - \delta\right)\right)\cos\left(\frac{n\pi}{b}y\right) \tag{20}$$

At the interface of x=a/2 E_y, E_z must be continuous:

$$\frac{1}{\varepsilon_1} F_1 \cos\left(f_1 \frac{a}{2}\right) = -\frac{1}{\varepsilon_2} F_2 \sin(f_2\delta) \tag{21}$$

At the interface of x=a/2 H_y, H_z must be continuous:

$$\frac{f_1}{\mu_1\varepsilon_1} F_1 \sin\left(f_1 \frac{a}{2}\right) = -\frac{f_2}{\mu_2\varepsilon_2} F_2 \cos(f_2\delta) \tag{22}$$

The solution to the Helmholtz wave equation requires:

$$\gamma^2 - f_1{}^2 - \left(\frac{n\pi}{b}\right)^2 + \omega^2\mu_1\varepsilon_1 = 0 \tag{23}$$

$$\gamma^2 - f_2{}^2 - \left(\frac{n\pi}{b}\right)^2 + \omega^2\mu_2\varepsilon_2 = 0 \tag{24}$$

Subtracting Equation 23 from Equation 23 results in:

$$f_2{}^2 = f_2{}^2 + \omega^2\mu_1\varepsilon_1\left(\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1} - 1\right) = 0 \tag{25}$$

Dividing Equation 22 by Equation 21 results in:

$$\frac{f_1}{f_2}\frac{\mu_2}{\mu 1} \tan\left(f_1 \frac{a}{2}\right)\tan(f_2\delta) = 1 \tag{26}$$

The attenuation is:

$$\alpha = \left| IM\left(\sqrt{\omega^2\mu_1\varepsilon_1 - f_1{}^2 - \left(\frac{n\pi}{b}\right)^2}\right)\right| \tag{27}$$

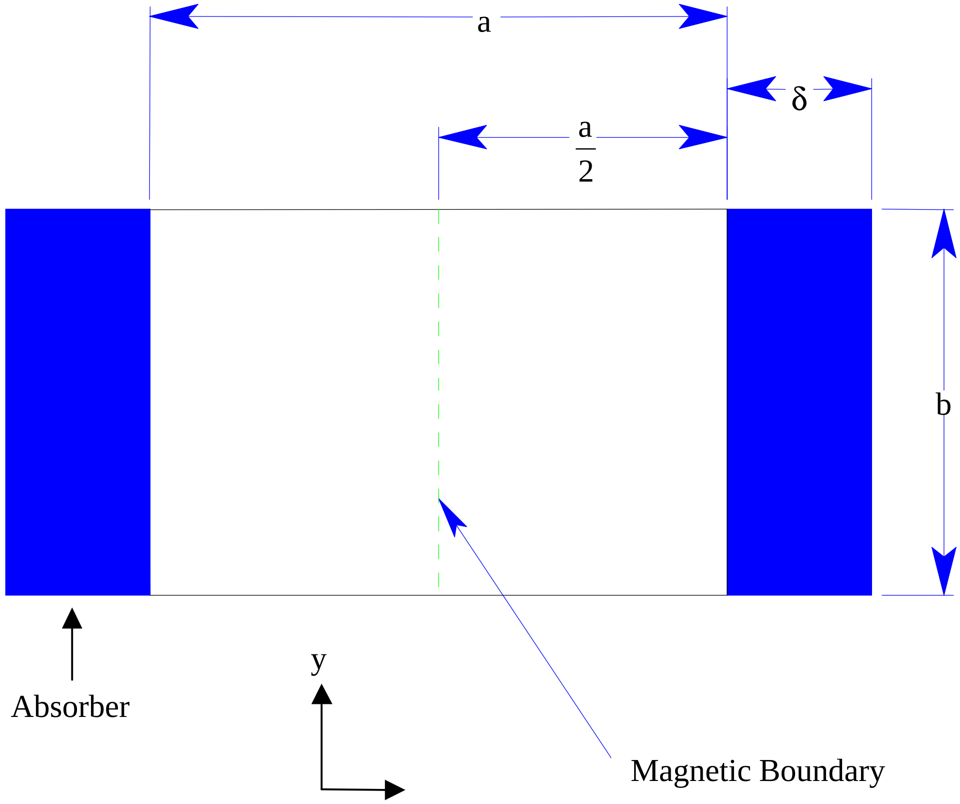

Figure 1. Geometry for sidebar absorbers.

<u>Power Loss in the Side-Bar Absorber</u>

This section deals with the relative power loss in the absorber. This information can be used to determine where most of the heat will be generated in the absorber. We will consider only the fundamental mode propagating in the waveguide ($TE^X{}_{10}$). The power density (power per unit volume) that is lost in the absorber is:

$$P_{loss} = -\frac{1}{2}\omega IM(\varepsilon_2)\left|\vec{E}\right|^2 + \frac{1}{2}\omega IM(\mu)\left|\vec{H}\right|^2 \tag{28}$$

The fields in the absorber are given by Equations 14-20. The fundamental mode has n=0. Because we are only interested in the relative power loss let

$$\left|\frac{\gamma F_2}{\varepsilon_2}\right|^2 = \frac{2}{\omega\varepsilon_1} \tag{29}$$

Also define:

$$\varepsilon_2 = \varepsilon_r \varepsilon_1 \tag{30}$$

$$\mu_2 = \mu_r \mu_1 \tag{31}$$

$$u = x - \frac{a}{2} \tag{32}$$

Then the power loss is given as:

$$P_{loss} = IM(\varepsilon_r)\left|\sin(f_2(u-\delta))\right|^2 + \frac{1}{\omega^2\mu_1\varepsilon_1}\frac{IM(\mu_r)}{|\mu_r|^2}\left(\left|\gamma\sin(f_2(u-\delta))\right|^2 + \left|f_2\cos(f_2(u-\delta))\right|^2\right) \tag{34}$$

where it has been assumed that ε_1 and μ_1 have only real parts (no loss.)

ABSORBER ON THE TOP AND BOTTOM WALLS OF THE WAVEGUIDE

The geometry of the problem is shown in Figure 2. Without the absorber, the dominant mode in the waveguide will be the Transverse Electric to Z 1,0 mode ($TE^Z{}_{10}$). With the introduction of the absorber, the waveguide modes can no longer be classified as transverse to Z (TE^Z, TM^Z) but can be classified as transverse to Y (TE^Y, TM^Y). Since the $TM^Y{}_{10}$ mode is the same as the $TE^Z{}_{10}$ mode in the absence of absorber, and the incident mode on the absorbing section of waveguide will be $TE^Z{}_{10}$, we will consider TM^Y modes which are even in Y only. TM^Y modes can be derived from a y-directed magnetic vector potential:

$$\vec{A}(x,y,z) = \hat{y}A_y(x,y)e^{-\gamma z} \tag{35}$$

The magnetic field is given as:

$$\vec{H} = \frac{1}{\mu}\vec{\nabla}\times\vec{A} \tag{36}$$

The electric field is given as:

$$\vec{E} = \frac{1}{j\omega\mu\varepsilon}\vec{\nabla}\times\vec{H} \tag{37}$$

From Equations 35-37, magnetic field is:

$$\vec{H} = \left(\frac{\gamma}{\mu}A_y\hat{x} + 0\hat{y} + \frac{1}{\mu}\frac{\partial A_y}{\partial x}\hat{z}\right)e^{-\gamma z} \tag{38}$$

The electric field is:

$$\vec{E} = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2 A_y}{\partial x\partial y}\hat{x} - \left(\frac{\partial^2 A_y}{\partial x^2} + \gamma^2 A_y\right)\hat{y} - \gamma\frac{\partial F_y}{\partial y}\hat{z}\right)e^{-\gamma z} \tag{39}$$

To meet the boundary conditions in Region I of Figure 2, the magnetic vector potential must be of the form:

$$A_y(x,y) = A_1\cos\left(\frac{m\pi}{a}x\right)\cos(a_1 y) \tag{40}$$

The magnetic field in Region I is:

$$H_x(x,y) = \frac{\gamma}{\mu_1} A_1 \cos\left(\frac{m\pi}{a}x\right)\cos(a_1 y) \tag{41}$$

$$H_y(x,y) = 0 \tag{42}$$

$$H_z(x,y) = -\frac{1}{\mu_1}\left(\frac{m\pi}{a}\right) A_1 \sin\left(\frac{m\pi}{a}x\right)\cos(a_1 y) \tag{43}$$

The electric field in Region I is:

$$E_x(x,y) = \frac{1}{j\omega\mu_1\varepsilon_1} a_1 \frac{m\pi}{a} A_1 \sin\left(\frac{m\pi}{a}x\right)\sin(a_1 y) \tag{44}$$

$$E_y(x,y) = -\frac{1}{j\omega\mu_1\varepsilon_1}\left(\gamma^2 - \left(\frac{m\pi}{a}\right)^2\right) A_1 \cos\left(\frac{m\pi}{a}x\right)\cos(a_1 y) \tag{45}$$

$$E_z(x,y) = \frac{\gamma}{j\omega\mu_1\varepsilon_1} a_1 A_1 \cos\left(\frac{m\pi}{a}x\right)\sin(a_1 y) \tag{46}$$

To meet the boundary conditions in Region II of Figure 2, the magnetic vector potential must be of the form:

$$A_y(x,y) = A_2 \cos\left(\frac{m\pi}{a}x\right)\cos\left(a_2\left(y - \frac{b}{2} - \delta\right)\right) \tag{47}$$

The magnetic field in Region II is:

$$H_x(x,y) = \frac{\gamma}{\mu_2} A_2 \cos\left(\frac{m\pi}{a}x\right)\cos\left(a_2\left(y - \frac{b}{2} - \delta\right)\right) \tag{48}$$

$$H_y(x,y) = 0 \tag{49}$$

$$H_z(x,y) = -\frac{1}{\mu_2}\left(\frac{m\pi}{a}\right) A_2 \sin\left(\frac{m\pi}{a}x\right)\cos\left(a_2\left(y - \frac{b}{2} - \delta\right)\right) \tag{50}$$

The electric field in Region II is:

$$E_y(x,y) = -\frac{1}{j\omega\mu_2\varepsilon_2}\left(\gamma^2 - \left(\frac{m\pi}{a}\right)^2\right) A_2 \cos\left(\frac{m\pi}{a}x\right)\cos\left(a_2\left(y - \frac{b}{2} - \delta\right)\right) \tag{51}$$

$$E_x(x,y) = \frac{1}{j\omega\mu_2\varepsilon_2} a_2 \frac{m\pi}{a} A_2 \sin\left(\frac{m\pi}{a}x\right)\sin\left(a_2\left(y - \frac{b}{2} - \delta\right)\right) \tag{52}$$

$$E_z(x,y)=\frac{\gamma}{j\omega\mu_2\varepsilon_2}a_2A_2\cos\left(\frac{m\pi}{a}x\right)\sin\left(a_2\left(y-\frac{b}{2}-\delta\right)\right) \tag{53}$$

At the interface of y=b/2 H_x, H_z must be continuous:

$$\frac{1}{\mu_1}A_1\cos\left(a_1\frac{b}{2}\right)=\frac{1}{\mu_2}A_2\cos(f_2\delta) \tag{54}$$

At the interface of y=b/2 E_x, E_z must be continuous:

$$\frac{a_1}{\mu_1\varepsilon_1}A_1\sin\left(a_1\frac{b}{2}\right)=-\frac{a_2}{\mu_2\varepsilon_2}A_2\cos(a_2\delta) \tag{55}$$

The solution to the Helmholtz wave equation requires:

$$a_2{}^2=a_2{}^2+\omega^2\mu_1\varepsilon_1\left(\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}-1\right)=0 \tag{56}$$

Dividing Equation 55 by Equation 54 results in:

$$\frac{a_1}{a_2}\frac{\varepsilon_2}{\varepsilon_1}\tan\left(a_1\frac{b}{2}\right)\cot(a_2\delta)=-1 \tag{57}$$

The attenuation constant is:

$$\alpha=\left|\mathrm{IM}\left(\sqrt{\omega^2\mu_1\varepsilon_1-\left(\frac{m\pi}{a}\right)^2-a_1{}^2}\right)\right| \tag{58}$$

POWER LOSS IN THE TOP WALL ABSORBER

This section deals with the relative power loss in the absorber. This information can be used to determine where most of the heat will be generated in the absorber. We will consider only the fundamental mode propagating in the waveguide ($TM^Y{}_{10}$). The power density (power per unit volume) that is lost in the absorber is:
Because we are only interested in the relative power loss let

$$\left|\frac{\gamma A_2}{\mu_2}\right|^2=\frac{2}{\omega\mu_1} \tag{59}$$

Also define:

$$u=y-\frac{b}{2} \tag{60}$$

$$\kappa_0=\omega\sqrt{\mu_1\varepsilon_1} \tag{61}$$

Then the power loss is given as:

$$
\begin{aligned}
P_{loss}(x,u) = & IM(\mu_r)\left(\cos\left(\frac{\pi x}{a}\right)\right)^2 \left|\cos(a_2(u-\delta))\right|^2 \\
& + IM(\mu_r)\left(\sin\left(\frac{\pi x}{a}\right)\right)^2 \left|\frac{1}{\gamma}\frac{\pi}{a}\cos(a_2(u-\delta))\right|^2 \\
& + \frac{IM(\varepsilon_r)}{|\varepsilon_r|^2}\left(\sin\left(\frac{\pi x}{a}\right)\right)^2 \left|\frac{1}{\gamma}\frac{\pi}{a}\frac{a_2}{\kappa_0}\sin(a_2(u-\delta))\right|^2 \\
& + \frac{IM(\varepsilon_r)}{|\varepsilon_r|^2}\left(\cos\left(\frac{\pi x}{a}\right)\right)^2 \left|\frac{1}{\gamma\kappa_0}\left(\gamma^2-\left(\frac{\pi}{a}\right)^2\right)\cos(a_2(u-\delta))\right|^2 \\
& + \frac{IM(\varepsilon_r)}{|\varepsilon_r|^2}\left(\cos\left(\frac{\pi x}{a}\right)\right)^2 \left|\frac{a_2}{\kappa_0}\sin(a_2(u-\delta))\right|^2
\end{aligned}
\qquad (62)
$$

where it has been assumed that ε_1 and μ_1 have only real parts (no loss.)

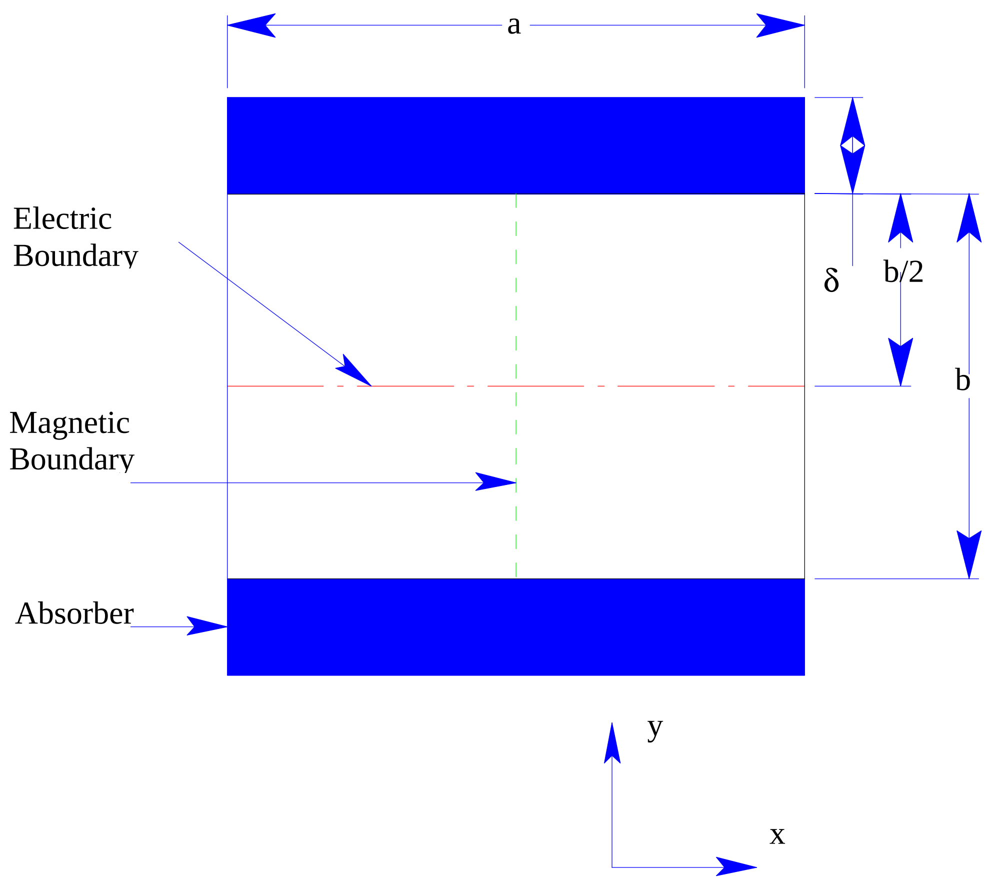

Figure 2. Geometry for absorber on the top and bottom of the waveguide.