

Electron Cooling for the Antiproton Accumulator

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Abstract

A possibility to utilize electron cooling in the Fermilab Antiproton Accumulator (AA) is studied. It is shown that, for the existing stochastic cooling system and 0.5 Ampere \times 10 m of electron cooler, the accumulated antiproton current could be increased up to 0.5-1.0 Ampere with required or lower longitudinal and transverse emittances.

I. INTRODUCTION

Efficiency of the antiproton stochastic cooling (SC) in the Accumulator is currently limited by several beam intensity factors.

- Back Diffusion. Antiproton accumulation at given (optimal) longitudinal gain and limited energy aperture leads to increase of a back flux of the accumulated antiprotons due to a diffusion, intrinsic for the longitudinal SC. As a result, accumulation above a certain threshold is possible only at a price of the flux reduction.
- Intrabeam Scattering. When the number of accumulated antiprotons increases, their mutual Coulomb scattering gets more and more significant. This mainly affects the beam horizontal emittance, which grows with the beam current due to the IBS.
- Longitudinal Instability. Longitudinal SC provides a special mechanism for the beam to feel its own coherent fluctuations. This means that a product of the gain and the beam current is limited by the stability requirement. Thus, the gain (and the flux) must be reduced when the current exceeds a certain threshold.

All the listed factors limit a performance of the antiproton cooling-stacking in the Accumulator, starting from 50 – 100 mA of the accumulated particles.

These limits could be significantly relaxed if the electron cooling (EC) were applied. Indeed, sufficient longitudinal EC would allow to switch off the longitudinal SC of the beam core and thus to nullify the back diffusion. The transverse EC is also mostly efficient for the core particles, where it could successfully counteract to IBS. What relates to the longitudinal instability, one might note that electron cooling would make beam even more unstable due to its lack of intrinsic diffusion and thus a tendency for overcooling. To avoid this obstacle, an additional longitudinal 'noiser' could be applied to the beam core, to keep its energy width at optimal level.

Due to inefficiency of EC for 'hot' particles, injected antiprotons have to be significantly pre-cooled (longitudinally and transversely) by SC, making EC being able to get them. Thus, electron and stochastic cooling have to work in tandem, being complimentary to each other.

Below, it is shown that combination of the existing SC system [1] with the same EC as assumed for the Recycler [2] would be able to hold up to 0.5 – 1 A of the antiproton current

in the Accumulator at normalized r. m. s. emittance 1.6 mm mrad or lower.

II. TRANSVERSE STOCHASTIC COOLING

The Courant-Snyder parameters (normalized actions)

$$J = \frac{\gamma\beta}{2\beta_x} \left[x^2 + (\alpha_x x + \beta_x x')^2 \right] \quad (1)$$

are damped by SC with rates

$$\lambda_{st} = \frac{4W_t}{N_p M} \frac{gM}{1 + gM/2}, \quad (2)$$

where W_t is the cooling band width, N_p is the number of particles, and g is the transverse gain. The mixing factor M describes transverse decoherence due to the revolution frequency spread (see e. g. [3]):

$$M \approx \frac{1}{3.3W_t T_0 \eta \sigma_\delta}. \quad (3)$$

with T_0 as the revolution time, $\eta = 1/\gamma_t^2 - 1/\gamma^2$ as the slippage factor, $\delta = \Delta p/p$ as the relative momentum deviation and σ_δ as its r. m. s. value. The numerical factor in the denominator of Eq. (3) is sensitive to details of the cooling scheme, its value here has been found from comparison with experimental results at the Accumulator (see below in more details).

Mutual influence of the cooled particles gives rise to diffusion in SC. The diffusion coefficient D_{st} defined in terms of the emittance growth through

$$\left. \frac{d\varepsilon}{dt} \right|_{\text{diff}} = D_{st} \varepsilon \quad (4)$$

can be written as

$$D_{st} = \frac{2W_t}{N_p M} \left(\frac{gM}{1 + gM/2} \right)^2. \quad (5)$$

Altogether, the transverse stochastic cooling plus diffusion lead to the action evolution with the total action-dependent rate:

$$\Lambda_{st}(J) \equiv \frac{1}{J} \frac{dJ}{dt} = -\frac{4W_t}{N_p M} \frac{gM}{1 + gM/2} \left(1 - \frac{\varepsilon}{2J} \frac{gM}{1 + gM/2} \right), \quad (6)$$

where emittance definition as the beam-averaged action, $\varepsilon \equiv \bar{J}$, has been applied.

Making average from Eq. (6) gives the emittance evolution as

$$\frac{1}{\varepsilon} \frac{d\varepsilon}{dt} = -\frac{4W_t}{N_p M} \frac{gM}{1 + gM/2} \left(1 - \frac{gM/2}{1 + gM/2} \right). \quad (7)$$

TABLE I: Main parameters of the Accumulator

Energy	E_p	8.9 MeV
Circumference	C	474 m
Slippage factor	$\eta \equiv 1/\gamma_t^2 - 1/\gamma^2$	0.12
Transverse SC band width	W_t	3.5 GHz
Longitudinal SC band width	W_l	2 GHz

Emittance damping rate (7) is maximized when the gain takes its optimal value, $g = g_o$,

$$g_o = 2/M, \quad (8)$$

in which case

$$\frac{1}{\varepsilon} \frac{d\varepsilon}{dt} = -\frac{2W_t}{N_p M}. \quad (9)$$

Comparison of this rate with one experimentally achieved at the Accumulator allowed to specify the number in the mixing factor definition (3). To be safe from the transverse coherent instability associated with too strong interaction via SC system, the gain g should not be much higher than its optimal value (8).

If initial action of a particle J_i significantly exceeds the core emittance ε , the diffusion term can be neglected, and the cooling time τ_{st} to a final action J_f be calculated with the action-independent rate (2) only:

$$\tau_{st} = \lambda_{st}^{-1} \ln(J_i/J_f). \quad (10)$$

Main parameters of the FNAL Antiproton Accumulator and its SC are listed in the Table I. Assuming number of antiprotons $N_p = 5 \cdot 10^{12}$, energy width $\sigma_\delta = 1 \cdot 10^{-3}$, initial action $J_i = 30$ mm mrad (acceptance), final action $J_f = 3.5$ mm mrad, and the gain at its optimal value ($=0.45$ in this case), it gives $\tau_{st} = 50$ min. Note that setting the specific final action for SC time calculation does not mean anything for the core emittance. The action J_f is "final" only in a sense of stochastic pre-cooling, while the core emittance is mainly determined by interplay of cooling, misalignments and IBS.

III. LONGITUDINAL STOCHASTIC COOLING AND STACKING

Longitudinal stochastic cooling and stacking in the Accumulator is provided according to Van der Meer's scheme with exponential longitudinal gain function [4], see also e. g. [5]. The total available energy width of the FNAL Accumulator ΔE can be presented as shared by the accumulated core occupying $\Delta E_c \approx 4\sigma_\delta E_p$ and tail particles taking the rest of it $\Delta E_t \approx \Delta E - 4\sigma_\delta E_p$. At injection side of the stack, the required stochastic acceleration \dot{E}_i is determined by the total energy width of the injected batch from the Debuncher ΔE_D and the repetition time τ_{rep} :

$$\dot{E}_i = \Delta E_D / \tau_{\text{rep}}. \quad (11)$$

The repetition time τ_{rep} is determined by the stack-tail flux ϕ and the Debuncher's batch population N_D ,

$$\tau_{\text{rep}} = N_D / \phi, \quad (12)$$

while the flux is determined by the longitudinal SC band width W_l and the e-fold energy of the longitudinal gain \mathcal{E}_d :

$$\phi = 0.7 \frac{\mathcal{E}_d W_l^2 T_0 |\eta|}{E_p \ln(W_{\text{max}}/W_{\text{min}})}. \quad (13)$$

Here T_0 is the revolution time, W_{max} and W_{min} are the stack-tail band boundaries, $W_l = W_{\text{max}} - W_{\text{min}}$, and the numerical factor (0.7) is empirical.

Injected particles are cooled inside the tail for time

$$\tau_{sl} = \frac{\mathcal{E}_d}{\dot{E}_i} \exp(\Delta E_t / \mathcal{E}_d). \quad (14)$$

After that, their transverse actions have to be as small as necessary for a longitudinal electron cooling to drag these particles in the core. In other words, the longitudinal pre-cooling time τ_{sl} may not be smaller than the transverse pre-cooling time τ_{st} , Eq. (10), and this sets the upper limit for the e-fold energy interval \mathcal{E}_d . From other side, this interval determines the maximal flux ϕ (13), and so should be taken as high as possible. Therefore, the e-fold interval \mathcal{E}_d is determined from the longitudinal-transverse matching condition $\tau_{sl} = \tau_{st}$, or

$$\mathcal{E}_d = \Delta E_t / \ln(\tau_{st} \dot{E}_i / \mathcal{E}_d). \quad (15)$$

Taking these all together, accepting $W_{\text{max}}/W_{\text{min}} = 2$, $N_D = 1.2 \cdot 10^8$ pbars (corresponds to 12 μA of the batch current in the Accumulator), $\Delta E_D = 8$ MeV, and mentioned above $\sigma_\delta = 0.001$, the parameters are found as $\mathcal{E}_d = 6.6$ MeV, $\phi = 22$ mA/hour, $\dot{E}_i = 4$ MeV/s.

The drag force exponentially drops from the injection to the core side of the tail stack, so that pbars spend most of their pre-cooling time at the last e-fold interval. The drag force at the core side of the tail is

$$\dot{E}_f = \mathcal{E}_d / \tau_{st}, \quad (16)$$

which results in $\dot{E}_f = 9$ MeV/hour. To avoid back diffusion, longitudinal electron cooling has to be strong enough to give approximately equal drag force at this tail-core boundary.

IV. LONGITUDINAL ELECTRON COOLING

Longitudinal EC rate is calculated as

$$\lambda_{el} \equiv -\frac{1}{\Delta p/p} \frac{d\Delta p/p}{dt} = \frac{4\pi(I_e/e)r_e r_p \eta_c L_C}{\beta\gamma^2} \left\langle \frac{n_e(r)}{u^3} \right\rangle. \quad (17)$$

Here I_e is electron current r_e, r_p are electron and proton classical radii, η_c is a portion of the antiproton orbit occupied by the electron beam, β, γ are the relativistic factors, u is the total pbar velocity in the beam frame, electron velocities in the beam frame are assumed to be negligible, L_C is the EC Coulomb logarithm, and the brackets $\langle \dots \rangle$ stay for averaging over the antiproton betatron oscillations. Electron current is DC, with the transverse profile described by a 2D density $n_e(r)$ normalized by $\int n_e(r) d^2r = 1$. Below, a rectangular electron profile is assumed with $n_e(r) = \theta(a_e - r) / (\pi a_e^2)$ with $\theta(a_e - r)$ as the Heaviside step function.

For a given electron current, both longitudinal and transverse cooling rates are maximized if the electron beam radius is equal to the maximal offset of the cooled particle. Assuming that after transverse stochastic pre-cooling all the antiprotons are inside the electron beam, and relative pbar-electron velocities are dominated by the antiproton velocities, the longitudinal EC rate can be approximately presented as in Ref. [6]:

$$\lambda_{el} = \frac{8(I_e/e)r_e r_p \eta_c L_C}{\pi\beta^2\gamma^2 a_e^2 v_z \sqrt{(v_x^2 + 2v_z^2/\pi)(v_y^2 + 2v_z^2/\pi)}}, \quad (18)$$

where $v_z = \beta\Delta p/p$ is the longitudinal pbar velocity in the beam frame, and $v_{x,y}$ are the transverse velocity amplitudes in the beam frame, $u_x = v_x \cos \psi_x$; all the velocities are taken in units of c . Inaccuracy of this approximation is 10% or better. The transverse amplitudes $v_{x,y}$ can be expressed in terms of the normalized actions $J_{x,y}$ and the beta-function in the cooler β_c as $v_{x,y} = \sqrt{2\gamma\beta J_{x,y}/\beta_c}$.

Let it be assumed that after transverse stochastic pre-cooling all the pbars are inside the surface

$$J_x + J_y \leq J_{\max}. \quad (19)$$

In terms of the pbar amplitudes $a_{x,y} = \sqrt{2J_{x,y}\beta_c/(\beta\gamma)}$ it can be expressed as

$$a_x^2 + a_y^2 \leq 2J_{\max}\beta_c/(\beta\gamma). \quad (20)$$

Thus, in this case the covering electron beam has to be of the radius

$$a_e = \sqrt{2J_{\max}\beta_c/(\beta\gamma)}. \quad (21)$$

According to results of Section II, it would take 50 min to cool a pbar transversely from the acceptance boundary to 3.5 mm mrad of the normalized action. Thus, after this time all the antiprotons would be inside the surface with $J_{\max} = 7$ mm mrad, which corresponds to electron beam radius $a_e = 5.5$ mm.

The EC drag force $F_e = \lambda_{el}\Delta p$ for the boundary antiprotons with $J_{x,y} = 3.5$ mm mrad, $\Delta p/p = 2\sigma_\delta = 2 \cdot 10^{-3}$ has to be not smaller than the SC drag force (16) calculated in the Section III as 9 MeV/h. Assuming $I_e = 0.5$ A, $a_e = 5.5$ mm, the EC rate (18) is found as $\lambda_{el} = 1.1$ h⁻¹ which corresponds to the drag force $F_e = 20$ MeV/h, which exceeds twice the SC force here, showing that the back core-tail diffusion should not happen.

Dependence of the EC drag force on the momentum offset $\Delta p/p$ is presented in Fig. 1 for various actions. It can be concluded that the force is almost constant on the surface $J_x + J_y = J_{\max}$.

To suppress coherent instabilities, energy width of the accumulated beam core has to be controlled. However, EC by itself does not introduce any significant diffusion, so the core could be easily overcooled. To avoid this, a controlled source of pbar core diffusion has to be installed together with the electron cooler. This longitudinal 'noiser' is discussed in the Section VII.

V. TRANSVERSE ELECTRON COOLING

Main purpose of the transverse EC is to counteract intrabeam scattering (IBS) of pbars blowing up their horizontal emittance. Transverse EC rate for the horizontal direction is

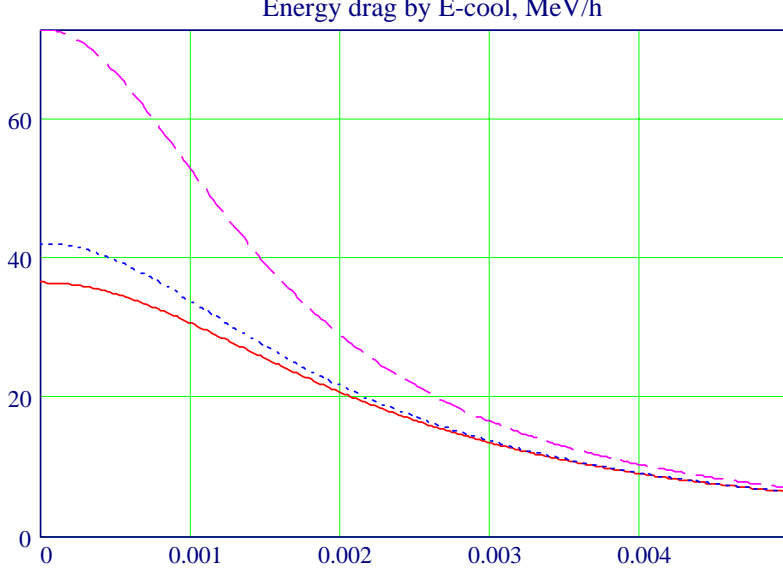


FIG. 1: EC drag force as a function of the relative momentum offset $\Delta p/p$ for $J_x = J_y = 3.5$ mm mrad (red solid line), $J_x = 1.75, J_y = 5.25$ mm mrad (blue dot line), $J_x = J_y = 1.75$ mm mrad (magenta dash line).

defined and can be calculated as

$$\lambda_{ex} \equiv -\frac{1}{J_x} \frac{dJ_x}{dt} = -\frac{2}{\gamma v_x^2} \langle F_x u_x \rangle = \frac{4(I_e/e)r_e r_p \eta_c L_C}{\beta \gamma^2 a_e^2} \left\langle \frac{1}{u^3} \frac{2u_x^2}{v_x^2} \right\rangle, \quad (22)$$

where F_x is the horizontal EC force, while all the other notations are described in the previous section.

Sufficiently accurate approximation for the transverse EC rate follows from

$$\left\langle \frac{1}{u^3} \frac{2u_x^2}{v_x^2} \right\rangle \approx \frac{1}{(v_x^2 + v_z^2)^{3/2}} g(q) \quad (23)$$

with

$$g(q) = q \frac{q \ln q + q + 1}{q^2 + 1}, \quad q = \sqrt{\frac{v_x^2 + v_z^2}{v_y^2 + v_z^2}},$$

and accuracy 10% or better.

Due to EC, the beam emittances shrink with rate

$$\overline{\lambda}_{ex} \equiv -\frac{1}{\varepsilon_x} \frac{d\varepsilon_x}{dt} = \overline{J_x \lambda_{ex}} / \varepsilon_x, \quad (24)$$

where over-lining stands for the beam average. Assuming Gaussian distribution with $\sigma_\delta = 0.001$ and r.m.s. normalized emittances $\varepsilon_x = \varepsilon_y = 1.7$ mm mrad, it comes out $1/\overline{\lambda}_{ex} = 9$ min.

The transverse distribution reaches an equilibrium when the cooling is balanced by the IBS heating.

VI. INTRABEAM SCATTERING

For an uncoupled beam, IBS mainly leads to a horizontal diffusion, the vertical IBS diffusion is very small if any. It means that vertical emittance shrinking would not stopped. As a result, space density of the cooled beam would increase, IBS diffusion in the horizontal plane amplified, and more and more electron cooling would be needed to hold the beam within the given horizontal emittance.

A possible way to overcome this obstacle is to introduce coupling between transverse planes. Taking into account that the normal working point of the Accumulator is almost at the coupling resonance ($\{\nu_x - \nu_y\} = 0.002$), relatively weak skew quad or solenoid would be sufficient to do this job. Proper coupling would equally share sum of the transverse IBS rates between the two degrees of freedom. Thus, the resulting transverse IBS growth rate $\bar{\lambda}_p$ can be expressed in terms of the uncoupled transverse IBS rates $\bar{\lambda}_{px}, \bar{\lambda}_{py}$ as

$$\bar{\lambda}_p \equiv \frac{1}{\sqrt{\varepsilon_{4D}}} \frac{d\sqrt{\varepsilon_{4D}}}{dt} = \frac{\bar{\lambda}_{px} + \bar{\lambda}_{py}}{2}. \quad (25)$$

where ε_{4D} is the 4D beam r.m.s. emittance; for uncoupled case $\varepsilon_{4D} = \varepsilon_x \varepsilon_y$.

The uncoupled IBS rates $\bar{\lambda}_{px}, \bar{\lambda}_{py}$ can be calculated with A. Piwinski formulae [7], modified to include the dispersion and envelope derivatives, as it was recently suggested by K. Bane [8]. It leads to

$$\bar{\lambda}_p = \frac{A}{2} \left\langle f(1/a, b/a) + f(1/b, a/b) + \frac{\gamma \mathcal{H}_x \sigma_H^2}{\varepsilon_x} f(a, b) \right\rangle \quad (26)$$

with

$$A = \frac{N_p r_p^2 c}{16\pi^{3/2} \gamma^2 \varepsilon_x \varepsilon_y \sigma_\delta C} \quad (27)$$

$$\mathcal{H}_x = \frac{D_x^2 + (\beta_x D'_x + \alpha_x D_x)^2}{\beta_x}, \quad \frac{1}{\sigma_H^2} = \frac{1}{\sigma_\delta^2} + \frac{\gamma \mathcal{H}_x}{\varepsilon_x}, \quad a = \sigma_H \sqrt{\frac{\beta_x}{\gamma \varepsilon_x}}, \quad b = \sigma_H \sqrt{\frac{\beta_y}{\gamma \varepsilon_y}}, \quad (28)$$

$$f(a, b) = 8\pi L_{IBS} \int_0^1 du \frac{1 - 3u^2}{P(a, u)P(b, u)}, \quad P(a, u) = \sqrt{a^2 + (1 - a^2)u^2}$$

Here β_x, α_x are the Courant-Snyder optical functions, D_x, D'_x is the dispersion with its derivative, $L_{IBS} = \ln(\sigma_y v_y^2 / r_p) \approx 20$ is the IBS Coulomb logarithm, and the brackets in Eq. (26) stay for the orbit averaging.

Substituting the parameters of the Antiproton Accumulator for the beam with $N_p = 5 \cdot 10^{12}$, $\sigma_\delta = 0.001$, $\varepsilon_x = \varepsilon_y = 1.6$ mm mrad leads to $1/\bar{\lambda}_p = 40$ min, which is about 4

times slower than EC time $1/\bar{\lambda}_{ex} = 9$ min found in the previous section. So it may be concluded that for the accepted electron current $I_e = 0.5$ A, IBS allows either to reduce the core emittance to ≈ 1.0 mm mrad (r. m. s., normalized), or to increase the number of antiprotons up to $2 \cdot 10^{13}$.

VII. CONTROLLED CORE ENERGY DIFFUSION

There are two factors forcing to avoid longitudinal overcooling of the beam core. First, when the energy spread is too narrow, the Landau damping on the revolution frequency spread is too small and the beam becomes unstable. And second, with the energy narrowing, the transverse stochastic cooling gets inefficient (the transverse SC rate goes as $\propto 1/M \propto \sigma_\delta$). From another side, the core energy spread has to be small enough to leave a sufficient portion of the longitudinal phase space to the beam stack. Thus, the core energy spread has to be kept at an optimal level, which approximately is taken here as $\sigma_\delta = 0.001$. The question is how to regulate this spread? Due to insignificance of the stochastic cooling and diffusion for the core particles, the beam has a tendency to get overcooled longitudinally. To avoid this, a source of controlled diffusion is required.

To avoid interference with the SC, a frequency band of this 'noiser' has to stay well below the SC band. Also, it has to be narrow-band, with the width determined by the core frequency spread, to avoid drag reduction outside the core. Assuming that its frequency ω_d corresponds to a harmonic number n_d , $\omega_d = n_d\omega_0$, this gives for its band width $\delta\omega_d = n_d\Delta\omega_0$ with $\Delta\omega_0 = \eta\sigma_\delta\omega_0$ as the core spread of the revolution frequencies. From here, the required merit factor of the noiser is $Q_d = \omega_d/\delta\omega_d = 1/(\eta\sigma_\delta) \approx 1 \cdot 10^5$.

This core diffusion can be presented in terms of the random force $F(t)$ changing particle energy as

$$\dot{E} = F(t) + \text{cooling terms} . \quad (29)$$

Note that the force F depends also on the particle energy, which is described by a resonance curve with a width $\sigma_\delta E_p$. This random force gives rise to a diffusion coefficient

$$D_E = \frac{d}{dt} \int_0^t \int_0^t dt dt' \langle F(t)F(t') \rangle \simeq \frac{F_0^2}{2\delta\omega_d} , \quad (30)$$

with F_0 as the force amplitude. To provide a given energy width $\sigma_\delta E_p$, the diffusion has to

be taken as

$$D_E = 2\lambda_{el}(\sigma_\delta E_p)^2, \quad (31)$$

where λ_{el} is the longitudinal (electron) cooling rate. This leads to the required force amplitude

$$F_0 = 2\sigma_\delta E_p \sqrt{\lambda_{el} \delta\omega_d}. \quad (32)$$

To see how much could it be, the noiser frequency can be assumed as $f_d = 2\pi\omega_d = 70$ MHz, which results in $F_0 = 2$ eV/turn.

One more issue here is that the noiser may not build any significant coherent perturbation. Taking into account that the mode number n_d is well inside the Keil-Schnell circle, its coherent damping time is estimated as $\simeq (n_d \Delta\omega_0)^{-1} = \delta\omega_d^{-1}$, i. e. it is same as the self-correlation time of the noiser.

For this time, the noiser builds an amplitude of the coherent energy modulation $\Delta E_c = F_0/\delta\omega_d$, which is associated with the relative current perturbation

$$\delta I_p/I_p = (\Delta E_c/E_p)\eta\omega_0/\delta\omega_d,$$

resulting after all the substitutions in

$$\delta I_p/I_p = 2\sqrt{\lambda_{el}/\delta\omega_d}$$

which gives as small number as $\delta I_p/I_p = 6 \cdot 10^{-4}$.

Being also well below the SC band, this very low coherent signal cannot interfere with the SC system.

Note that the noiser application does not mean energy widening of the extracted pbars: before extraction, when the accumulation is finished, both stack-tail stochastic cooling and the noiser could be switched off, and the energy width be reduced with the stochastic core cooling (if no EC), or with EC, if it is applied.

VIII. LONGITUDINAL INSTABILITY DRIVEN BY STOCHASTIC COOLING

Conventionally, the longitudinal distribution function is presented as a sum of a main stationary term and a small time-dependent perturbation:

$$\psi(E, \theta, t) = \psi_0(E) + \psi_l(E) \exp(i l \theta - i l \omega_0 t - i \Omega_l t) \equiv \psi_0 + \tilde{\psi}. \quad (33)$$

with Ω_l as a coherent frequency shift and E as the energy offset. The distribution perturbation drives the energy change through the longitudinal gain function $G_l(E)$:

$$\dot{E} = \int dE' \tilde{\psi}(E') G_l(E'). \quad (34)$$

Substituted into the Vlasov equation, this allows to find the the coherent frequency shift Ω_l :

$$1 = -i \int \frac{dE \psi'_0(E) G_l(E)}{\Omega_l - l\omega'_0 E + i0}, \quad (35)$$

where the prime ' stands for a derivative over energy E , $\omega'_0 = -\eta\omega_0/E_p$.

If for any energy $\psi'_0(E) \text{Re}G_l(E) \geq 0$, then the mode l is coherently damped by the gain G_l , i.e. $\text{Im}\Omega_l < 0$. Technically, however, this condition of gain phasing is hard to be satisfied for the whole frequency band [9]; usually, for some frequencies it is violated.

The threshold intensity N_{th} determined with Eq. (35) scales as the energy spread in some power,

$$N_{th} \propto \sigma_\delta^q. \quad (36)$$

This power q is determined by energy dependence of the partial gain $G_l(E)$ and can be expected as $1 \leq q \leq 2$. Taking into account that the partial gain $G_l(E)$ scales in the same way as the stack-tail drag at injection, \dot{E}_i , the stability condition can be expressed as

$$\dot{E}_i N \leq (\dot{E}_i N)_{th}. \quad (37)$$

This means that above a certain threshold, the accumulated current can be increased only by means of the flux $\phi = \dot{E}_i N_D / \delta E_D$ suppression, where N_D is the number of particles injected from Debuncher inside the energy width δE_D with the repetition time $\tau_{\text{rep}} = N_D / \phi$. At current conditions with a rather narrow core, $\sigma_\delta = 0.4 \cdot 10^{-3}$, this threshold is seen experimentally as

$$(\dot{E}_i N)_{th} = 280 \cdot 10^{10} \text{MeV/s}, \quad (38)$$

which significantly limits the Accumulator performance. Indeed, $N_D = 15 \cdot 10^7$, $\delta E_D = 12 \text{ MeV}$ and $\phi = 5.6 \cdot 10^7 \text{ s}^{-1}$ requires $\dot{E}_i = 4.5 \text{ MeV/s}$ which puts $N_{th} = 70 \cdot 10^{10}$. To accumulate more than that number of antiprotons under the current conditions, the flux has to be reduced.

In principle, there are several ways to increase the threshold (38):

- to reduce the weight of the wrong-phased modes G_l / \dot{E}_i ;

- to increase the longitudinal phase density at injection $N_D/\delta E_D$;
- to increase the core energy width σ_δ .

While the first two means are always beneficial, although not easy realized, use of the third one is limited by a finite energy aperture, currently ≈ 84 MeV. In estimations of previous sections it was assumed $\sigma_\delta = 1 \cdot 10^{-3}$, which is 2 – 3 times higher than the current width. Depending on the power q , Eq. (36), this would result in 2-9 times increase of the threshold (38). However, at this core width, it occupies a significant portion of the whole aperture, $4\sigma_\delta E_p = 35$ MeV, which limits the flux by $\phi = 5.6 \cdot 10^7$ s⁻¹ corresponding to 20 mA/h in terms of current (see Section III).

IX. TRANSVERSE INSTABILITY ON THE RING IMPEDANCE

Coherent motion is stable if the coherent peaks lie within the incoherent frequency spread. For low and medium energy rings, the main factor of the transverse coherent-incoherent frequency split is the incoherent space charge tune shift

$$\Delta\nu_{sc} = -\frac{N_p r_p}{4\pi\beta\gamma^2\varepsilon}; \quad (39)$$

for the number of particles $N_p = 5 \cdot 10^{12}$ and the normalized r. m. s. emittance $\varepsilon = 1.6$ mm mrad it gives $\Delta\nu_{sc} = 4.4 \cdot 10^{-3}$. The coherent complex frequency shift is related to the transverse impedance $Z_\perp(\omega)$:

$$\Delta\nu_c \equiv \nu_c - \nu_x - n = -i \frac{N_p r_p}{\gamma C} \frac{\langle \beta_x Z_\perp((n + \nu_x)\omega_0) \rangle}{Z_0}, \quad (40)$$

where $Z_0 = 4\pi/c = 377$ Ohm and the brackets stay for the ring averaging, see e. g. Ref. [10]. Estimating $Z_\perp \leq 4M\Omega/m$, it comes out $\Delta\nu_c \leq 3 \cdot 10^{-4} \ll \Delta\nu_{sc}$, strongly supporting the above statement about the space charge domination in the coherent-incoherent tune split. The stability requires the particle spread to be at least 1/3 – 1/5 of the space charge tune shift, what is known as the stability limit for space charge dominated tune splits.

The related single-particle spread $\delta\nu_p$ is contributed by two independent terms: an energy-spread-related $\delta\nu_\delta = | -n\eta + \xi|\sigma_\delta$ and an emittance, or octupoles-related $\delta\nu_\varepsilon$, with ξ as the chromaticity.

Note that, according to Eq. (40), the instability might occur only at $n + \nu_x < 0$, where $\text{Re}Z_\perp < 0$ and thus $\text{Im}\Delta\nu_c > 0$.

Taking all these considerations into account, the stability condition can be expressed as

$$| -n\eta + \xi|\sigma_\delta + \delta\nu_\varepsilon \geq \Delta\nu_{sc}/F_{st} \quad \text{for } n < -\nu_x < 0, \quad (41)$$

where $F_{st} \simeq 3 - 5$ is the above-mentioned factor for the space-charge dominated tune splits.

According to the accepted definition, the slippage factor $\eta > 0$ above transition. Thus, the two terms of the energy-related spread $| -n\eta + \xi|\sigma_\delta$ are never subtracted at the dangerous region $n < 0$, if the chromaticity is positive. For positive chromaticities, the stability is guaranteed if $\xi\sigma_\delta \geq \Delta\nu_{sc}/F_{st}$; for the accepted parameters it gives $\xi \geq 1 - 2$. The situation is very different if the above-transition chromaticity is negative. In this case, the two terms in the energy-related spread almost cancel each other at mode numbers $n \simeq \xi/\eta$. Thus, sufficiently strong octupoles-related tune spread is required in this case to stabilize these transverse beam modes: $\delta\nu_\varepsilon \geq \Delta\nu_{sc}/F_{st}$ or $\delta\nu_\varepsilon \geq (1-2) \cdot 10^{-3}$ for the proclaimed parameters.

Currently, the Accumulator is operated with $N_p \leq 1.5 \cdot 10^{12}$, with both chromaticities negative; the transverse instability has never been observed. From here, it can be concluded that the emittance-related tune spread has a lower limit as $\delta\nu_\varepsilon \geq (3 - 5) \cdot 10^{-4}$.

If this octupolar spread is presently near the threshold, it means that rising of the accumulated current requires either positive chromaticity $\xi \geq 1 - 2$ or increase of the octupolar spread up to its higher threshold value.

X. SUMMARY

A main purpose of this paper was to show that a potential of the Fermilab Antiproton Accumulator would be significantly risen if the commissioned electron cooling system were applied there, even without any significant changes of the Accumulator or its stochastic cooling systems.

Also, two relatively easy-doable meliorations was shown as significantly improving the cooling-stacking parameters, without EC, as well as with it.

First, installation of the longitudinal 'noiser' should significantly increase the threshold of the longitudinal coherent instability, currently one of the most serious limiting factors. Note that this does not mean energy widening of the extracted pbars: before extraction, when the accumulation is finished, stack-tail stochastic cooling and the noiser could be switched off, and the energy width be reduced with the stochastic core cooling (if no EC), or with

EC, if it is applied.

Second, near the coupling resonance, where the Accumulator is operated, even a weak skew-quad or solenoid provides a full multi-turn transition of vertical into horizontal oscillations and back, leaving a single-turn trajectories almost untouched. With this coupling, the total transverse IBS diffusion is not changed, but gets to be equally shared between the two transverse degrees of freedom. Both with and without EC, it introduces significant amelioration in the cooling process: it reduces twice the requirement for the horizontal cooling, and prevents the vertical beam shrinking; this shrinking is undesirable, being a factor of stronger horizontal widening.

With these two meliorations and existing stochastic cooling system, application of the Pelletron-based electron cooler of 10 m length and 500 mA of the electron beam of 0.5 cm radius would allow to accumulate up to 0.5 – 1.0 A of antiproton current, keeping the flux 20 mA/h. The scheduled doubling of the stack-tail band width would allow to rise the flux to 40 mA/h.

One more beneficial feature of the proposed EC application is that the final pbar emittance is actually determined by electron beam misalignments only: until they are small, the equilibrium antiproton emittance is proportional to the electron beam radius squared. Thus, squeezing of the electron beam cross-section at the final stage would result after a while in the same reduction of the antiproton emittance.

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