## Checking the Beam Energy Calculation from the June 14, $2000 \psi^{\prime}$ Scan

No magnetic field change for most of the scan
$\Rightarrow$ Three checks of the beam energy calculation can be made:

1) The change in beam momentum ( $\Delta p$ ) of each point relative to any other can be calculated from the change in the revolution frequency of the beam $\left(f_{\text {rev }}\right)$ :

$$
\begin{aligned}
& \frac{\Delta p}{p}=-\frac{1}{\eta} \frac{\Delta f_{\text {rev }}}{f_{\text {rev }}} \\
& \text { where } \eta=\frac{1}{\gamma_{t}^{2}}-\frac{1}{\gamma^{2}}
\end{aligned}
$$

2) From the $\Delta p$ calculated in 1) the relative change in orbit length ( $\Delta L$ ) can be calculated:

$$
\frac{\Delta L}{L}=\frac{1}{\gamma_{t}^{2}} \frac{\Delta p}{p}
$$

3) The radial movement of the beam on the high dispersion BPMs can be calculated:

$$
\Delta x=D \frac{\Delta p}{p}
$$

where $D$ is the dispersion function at the BPMs.


## The Reference Run:

The reference for these calculations is Run 5827.
This means two things:

1) The changes in the various quantities of interest in this talk are always relative to Run 5827.
Example: $\quad \Delta p=p-p_{5827}$
2) The length assigned to the BPM orbit for Run 5827 was chosen to make $M_{\psi^{\prime}}=3686.000 \mathrm{MeV} / c^{2}$

## The Checks:

Check \#1: Checking the beam energy

- Recall:

$$
\frac{\Delta p}{p}=-\frac{1}{\eta} \frac{\Delta f_{r e v}}{f_{r e v}}
$$

- Requires a knowledge of $\eta$. $\eta=0.0216 \pm 0.0022$ at the $\psi$ '. The $\sim 10 \%$ uncertainty in $\eta$ gives rise to a $10 \%$ uncertainty in the change in $E_{c m}(\sim 20 \mathrm{keV}$ for this scan).
- $\eta$ was measured during ramp developement (Fall '99). A recent (April 2000) measurement by Giulio Stancari verified the earlier measurements.
- Notation: $E_{B P M}$ will denote the center of mass energy measured in the usual way (using the BPMs and the orbit length calculation).
- $\quad \boldsymbol{E}_{\eta}$ will denote the center of mass energy determined from $\eta$ and $\Delta f_{\text {rev }} . E_{\eta}$ is given by:

$$
E_{\eta}=E_{5827}-\left(\frac{d E_{c m}}{d p}\right) \frac{p}{\eta} \frac{\Delta f_{R F}}{f_{R F}}
$$

$f_{R F}$ is $1 / 2 \times$ the frequency of the RF modulation on the beam (i.e. it is the revolution frequency of the beam detected by the BPMs).
$E_{c m}$ is related to $p$ (the beam momentum in lab frame) by:

$$
\begin{aligned}
\frac{d E_{c m}}{d p} & =\frac{\beta m_{p}}{E_{c m}} \\
& =0.252 \text { at the } \psi
\end{aligned}
$$

The table below compares $E_{\eta}$ and $E_{B P M} . \Delta \mathrm{E}_{\text {err }}=\mathrm{E}_{\eta}-\mathrm{E}_{B P M}$

| Run | $\boldsymbol{E}_{\eta}$ <br> $(\mathrm{MeV})$ | $\boldsymbol{E}_{\text {BPM }}$ <br> $(\mathrm{MeV})$ | $\Delta \boldsymbol{E}_{\text {err }}$ <br> $(\mathrm{keV})$ |
| :---: | :---: | :---: | ---: |
| 5818 | 3686.607 | 3686.781 | -174.00 |
| 5819 | 3686.621 | 3686.810 | -189.00 |
| 5821 | 3686.378 | 3686.497 | -119.00 |
| 5822 | 3686.380 | 3686.510 | -130.00 |
| 5824 | 3686.142 | 3686.191 | -49.00 |
| 5825 | 3686.148 | 3686.198 | -50.00 |
| 5827 | 3685.960 | 3685.960 | 0.00 |
| 5828 | 3685.960 | 3685.961 | -1.00 |
| 5830 | 3685.721 | 3685.645 | 76.00 |
| 5831 | 3685.721 | 3685.654 | 67.00 |

The error in the determination of $E_{c m}$ from the BPMs ( $=\Delta \mathrm{E}_{\text {err }}$ ) depends linearly on the $\Delta p / p$ relative to the reference point in the scan. The error is $44.5 \mathrm{keV} / 10^{-4}$.


A note about $\Delta p / p$ : the orbit length calculation outputs its own estimate of $\Delta p / p$. This estimate will virtually always be wrong. The orbit length calculation gets this wrong because it can't distinguish between a $\Delta p / p$ and a $\Delta B / B$ error - (i.e. a bend bus error).

- This error has an enormous impact on the $\psi^{\prime}$ width measurement.
When $E_{\eta} \quad$ is used, $\Gamma_{\psi^{\prime}}=345.5 \mathrm{keV}$.
When $E_{B P M}$ is used, $\Gamma_{\psi^{\prime}}=647.3 \mathrm{keV}$.

Check \#2: Checking the orbit length calculation.

- Recall:

$$
\frac{\Delta L}{L}=\frac{1}{\gamma_{t}^{2}} \frac{\Delta p}{p}
$$

This check requires a knowledge of $\gamma_{t}$. The value of $\gamma_{t}$ indicates the transition energy $\left(E_{t}\right)$ of the accelerator via $E_{t}=\gamma_{t} m_{p} c^{2}$.

- $\quad \gamma_{t}$ changes with energy on the deceleration ramps in a way that keeps the energies of interest to E835 above transition.
At the $\psi^{\prime}, \gamma_{\mathrm{t}}=4.778 \pm 0.005$.
- $\quad \gamma_{t}$ is determined from the measurements of $\eta$. Recall:

$$
\eta=\frac{1}{\gamma_{t}^{2}}-\frac{1}{\gamma^{2}}
$$

- The orbit length is an essential ingredient of the beam energy calculation. The beam energy is derived from a measurement of the velocity of the beam via $v=f_{\text {rev }} L$. Ordinarily $L$ is calculated from a fit to the orbit of the beam as measured by the Beam Position Monitoring system (BPMs).
- For this check, the orbit length is calculated by:

$$
L_{\eta}=L_{5827}\left(1+\frac{1}{\gamma_{t}^{2}} \frac{\Delta p}{p}\right)
$$

- Again, there is a linear relationship between the error in the orbit length calculation $\left(\Delta L_{e r r}\right)$ and $\Delta p / p$. The error plotted here is $\Delta L_{e r r}=L_{\eta}-L_{B P M}$.

- The error in orbit length is $0.3 \mathrm{~mm} / 10^{-4}$. At the $\psi^{\prime}$, a change in orbit length of 1 mm corresponds to a 149 keV change in the center of mass energy.
- In terms of center of mass energy the orbit length error is $44.5 \mathrm{keV} / 10^{-4}$. This is the same error determined in the first check.

Check \#3: Checking the BPM measurement of the radial movement of the beam.

- Recall:

$$
\Delta x=D \frac{\Delta p}{p}
$$

- This check requires a knowledge of the dispersion at the BPMs
- The only measurements of the dispersion function involve the use of the BPMs. Therefore this check depends on a lattice model of the Accumulator for its dispersion values.

Dispersion function at the BPMs from Acculator lattice model at the $\psi^{\prime}$


## $\psi^{\prime}$ Scan Orbit Differences Relative to Run 5827 - Measured and Calculated -




- For the high dispersion BPMs the "error" in $\Delta x$ is proportional to $\Delta \mathrm{x}$
If one believes the lattice model, a correction factor can calculated for the high dispersion BPMs. For example:


Applying these BPM corrections and re-doing the $\psi^{\prime}$ maximum liklihood fit for the width gives:
$\Gamma_{\psi^{\prime}}=416.4 \mathrm{keV}$.

## June 14, 2000 ' Scan BPM Corrections Applied



- The width is still too large (but not by a factor of 2 as before).
- The BPM corrections were also applied to the last three runs of the scan. These runs were not part of the constant field part of the scan. Ramping the magnets to get to this point of the scan keeps the beam on the central orbit, which also generally means near the reference orbit for the beam energy calculation.
The $E_{c m}$ of each of these runs was increased by only 4 or 5 keV .
As expected, the BPM corrections have little effect if the orbit is close to the reference orbit.
- The Mass of the $\psi^{\prime}$ did not change so the reference orbit obtained from this scan is probably valid.


## The Garzoglio paradox: Why do we get the right $\psi^{\prime}$ width when the beam energies are calculated with "Quad Steering" off?

- The normal proceedure is to turn Quad Steering ON. Accounting for quad steering in the beam energy calculation is a way to accomodate differences in the lattice between the point in the deceleration ramps where the reference orbit was measured and the point where you are trying to measure the beam energy. (See Pbar Note 633 for the details)
- At the $\psi^{\prime}$ both calculations should give the same result since the reference orbit is measured at the $\psi^{\prime}$.
- When quad steering is OFF the orbit length calculation fits the BPM orbit to a superposition of kicks from all of the dipole elements in the Accumulator. This is an under determined problem since there are more BPMs than dipoles. Therefore, in general, the modeled orbit doesn't exactly match the BPM measurements.
- When quad steering is ON the orbit length calculation tries to determine kicks from all of the dipoles plus all of the quadrupoles. This problem is greatly over determined. In this case the modeled orbit always fits the BPM orbit exactly (unless one does something silly with the SVD threshold).
- The Quad Steering ON calculation will readily turn any errors in the BPM readouts into kicks that aren't really there. However, the Quad Steering OFF calculation does not have the degrees of freedom to do serious damage to the orbit model for small BPM errors.


## Conclusions / Recommendations:

1) It is very important to keep the orbit close to the reference orbit.
2) It is likely that BPMs are not perfectly calibrated.
3) The orbit length calculation with Quad Steering ON is more sensitive to errors in the BPM readout than with Quad Steering OFF. However, unless we are at the $\psi^{\prime}$, Quad Steering should be ON.
4) Question: Should we use the BPM corrections derived from this scan?

Answer: I don't know. I would prefer not to. If we keep the orbit close to the reference, we don't need the corrections. For cases where the orbit differs appreciably from the reference orbit, we should do the energy calculation both ways. (Perhaps with Quad steering ON and OFF too).
5) We should use the reference orbit derived from this scan. However, if there is the time and the man power, it would be desireable to do a proper scan of the $\psi^{\prime}$.

