PBAR Note 669

RF CURVES FOR EXTRACTION FROM THE ACCUMULATOR

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INTRODUCTION

Since the start of Run IIa, the RF curves for the extraction process from the Accumulator have been based on an algorithm described in Pbar Note 636. There are a number of problems with this procedure that result in a dilution of the longitudinal phase space of the extracted beam.

The procedure consists of a number of steps in which the frequency curve during each process is a linear time ramp. For a constant bend field, the synchronous phase angle is given as:

$$\Gamma = \sin(\phi_s) = -\frac{h}{\eta}\frac{\left(\frac{1}{f_{rf}}\right)^2 \frac{df_{rf}}{dt}}{\frac{qV}{pc}} \qquad (1)$$

where h is the harmonic number of the RF. Equation (1) shows that if the frequency curve consists of a number of linear time ramps with different slopes, there will be discontinuities in the synchronous phase. These discontinuities in the synchronous phase will lead to dipole oscillations of the beam in the RF bucket. The discontinuities observed for the present RF curves are about 10 degrees.

In the procedure outlined in Pbar Note 636, the RF bucket is formed on the high energy edge of the rectangular momentum distribution. As the RF bucket is pulled away from the core, it is also programmed to increase in area. If the distribution is not perfectly rectangular, or if the bucket is not formed at the edge of the distribution, the growing bucket will gather up more particles at the edges of the bucket resulting in a substantial increase of longitudinal emittance.

Finally, it is fairly difficult to prepare a rectangular momentum distribution and keep it rectangular for extended periods of time. Once the rectangular distribution is prepared, the core momentum cooling must be turned off. If there is a delay in the extraction process, the sharp edges of the rectangular distribution will soon diffuse. With the momentum cooling disabled, the longitudinal emittance of the core will grow resulting in larger longitudinal emittances for the extracted beam.

THE FREQUENCY RAMP

The first change to the extraction process is to replace the linear piecewise continuous frequency ramp with a ramp in which the first derivative is also continuous. The extraction process consists of a number of steps. The frequency ramps defined below will be given in units of the revolution frequency (h=1)

A. *Growing the bucket in the core.* ($0 < t < t_1$)

At this point the frequency ramp is constant:

$$f = f_1 \qquad (2)$$

The frequency is determined by the energy the beam is to be captured. The user specifies the high-energy edge (pc_L) of the momentum slice to be captured. The revolution frequency corresponding to this edge is:

$$f_L = -\eta \frac{pc_L - pc_o}{pc_o} f_o + f_o \tag{3}$$

The center frequency of this slice is given by the amount of longitudinal phase space to be captured hA_c.

$$f_1 = f_L + \frac{hA_c}{2} |\eta| \frac{f_o^{\,2}}{pc_o} \tag{4}$$

B. *Slowly removing the bucket from the core.* ($t_1 < t < t_2$)
Since the first derivative of the ramp in Step A is zero, this first derivative of this curve must also be zero at $t = t_1$.

$$f = f_1 + (f_2 - f_1)\frac{(t - t_1)^2}{(t_2 - t_1)^2} \tag{5}$$

C. *Bringing the beam to the extraction orbit.* ($t_2 < t < t_3$)
The initial value and slope of this curve must match the final value and slope of the curve in Step B. The final value of this curve must be the extraction frequency and the slope when the beam reaches the extraction frequency must be zero. Because of these four boundary conditions, this curve must be at least a third order polynomial.

$$f = f_2 + a_1 \frac{(t - t_2)}{(t_3 - t_2)} + a_2 \frac{(t - t_2)^2}{(t_3 - t_2)^2} + a_3 \frac{(t - t_2)^3}{(t_3 - t_2)^3} \tag{6}$$

where:

$$a_1 = 2(f_2 - f_1)\frac{t_3 - t_2}{t_2 - t_1} \tag{7}$$

$$a_2 = 3(f_3 - f_2) - 4(f_2 - f_1)\frac{t_3 - t_2}{t_2 - t_1} \tag{8}$$

$$a_3 = -2(f_3 - f_2) + 2(f_2 - f_1)\frac{t_3 - t_2}{t_2 - t_1} \tag{9}$$

D. *Preparing for Main Injector phase lock and growing the extraction bucket area.* ($t_3 < t$)

$$f = f_3 \tag{10}$$

THE VOLTAGE RAMPS

The voltage ramps are broken into a number of steps:

A.	Growing the capturing bucket	($0 < t < t_1$)
B.	Moving the bucket away from the core.	($t_1 < t < t_2$)
C.	Increasing the bucket area.	($t_2 < t < t_{2p}$)
D.	Moving to the extraction orbit with a constant bucket area	($t_{2p} < t < t_3$)
E.	Growing the bucket to the final bucket area.	($t_4 < t < t_5$)

In most of these steps, the bucket area will be specified and the voltage must be calculated. The bucket area is a function of the voltage and the synchronous phase angle.

$$A = A_s(V)\alpha(\Gamma) \tag{11}$$

where A_s is the stationary bucket area:

$$A_s = A_o\sqrt{qV} \tag{12}$$

where:

$$A_o = \frac{8}{\pi f_{rf}}\sqrt{\frac{pc}{2\pi h|\eta|}} \tag{13}$$

The synchronous phase angle can be written as:

$$\Gamma = \Gamma_o \frac{1}{qV} \tag{14}$$

where

$$\Gamma_o = -\frac{h}{\eta} pc \left(\frac{1}{f_{rf}}\right)^2 \frac{df_{rf}}{dt} \tag{15}$$

The moving bucket factor $\alpha(\Gamma)$ has to be calculated empirically. Reasonable polynomial fits can be obtained for this factor. However, if the moving bucket factor is expressed as:

$$\alpha(\Gamma) \approx 1 - k_1\sqrt{\Gamma} - k_2\Gamma \tag{16}$$

where

$$\begin{aligned} k_1 &= 0.24 \\ k_2 &= 1.137 \end{aligned} \tag{17}$$

then the voltage can be explicitly expressed as a function of the bucket area:

$$qV = \left[\left(\frac{k_1\sqrt{\Gamma_o}}{2} + \frac{A}{2A_o}\right) + \sqrt{\left(\frac{k_1\sqrt{\Gamma_o}}{2} + \frac{A}{2A_o}\right)^2 + k_2\Gamma_o}\right]^2 \tag{18}$$

Also, the bucket height is a function of the voltage and the synchronous phase angle.

$$\Delta f_{rf} = |\eta| f_{rf} \frac{\Delta pc}{pc} = H_o\sqrt{qV}\beta(\Gamma) \tag{19}$$

where:

$$H_o = \frac{\pi|\eta|}{4pc} f_{rf}{}^2 A_o \tag{20}$$

and $\beta(\Gamma)$ is the ratio between the moving bucket height to the stationary bucket height. The factor $\beta(\Gamma)$ also has to be calculated empirically. However, if $\beta(\Gamma)$ is expressed as:

$$\beta(\Gamma) \approx 1 - k_4\Gamma \tag{21}$$

where

$$k_4 = 0.801 \tag{22}$$

then the voltage can be explicitly expressed as a function of the bucket area:

$$qV = \left[\frac{\Delta f_{rf}}{2H_o} + \sqrt{\left(\frac{\Delta f_{rf}}{2H_o}\right)^2 + k_4\Gamma_o}\right]^2 \tag{23}$$

Step A. *Growing the capturing bucket* ($0 < t < t_1$)

At this point, the bucket is stationary (Γ=0) and the bucket area is grown linearly from zero to A_c.

$$A = A_c \frac{t}{t_1} \tag{24}$$

$$qV = \left(\frac{A_c}{A_o} \frac{t}{t_1} \right)^2 \tag{25}$$

Step B. *Moving the bucket away from the core* ($t_1 < t < t_2$)

The bucket is moved away from the core with a constant bucket area A_c until the low energy edge of the moving bucket is at the user-defined momentum (pc_U) at time t_2. The revolution frequency corresponding to this edge is:

$$f_U = -\eta \frac{pc_U - pc_o}{pc_o} f_o + f_o \tag{26}$$

At time t_2 the frequency f_2 must be:

$$f_2 = f_U - \frac{\Delta f_{rf}}{h} \tag{27}$$

but the bucket height Δf_{rf} at t_2 is also a function of f_2. The frequency f_2 can be determined by an iterative procedure:

i.) Set $f_2 = f_1$

ii.) Calculate the voltage knowing A_c, the RF frequency, and the derivative of the RF frequency.

iii.) Calculate the bucket height Δf_{rf}

iv.) Set the new value of f_2:

$$f_2\big|_{new} = f_U - \frac{\Delta f_{rf}\left(f_2\big|_{old}\right)}{h} \tag{28}$$

v.) go back to step ii and use the new value of f_2.

Once f_2 is determined, the voltage during this time interval is given by Eqn. (18) where $A = A_c$.

Step C. *Increasing the bucket area.* ($t_2 < t < t_{2p}$)

Up to this point, the time t_2 at which the frequency reaches f_2 is arbitrary. The constraint that this algorithm will use is to require the synchronous phase angle at t_2 to equal the synchronous phase angle at the time when the RF frequency is changing at the greatest rate. This time is determined when the second derivative of Equation 6 is equal to zero:

$$t_{2p} = t_2 + \frac{1}{3} \frac{\left(3(f_3 - f_2)(t_2 - t_1) - 4(f_2 - f_1)(t_3 - t_2)\right)}{\left(2(f_3 - f_2)(t_2 - t_1) - 2(f_2 - f_1)(t_3 - t_2)\right)} (t_3 - t_2) \tag{29}$$

At this time t_{2p}, the bucket area is given by the capture bucket area A_c multiplied by a user-defined multiplier M_c. A reasonable number for this multiplier is two. Since t_{2p} is a function of t_2, t_2 must be solved using an iterative procedure:

i.) Set $t_2 = t_1+\delta$
ii.) Calculate f_2 using the procedure outlined in Step. B
iii.) Determine the voltage at t_2 for a bucket area A_c
iv.) Determine the synchronous phase (Γ_{t2}) at t_2 using the voltage from iii.
v.) Determine the voltage at t_{2p} for a bucket area A_cM_c
vi.) Determine the synchronous phase (Γ_{t2p}) at t_{2p} using the voltage from Step v.
vii.) Calculate a new value of t_2

$$t_2\Big|_{new} = \left(1 - 2\frac{\Gamma_{t_{2p}} - \Gamma_{t_2}}{\Gamma_{t_{2p}} + \Gamma_{t_2}}\right) t_2\Big|_{old} \tag{30}$$

vi.) Go back to step ii.

Once t_2 and f_2 have been determined, the voltage is given as:

$$qV = \frac{\Gamma_o}{\Gamma_{t2}} \tag{31}$$

Step D. *Moving to the extraction orbit with a constant bucket area* ($t_{2p} < t < t_3$)

Now that f_2 and t_2 have been determined in Steps B and C, the voltage during this time interval is given by Eqn. (18) where $A = A_cM_c$.

Step E. *Growing the bucket to the final bucket area*. ($t_4 < t < t_5$)

The RF frequency has stopped changing at this time and the bucket area will be increased linearly from A_cM_c to A_h.

$$qV = \left(\frac{A_cM_c + (A_h - A_cM_c)\frac{t - t_4}{t_5 - t_4}}{A_o}\right)^2 \tag{32}$$

EXAMPLE CURVES

Below are some example curves using the following parameters

$$t_1 = 1.5 \text{ s}$$
$$t_3 = 18 \text{ s}$$
$$t_4 = 18.1 \text{ s}$$
$$t_5 = 22 \text{ s}$$
$$h = 0.012$$
$$pc = 8801 \text{ MeV}$$
$$f_L = 628{,}884 \text{ Hz}$$
$$f_U = 628{,}875 \text{ Hz}$$
$$f_3 = 628{,}765 \text{ Hz}$$
$$A_c = 0.457 \text{ eV-sec}$$
$$A_cM_c = 2.0$$
$$A_h = A_c \times 5.57 = 2.527 \text{ eV-sec}$$

The following are calculated from the above parameters:

$$f_1 = 628{,}883.5 \text{ Hz}$$
$$f_2 = 628{,}873.9 \text{ Hz}$$
$$t_2 = 6.29 \text{ s}$$
$$t_{2p} = 12.07 \text{ s}$$

The above parameters generate the following curves:

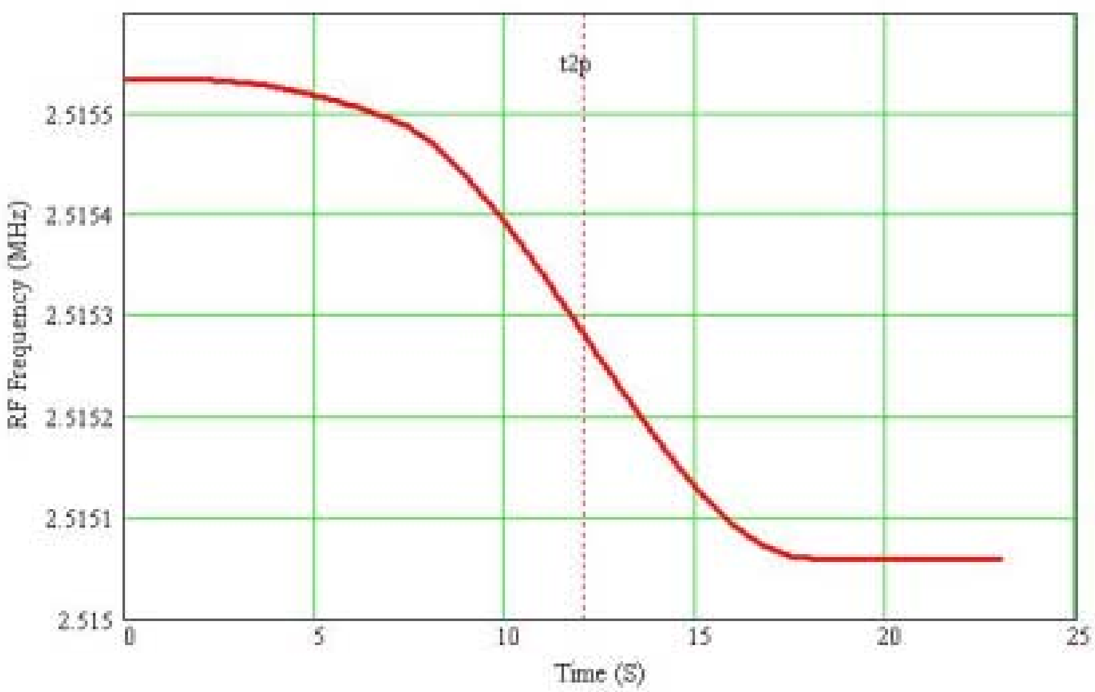

Figure 1. RF frequency Curve

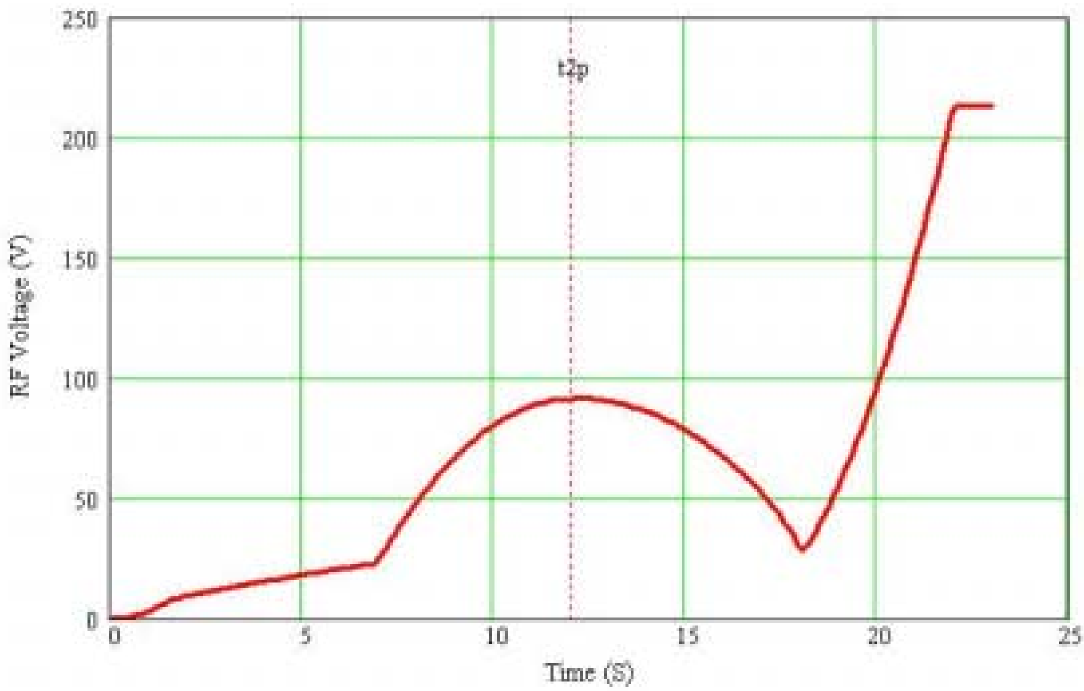

Figure 2. RF voltage curve

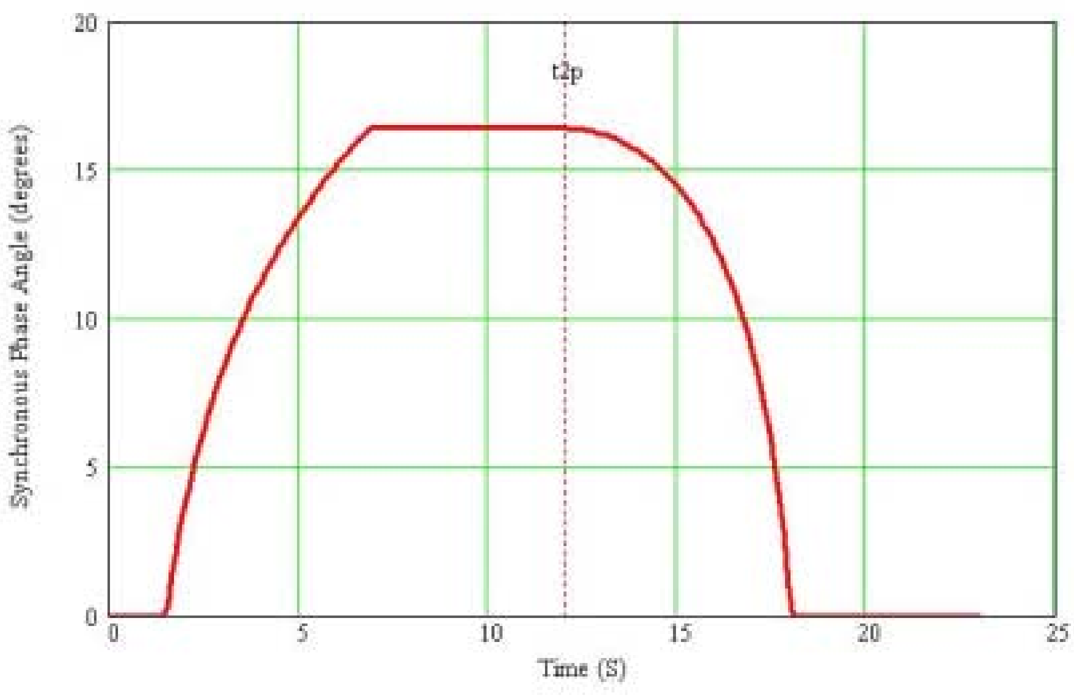

Figure 3. The synchronous phase vs. time

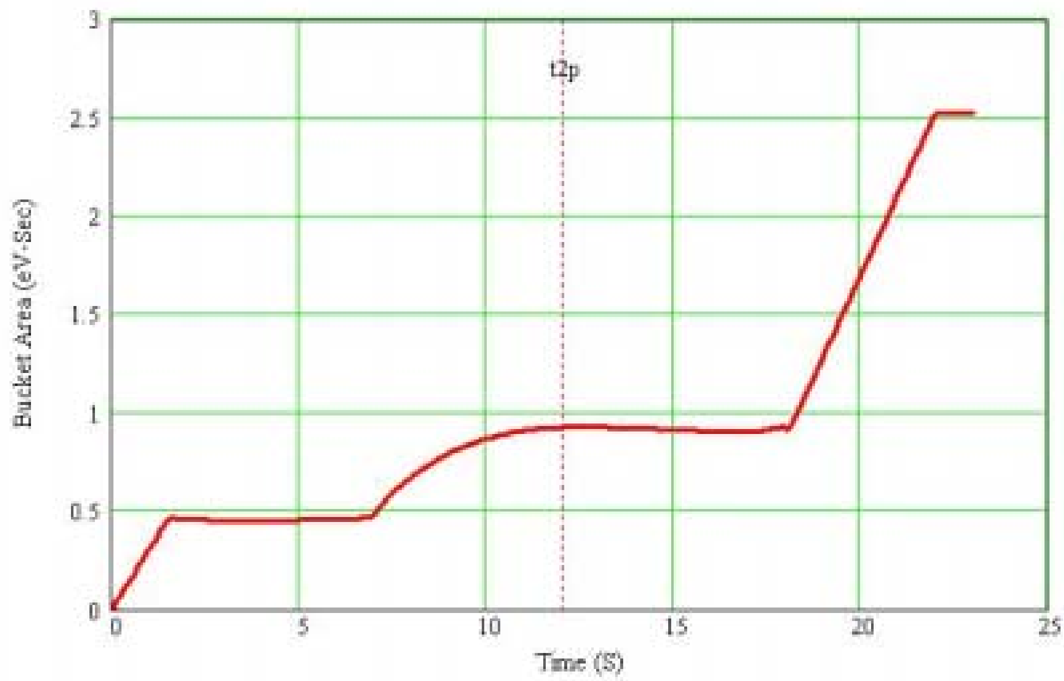

Figure 4. The h=4 bucket area as a function of time. The droop in the bucket area between t_{2p} and t_3 is due to the fit of the moving bucket parameter described in Eqn. 16

TRACKING SIMULATION

The RF curves shown in Figure 1 and Figure 2 were used in a particle tracking simulation of the RF extraction process. Figure 5 shows the initial phase space distribution. The momentum distribution is centered on a momentum of 8801 MeV with a revolution frequency of 628,886 Hz. The initial momentum spread is 13.9 Mev (95%). The horizontal axis is in units of RF phase and the vertical axis is in units of MeV. The green trace is the projection of the particle distribution on the momentum axis. Figure 6 shows the distribution 10 seconds into the simulation. The number of particles left in the core is 90.8%. The momentum spread of the core has shrunk from 13.9 MeV to 12.9 MeV. Figure 7 shows the distribution of the extracted beam on the extraction orbit at t=22.5 seconds. The bucket area at this time is 2.527 eV-sec. The blue trace in Figure 7 is the projection on the time axis. The total time spread of the beam is 116 degrees (h=4). The total momentum spread is about 5 MeV, which gives a longitudinal emittance of about 0.5 eV-sec.

Figure 8 shows the evolution of the momentum, profile during the first 7 seconds of the extraction process. Once the RF bucket is pulled away from the core, the core shrinks in size from 13.9 MeV to 12.9 MeV (7.2%). If there was no dilution of the longitudinal emittance of the core, the momentum spread should have been reduced to 12.6 MeV (9.2%). Figure 9 shows the evolution of the time profile during the entire extraction process.

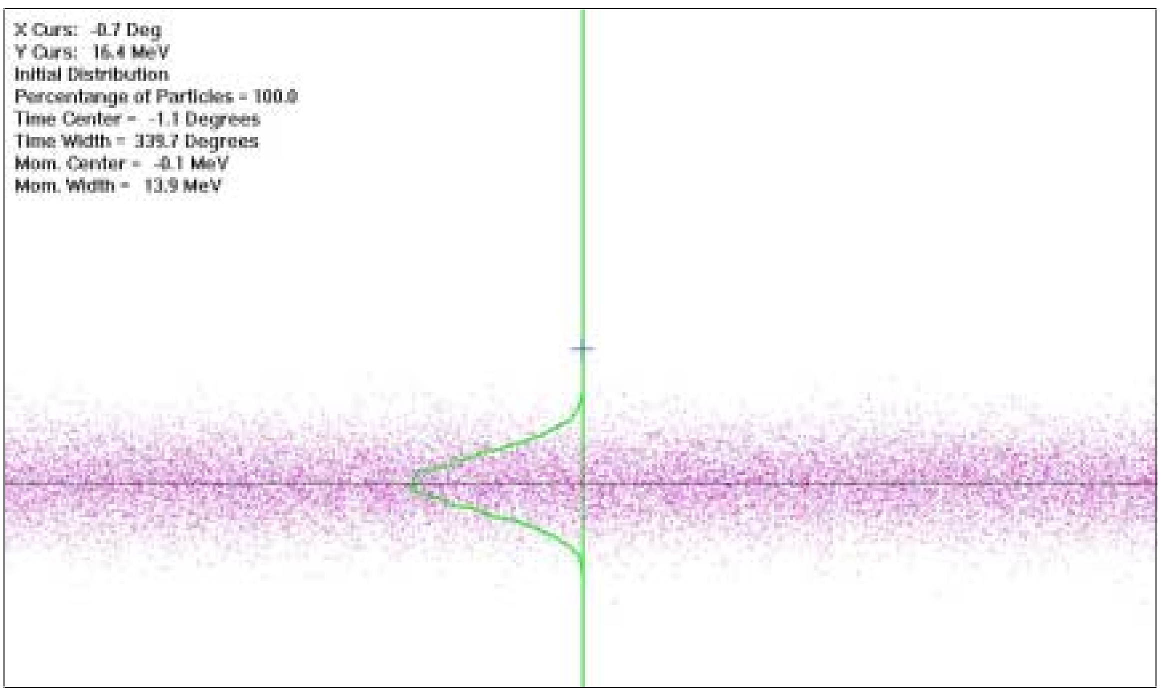

Figure 5. Initial phase space distribution for tracking simulation.

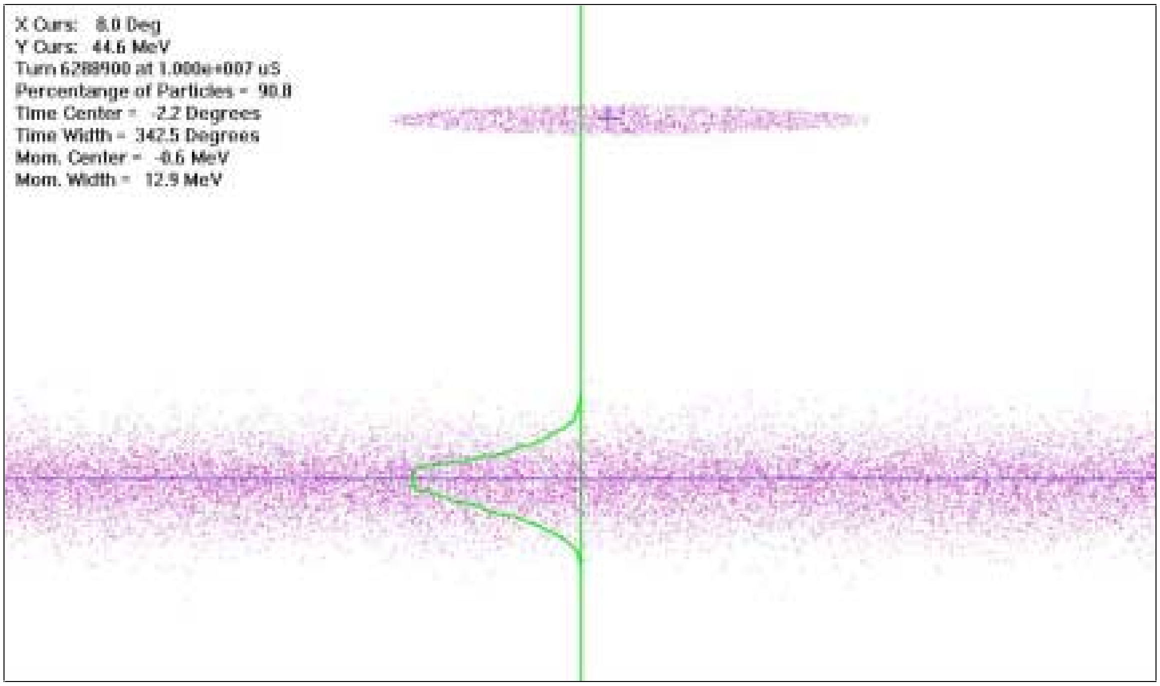

Figure 6. The particle distribution 10 seconds into the simulation.

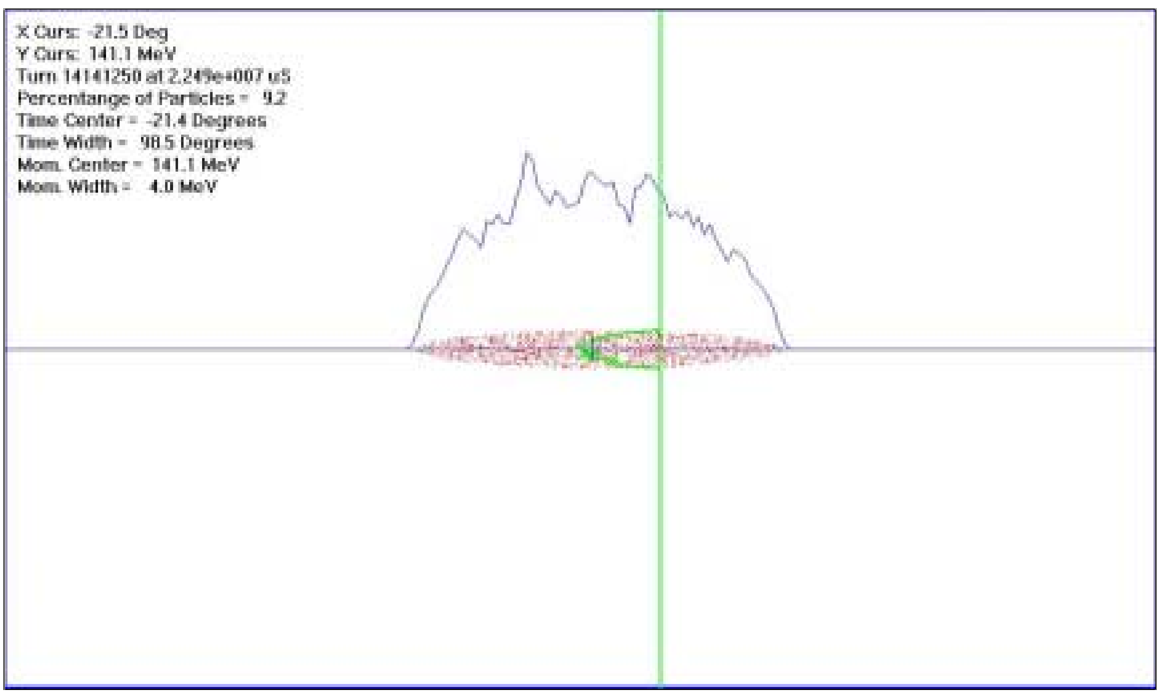

Figure 7. The particle distribution of the extracted beam on the extraction orbit.

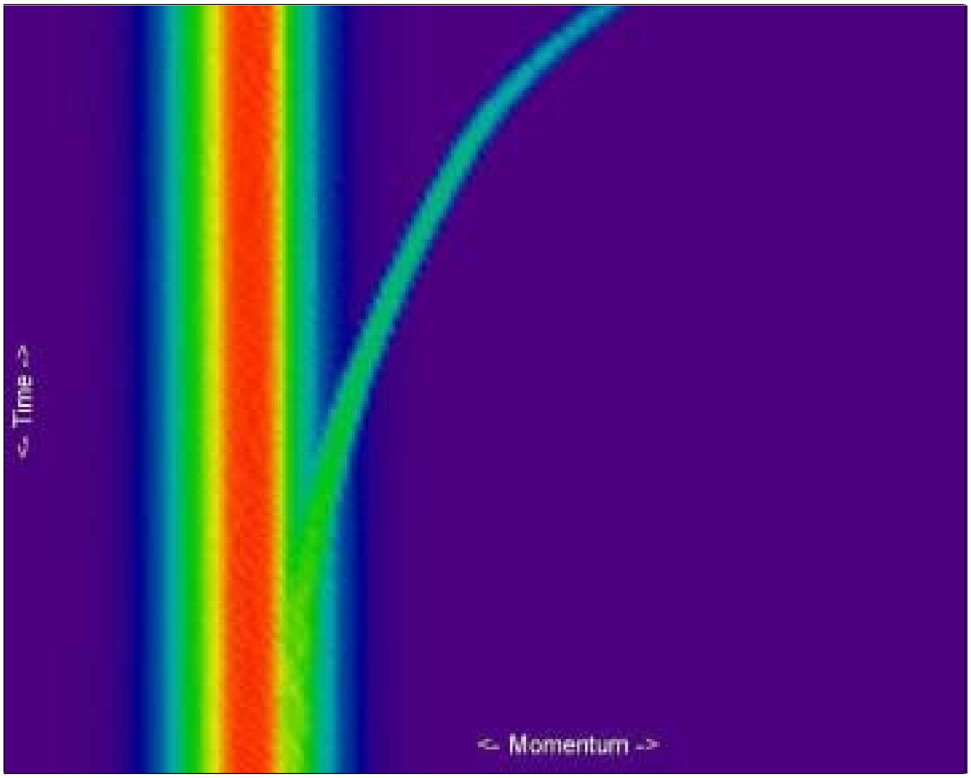

Figure 8. Evolution of the momentum profile during the first 7 seconds of the simulation.

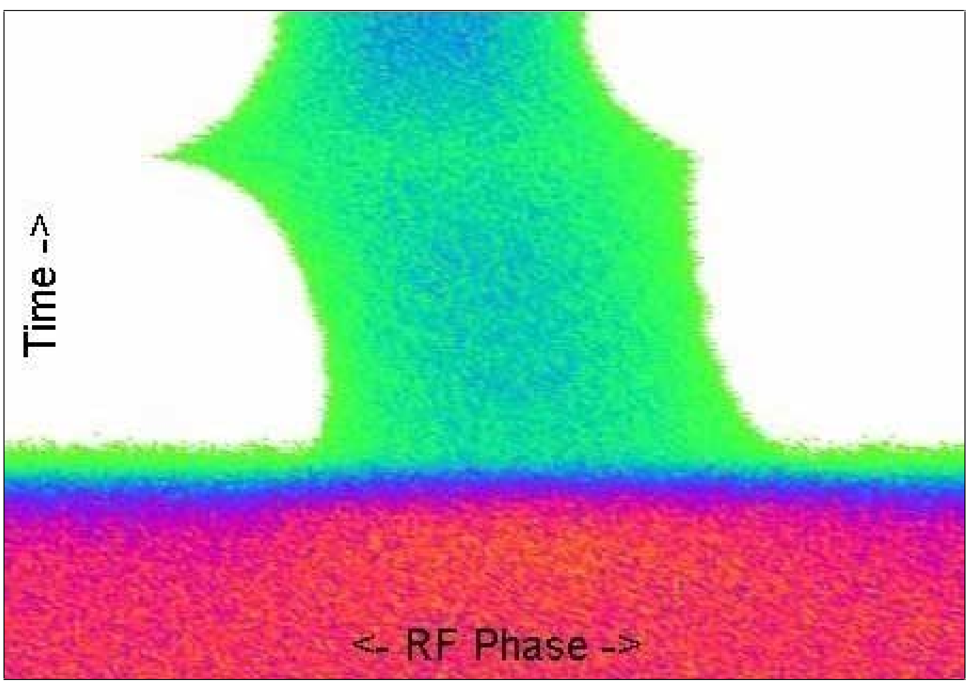

Figure 9. Evolution of time profile during the entire extraction process.

DETERMINING THE FREQUENCY MARKERS FOR EXTRACTION

In Step A of the Frequency Ramp generation, the high-energy edge of the momentum slice to be captured (pc_L) needs to be specified. To provide a small longitudinal emittance, it is desirable to grow the extraction bucket where the density of particles in the core is at a maximum. The phase space density is usually a maximum at the center of the core. However, while the extraction bucket is moving out of the core, it will phase displace the high-energy portion of the core. Since this phase displacement will result in some longitudinal dilution, it would also be desirable to start the extraction from the high-energy side of the core.

We will assume that the particle density as a function of energy is roughly trapezoid in shape. We propose that the location of the high-energy edge of the momentum slice to be captured (pc_L) should be the high-energy "corner" of the trapezoid. On the high-energy side of the distribution, the revolution frequency in which the derivative of the particle density as a function of revolution frequency is a maximum will be designated f_{rs}. The particle density at this location is ψ_s. A line going through this location with the same derivative as the particle distribution will be:

$$\psi = \left.\frac{d\psi}{df_r}\right|_{f_{rs}} (f_r - f_{rs}) + \psi_s \tag{33}$$

The location of the revolution frequency f_L will be where this line intersects a horizontal line that goes through the maximum particle density ψ_m.

$$f_L = \frac{\psi_m - \psi_s}{\left.\frac{d\psi}{df_r}\right|_{f_{rs}}} + f_{rs} \tag{34}$$

An estimate of where the particle density vanishes on the high-energy edge of the distribution would be:

$$f_{rY} = f_{rs} - \frac{\psi_s}{\left.\frac{d\psi}{df_r}\right|_{f_{rs}}} \tag{35}$$

A reasonable choice for f_U (the minimum low energy edge of the moving bucket before the bucket area can be increased) would be to have the distance from f_U to the center of the distribution, f_{ro}, be twice the distance between f_{rY} and f_{ro}.

$$f_{ro} - f_U = 2(f_{ro} - f_{rY}) \tag{36}$$

$$f_U = 2f_{rs} - 2\frac{\psi_s}{\left.\frac{d\psi}{df_r}\right|_{f_{rs}}} - f_{ro} \tag{37}$$

Figure 10 shows a gaussian density distribution for the core. The solid magenta line is the line described in Eqn. 33. The red dashed line frequency marker is the location of f_L. The blue dashed line frequency marker is the location of f_U and the brown marker is the location of f_{rY}. Figure 11 shows an actual longitudinal schottky spectrum with the extraction frequency markers. The red marker is the location of f_L. The blue marker is the location of f_U and the yellow marker is the location of f_{rY}

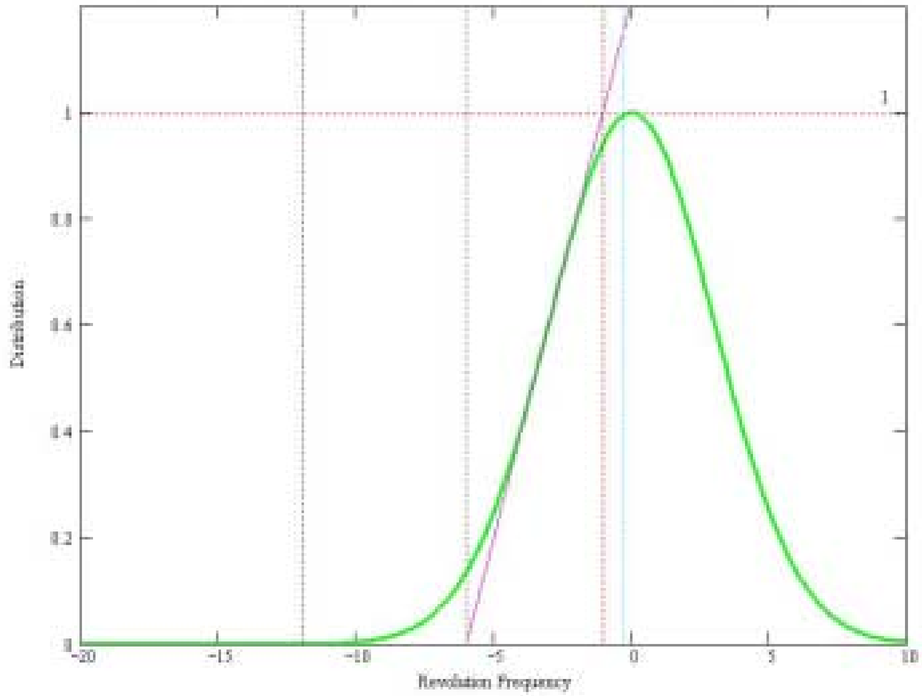

Figure 10. Core particle density distribution function with extraction frequency markers.

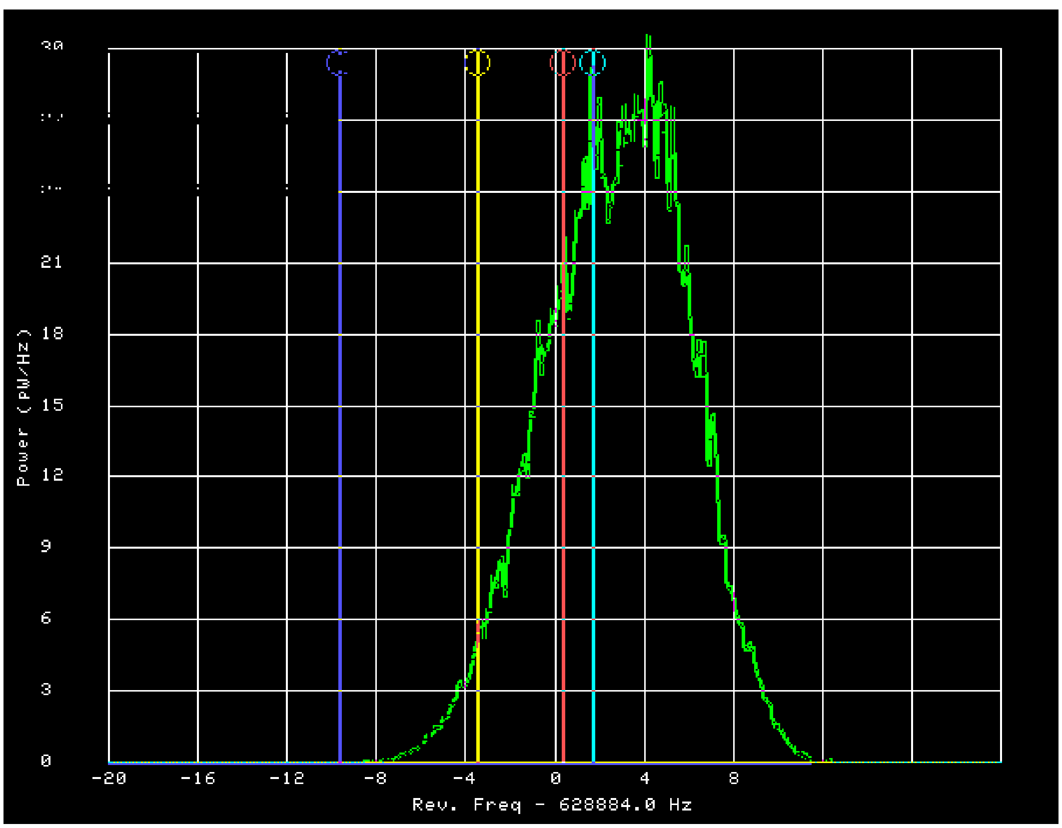

Figure 11. Actual core longitudinal schottky spectrum with extraction frequency markers.