Run II Upgrades: Stacktail Betatron cooling calculations Pbar Note 686

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Abstract

Calculations for a stacktail betatron cooling system are detailed. The relative improvement in transverse emittance by including such a system compared to the core betatron systems are shown.

1 Why Stacktail Betatron cooling?

As part of the Run II Upgrade plan [1], the Recycler is the final repository for $\bar{p}$'s with transfers occuring every 30 minutes with transverse emittance ≤ 10 π mm mr (95% normalized). The necessary changes for momentum stacking are detailed in references [1] and [2]. A model of transverse cooling during momentum stacking has been developed and explained in reference [1]. While the results of this model are within the desired transverse emittance for transfers, this model does not take into account alternate transverse heating sources (e.g., stacktail momentum system heating the beam transversely) which are known to have significant effects on the transverse cooling rates. Therefore, I investigate an additional betatron cooling system, with pickups and kickers located in high dispersion regions, to see what additional margin is available.

2 Calculations

I follow the calculation as described in reference [1]. The transverse cooling equation is usually written as:

$$\frac{\mathrm{d}\epsilon}{\mathrm{dt}} = -\frac{\mathrm{W}}{\mathrm{N}}[2\mathrm{Re}\{\mathrm{g}\} - |\mathrm{g}|^2(\frac{1}{\mathrm{n}_l}\sum_{\mathrm{n}}\frac{\mathrm{f}_0}{\Delta \mathrm{f}_{\mathrm{n}}}) - |\mathrm{g}|^2\mathrm{U}]\epsilon \tag{1}$$

where the sum is overall the Schottky bands inside the bandwidth W, n_l is the number of Schottky lines, and U is the average noise to average signal [3]. This equation is written in a form that is most useful for understanding cooling of a single core of particles. Since the $\bar{p}$'s moving through the stacktail see a wide range of particle densities, the equation can be rewritten in the following more useful form:

$$\frac{\mathrm{d}\epsilon(\mathrm{t},\mathrm{E})}{\mathrm{dt}} = -\frac{1}{\tau_c}(2\mathrm{Re}\{\mathrm{x}\} - |\mathrm{x}|^2\frac{\mathrm{M}(\mathrm{E})}{\mathrm{M}(\mathrm{E}_c)}))\epsilon(\mathrm{t},\mathrm{E}) + \frac{|\mathrm{x}|^2}{\tau_c}\frac{\mathrm{U}_0}{\mathrm{M}(\mathrm{E}_c)} \tag{2}$$

where for a uniform gain:

$$M(E) = \frac{pc}{\eta} \Psi(E) \frac{1}{n_l} \sum_n \frac{1}{n}, \tag{3}$$

τ_c is the optimum cooling time at energy E_c:

$$\frac{1}{\tau_c} = \frac{W}{M(E_c)}, \tag{4}$$

and x is the ratio of the gain to the optimum gain. U_0 is a constant of the system and can be determined by measuring the signal to noise S_m for a given emittance ϵ_m and mixing factor $M(E_m)$:

$$U_0 = \epsilon_m \frac{M(E_m)}{S_m}. \tag{5}$$

The present transverse cooling systems in the Accumulator are comprised of three bands (in both horizontal and vertical planes) with bandwidths of about 1.2 GHz centered at 4.8, 6.0, and 7.2 GHz respectively. The effective bandwidth of the core betatron systems is about 3.5 GHz. The pickups and kickers for these systems are located in low dispersion straight sections in the Accumulator. The proposed stacktail betatron system would be comprised of one band centered at 5 GHz with effective bandwidth of 1 GHz. The pickups and kickers in this system would be located in high dispersion straight sections, introducing an energy dependent sensitivity in the pickup and kicker response to the problem. For the present core betatron cooling systems, U_0 has an approximate value of $230 \times 10^{10} \pi$ mm mr. I will assume that the stacktail betatron system has a similar value [4].

It is assumed that the magnitude of the electronic gain of the cooling system does not vary as a function of energy and that the systems are phased for defined energies E_c (the edge of the stacktail for the core momentum systems and the pickup position for the stacktail momentum system). Particles with different energy will have a phase error at harmonic n between pickup and kicker given as:

$$\Delta\theta_n(E) = 2\pi n \frac{\eta}{3} \frac{E - E_c}{pc} \tag{6}$$

where the factor of $\frac{1}{3}$ is because the distance from pickup to kicker is $\frac{1}{3}$ the circumference of the Accumulator. The cooling term in equation 2 is replaced with

$$2\mathrm{Re}\{x\} = 2x_0 \frac{1}{n_l} \sum_n cos(\Delta\theta_n(E)). \tag{7}$$

To calculate the transverse emittance of a sample of particles as they travel through the stacktail, the energy of the sample at a given time must be known. The time t at which a sample of particles is at energy E is given as

$$t = \frac{1}{\Phi_0} \int_{E_1}^{E} \Psi(\zeta) d\zeta \tag{8}$$

where $\Psi(E)$ is the density as a function of energy. The density distribution is assumed to follow an exponential, with gain slope $E_d = 8$ MeV for the stacktail momentum and $E_d = 5$ MeV for the core momentum. The handoff from stacktail to core occurs at an energy of -42 MeV with respect to the initial dropoff position of the injected beam [1].

An important element of the calculation is the value of g, the gain with respect to optimal gain. The core betatron systems, which are phased to the transition point from stacktail momentum to core momentum (at -42 MeV), are able to run at optimum gain. The stacktail betatron system, which is working on much smaller densities, is power limited and is not able to run at optimum gain. I calculate the signal and noise power at the pickup, integrating the pickup sensitivity [5] convoluted with the density distribution $\Psi(E)$ over the energy range of the

stacktail momentum system. Noise power is based on an effective temperature of 125 K [6] and optimum gain follows the expressions derived in [7]. Results for the stacktail betatron are very similar to those derived for the core betatron planar pickups and kickers [8]. Depending upon the pickup position and the number of TWTs available, the stacktail betatron system could run at $1-6\%$ of optimum gain.

All calculations are done in *Mathematica*. The notebook is included as Appendix A.

3   Results

I have investigated 5 scenarios for transverse cooling, using the existing core systems only and then adding a stacktail betatron system at different energy positions. The following are the 5 scenarios:

1. Core betatron cooling only
2. Core betatron cooling plus a stacktail betatron system with pickups at 15 MeV, running at 1% of optimum gain
3. Core betatron cooling plus a stacktail betatron system with pickups at 22.5 MeV, running at 1% of optimum gain
4. Core betatron cooling plus a stacktail betatron system with pickups at 30 MeV, running at 3% of optimum gain
5. Core betatron cooling plus a stacktail betatron system with pickups at 30 MeV, running at 6% of optimum gain

In scenario 2, the pickups are the same physical pickups used in the stacktail momentum system, located in the A60 cooling tanks. The kickers would be placed in A20 and would require 8 TWTs. In scenarios 3, 4, and 5, an additional set of pickups, to be located in A20 would need to be built, with associated kickers in A40. 8-16 TWTs would be needed for these scenarios. Note that the original cost estimates for the system [2] used 2 TWTs and the A60 pickups, so all scenarios are a significant increase ($300 - $600K in TWTs plus additional tanks) in cost.

Figure 1 shows the time distribution of the 95% transverse emittance. Transfers are expected to take place after 30 minutes [1]. Table 1 contains the 95% transverse emittance after 30 minutes. The simplest scenario to build, which is scenario 2, offers a 2% improvement in the transverse emittance. Scenario 5, where we run with two TWTs per 32 loop board and use 300 W of transverse cooling power, gives a 15% improvement in transverse emittance.

4   Conclusions

I have investigated the additional cooling margin available from a stacktail betatron cooling system in various scenarios. Of the 4 scenarios considered, the best improvement is 15% in transverse emittance after 30 minutes of stacking. This scenario is considerably more expensive and more difficult to construct than what was originally proposed. The main problem is power limitations keeping the system from running near optimum gain.

References

[1] D. McGinnis, "Issues for Antiproton stacking and cooling", Report prepared for Department of Energy Review, July 2003 (unpublished). This document is available at http://www-bdnew.fnal.gov/doereview03/Current/02_Pbar_stacking_cooling.pdf

[2] P. Derwent and R. Pasquinelli, "Stacktail Cooling System Upgrade", Pbar Note 682 (unpublished), 2003. This document is available at http://www-bdnew.fnal.gov/pbar/documents/pbarnotes/pdf_files/PbarNote682.pdf
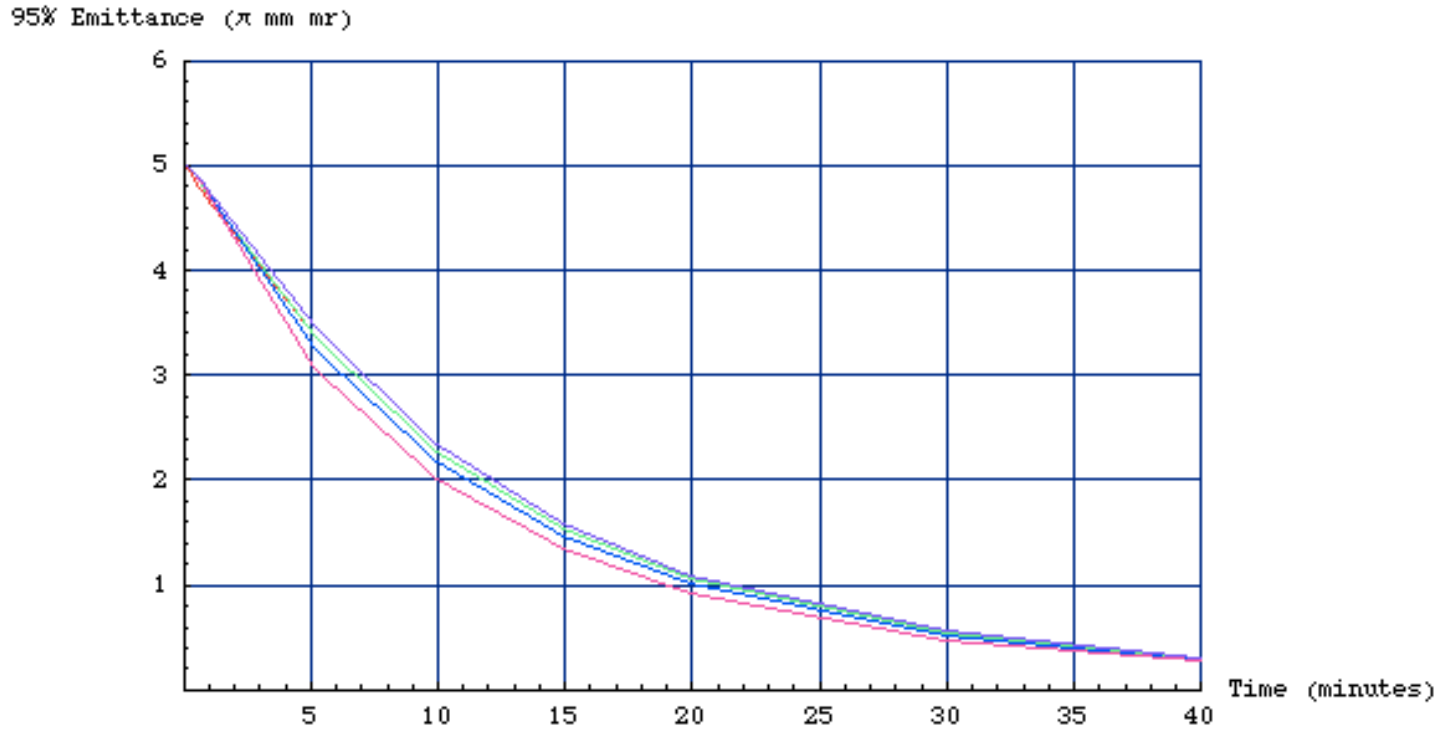
95% Emittance (π mm mr)

Figure 1: Results of the calculations for 5 scenarios, in order of decreasing emittance at the 30 minute mark: (1 – purple) core betatron only (2 & 3 – green) with stacktail pickup at 15 MeV and 1% of optimum gain and with stacktail pickup at 22.5 MeV at 1% of optimum gain (curves overlap almost entirely) (4 – blue) with stacktail pickup at 30 MeV and 3% of optimum gain (5 – pink) with stacktail pickup at 30 MeV and 6% of optimum gain.

Pickup Position	Fraction of optimum gain	95% transverse emittance at 30 minutes
Core Only		5.2 π mm mr
15 MeV	1%	5.1 π mm mr
22.5 MeV	1%	5.1 π mm mr
30 MeV	3%	4.8 π mm mr
30 MeV	6%	4.5 π mm mr

Table 1: 95% transverse emittance results for the 5 different scenarios considered.

[3] F. Sacherer, "Stochastic Cooling Theory", CERN-ISR-TH/78-11 (unpublished), 1978.

[4] The stacktail betatron system would use loop pickups, which have much lower sensitivity than the slotted waveguide pickups used in the core betatron systems. However, they would be cooled to liquid nitrogen temperatures in contrast to the room temperature waveguide pickups. I decided to take the two changes as cancelling. If anything, the noise to signal ratio for the loop pickups will be worse than the waveguide pickups, so using the same value for U_0 is an optimistic assumption.

[5] R. Shafer, "Notation for Loop Pickup Geometrical Sensitivity", Pbar Note 232 (unpublished), 1982.

[6] P. Derwent, "Effective Noise Temperature for the Accumulator Stacktail System", Pbar Note 602 (unpublished), 1998. This document is available at http://www-bdnew.fnal.gov/pbar/documents/pbarnotes/pdf_files/PB602.pdf

[7] D. Neuffer, "Parameters for Betatron Core Cooling in the Fermilab Accumulator", Pbar Note 186 (unpublished), 1981.

[8] P. Derwent, "Accumulator Betatron Core Cooling Scenarios", Pbar Note 623 (unpublished), 1999. This document is available at http://www-bdnew.fnal.gov/pbar/documents/pbarnotes/pdf_files/PB602.pdf

A   *Mathematica* Notebook used in calculations

Appended is the *Mathematica* notebook used in the calculations. The settings are for scenario 5, with pickup at 30 MeV and 6% of optimum gain.

for fitting purposes

     << Statistics`NonLinearFit`

Note: emittances in this notebook are un-normalized 95%, the Pbar standard, in contrast to FNAL standard of normalized 95%.

Input flux: 50 mA/hour

     ϕ_0 **= 50 10^{10} / 3600**

     $\dfrac{1250000000}{9}$

repetition rate: 2 seconds

     T$_{rep}$ = 2.0

     2.

input beam width from debuncher! (MeV)

     ΔE$_{bd}$ = 6

     6

average input density

     Ψ_1 = ϕ_0 T$_{rep}$ / ΔE$_{bd}$

     4.62963×10^7

stacktail gain slope (MeV)

     E$_d$ = 8

     8

core gain slope (MeV)

     E$_{dc}$ = 5

     5

beam energy (MeV)

     E$_{beam}$ = 8815

     8815

     η = 0.012

     0.012

how many betatron cooling systems exist (3 core, 1 stacktail!)

nbands = 4

4

stacktail betatron energy center (either 0 or 15)

SBE_c = 30

30

bandwidths of the systems

W = {1.167 10^9, 1.167 10^9, 1.167 10^9, 1 10^9}

$\{1.167 \times 10^9, 1.167 \times 10^9, 1.167 \times 10^9, 1000000000\}$

and central frequencies

F = {4.833 10^9, 6 10^9, 7.167 10^9, 5 10^9}

$\{4.833 \times 10^9, 6000000000, 7.167 \times 10^9, 5000000000\}$

f_0 = 628840

628840

SumHarmonics = $\left\{ N\left[\frac{1}{1855} \sum_{n=6757}^{8613} \frac{1}{n}\right], N\left[\frac{1}{1855} \sum_{n=8613}^{10468} \frac{1}{n}\right], N\left[\frac{1}{1855} \sum_{n=10468}^{12323} \frac{1}{n}\right], N\left[\frac{1}{3180} \sum_{n=6361}^{9541} \frac{1}{n}\right]\right\}$

$\{0.000130903, 0.000105205, 0.0000879961, 0.000127529\}$

average noise/signal term, lowered it for stacktail betatron as pickups are at liquid nitrogen!?! but sensitiviy not as good, so keep it the same?

U_0 = {240 10^9, 240 10^9, 240 10^9, 240 10^9}

$\{240000000000, 240000000000, 240000000000, 240000000000\}$

time to get to the edge of the core

T_2 = T_{rep} $\frac{E_d}{\Delta E_{bd}}$ Exp$\left[\frac{\Delta E_s [\Delta E_{bd}]}{E_d}\right]$ - 1

$-1 + 2.66667 \ e^{\frac{\Delta E_s [6]}{8}}$

$\Delta E_s [En\_]$:= $\frac{1}{\eta}$ $\frac{4}{5.3}$ $\frac{f_0}{7.2 \ 10^9}$ E_{beam} - En

core width, edge that systems are phased to

$$\Delta E_c = \left\{ \frac{1}{6\,\eta} \; \frac{f_0}{8.3\,10^9} \; E_{beam}, \; \frac{1}{6\,\eta} \; \frac{f_0}{8.3\,10^9} \; E_{beam}, \; \frac{1}{6\,\eta} \; \frac{f_0}{8.3\,10^9} \; E_{beam}, \; -\Delta E_s[\Delta E_{bd}] + SBE_c \right\}$$

{9.27581, 9.27581, 9.27581, -12.4209}

Sens[En_, i_] := 1 /; i < 4

The sensitivity for the stacktail betatron pickup falls exponentially, just like the sum mode. But...add in that the kicker also falls exponentially, so goes as 2*Energy/E_d.
If putting it not at 0, should probably use the real shape (for continuity purposes). But still want to square it!

Sens[En_, i_] :=
$$\left(ArcTan[Tan[\pi/33] \; (Tanh[10\,\pi/33 + (En - SBE_c)/E_d] - Tanh[-10\,\pi/33 + (En - SBE_c)/E_d])] \; \middle/ \right.$$
$$\left. ArcTan\left[2\,Tan\left[\frac{\pi}{33}\right]\,Tanh\left[\frac{10\,\pi}{33}\right]\right]\right)\wedge 2 \; /; \; i \geq 4$$

Sens[15, 4]

$$\frac{ArcTan[Tan[\frac{\pi}{33}] \; (-Tanh[\frac{15}{8} - \frac{10\,\pi}{33}] + Tanh[\frac{15}{8} + \frac{10\,\pi}{33}])]^2}{ArcTan[2\,Tan[\frac{\pi}{33}]\,Tanh[\frac{10\,\pi}{33}]]^2}$$

Plot[Sens[x, 4], {x, 0, 50}, PlotRange → {0, 1}]
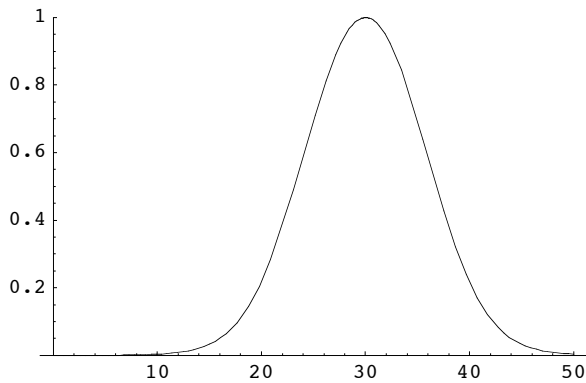
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beam energy as a function of time

$$\text{Energy}[t\_] := E_d \, Log[t/T_{rep} \, \Delta E_{bd}/E_d + 1] \; /; \; t < T_2$$

$$\text{Energy}[t\_] := \Delta E_s[\Delta E_{bd}] + E_{dc} \, Log\left[\frac{t - T_2}{T_{rep}} \; \frac{\Delta E_{bd}}{E_d} \; Exp\left[\frac{-\Delta E_s[\Delta E_{bd}]}{E_d}\right] + 1\right] \; /; \; t \geq T_2$$

Energy[T₂ - 0.000000001]

42.4458

almost perfectly matched, but close enough for government work

Energy[T$_2$] - Energy[T$_2$ - 0.000000001]

-0.0248542

Plot[Energy[t], {t, 0, 3600}]
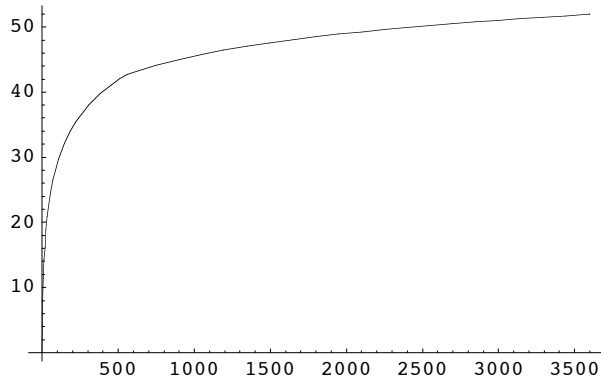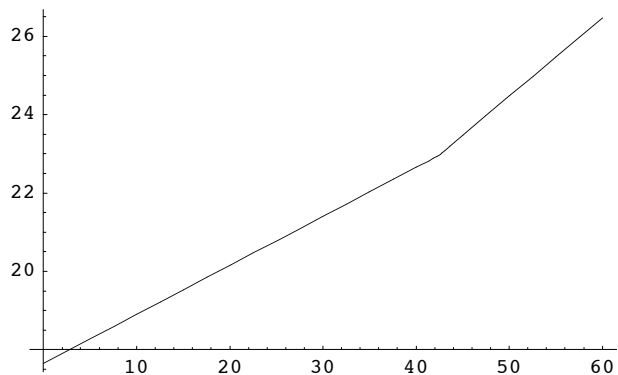

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Energy[1800]

48.4842

density as a function of energy

$$\psi[\text{En\_}] := \Psi_1 \, \text{Exp}\Big[\frac{\text{En}}{\text{E}_d}\Big] \; /; \; \text{En} < \Delta\text{E}_s \, [\Delta\text{E}_{bd}]$$

$$\psi[\text{En\_}] := \Psi_1 \, \text{Exp}\Big[\frac{\Delta\text{E}_s \, [\Delta\text{E}_{bd}]}{\text{E}_d}\Big] \, \text{Exp}\Big[\frac{\text{En} - \Delta\text{E}_s \, [\Delta\text{E}_{bd}]}{\text{E}_{dc}}\Big] \; /; \; \text{En} \geq \Delta\text{E}_s \, [\Delta\text{E}_{bd}]$$

Plot[Log[ψ[x]], {x, 0, 60}]

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ψ[15]

3.0189×10^8

$\psi[30]$

1.96857×10^9

Mixing factor

$$M[En\_, i\_] := \frac{E_{beam}}{\eta} \, \psi[En] \, SumHarmonics[[i]]$$

cooling time for optimum gain

$$\tau[En\_] := M[En] \, / \, W$$

input emittance

$$\varepsilon_0 := 5$$

width of stacktail

$\Delta E_s[\Delta E_{bd}]$

42.4209

$\beta_{pu} = 15$

15

$Gap_{pu} = 0.033$

0.033

$Z_{betatron} = 100$

100

$N_{loops} = 32$

32

this is induced current on one pickup!

$$I_{betatron} = \sqrt{\frac{2 \; 1.6 \; 10^{-19} \; W[[4]] \; \beta_{pu}}{6}} \; \frac{1}{Gap_{pu}} \; NIntegrate\left[\sqrt{Sens[x, 4]}\right.$$

$$\left. \sqrt{\left(9.5 \left(0.1 + 4.9 \, Exp\left[\frac{-0.1}{60} \; \frac{T_{rep}}{\Delta E_{bd}} \; E_d \left(Exp\left[\frac{x}{E_d}\right] - 1\right)\right]\right) 10^{-6} \; \psi[x] \; 10^{-13}\right)}, \{x, 0, 42\}\right] \Big/ 42$$

3.31907×10^{-8}

betatron signal power for stacktail system (in pWatts) for one pickup!

$$Power_{betatron} = I_{betatron}{}^2 \; Z_{betatron} \; 10^{12}$$

0.110162

noise power for stacktail system   effective temp of 125 K (in pWatts)

Power$_{noise}$ = 1.38 10^{-23} 125 2 10^9 10^{12}

3.45

for the stacktail betatron, mixing factor is about 5, so optimum gain = 1/(M+U) ~ 0.17
necessary kicker voltage for this gain is:

Lkick = 0.005

0.005

$$\text{optgain} = \frac{1}{\left(5.3 + \frac{Power_{noise}}{Power_{betatron}}\right)}$$

0.0273093

Npart = NIntegrate$\left[\psi[x] \sqrt{\text{Sens}[x, 4]}, \{x, 0, 50\}\right]$

6.73425×10^{10}

Kicker voltage = ebeam/charge (V!) * optgain * gap / 2 kicker length * square root term.... for one loop!

$$KV = \frac{E_{beam} \; 10^6 \; \text{optgain} \; Gap_{pu}}{2 \; \text{Lkick}} \sqrt{\frac{9.5 \; 5 \; 10^{-6} \; W[[4]]}{6 \; \beta_{pu} \; \text{Npart} \; f_0}}$$

88.6866

signal power = KV^2/Z ... and multiply by 32 to get total power to board

TWT$_{signal}$ = KV^2 / 100 N$_{loops}$

2516.9

$$\text{TWT}_{noise} = \frac{Power_{noise}}{Power_{betatron}} \; \frac{1}{N_{loops}} \; TWT_{signal}$$

2463.22

TWT$_{total}$ = TWT$_{signal}$ + TWT$_{noise}$

4980.12

Core betatron systems running at optimum gain for the core, stacktail betatron can't run at optimum gain as don't have 3 kW of TWT power! So lower it by two orders of magnitude
4 Nov 03...according to Ralph, we could probably put 300 W into a set of boards with appropriate splitting of inputs, so try 6% of optimum gain here...

gain = {1., 1., 1., 6. 10^{-2} }

{1., 1., 1., 0.06}

Mixing factor at central energy of the pickup (where it is phased). Note that the betatron system is phased for cooling at the edge of the stacktail.

```
M_core = Table[M[ΔE_s[ΔE_bd] + ΔE_c[[i]], i], {i, 1, nbands}]
```

$\{5.71637 \times 10^{12}, 4.59418 \times 10^{12}, 3.84267 \times 10^{12}, 1.84418 \times 10^{11}\}$

```
Table[ψ[ΔE_s[ΔE_bd] + ΔE_c[[i]]], {i, 1, nbands}]
```

$\{5.94468 \times 10^{10}, 5.94468 \times 10^{10}, 5.94468 \times 10^{10}, 1.96857 \times 10^{9}\}$

Phase difference as a function of energy for the 4 systems. Note that set ΔEc to cancel ΔEs for stacktail betatron and have it phased for zero energy.

$$\Delta\theta[En\_, freq\_, i\_] := 2\pi \frac{freq}{f_0} \frac{\eta}{3} \frac{En - \Delta E_s[\Delta E_{bd}] - \Delta E_c[[i]]}{E_{beam}}$$

the cooling term

$$Cool[En\_, xr\_] := 2 \, Sum\left[\frac{xr[[i]] \, W[[i]] \, Cos[\Delta\theta[En, F[[i]], i]] \, Sens[En, i]}{M_{core}[[i]]}, \{i, 1, nbands\}\right]$$

```
Cool[10, gain]
```

0.000596211

note that it is very large for small values of energy! but we don't spend much time there...

```
Plot[Cool[x, gain], {x, 0, 60}]
```
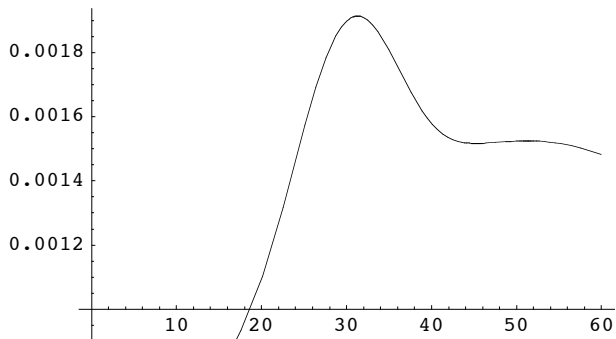

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the mixing term, which is a heating term

$$Mix[En\_, xr\_] := Sum\left[\frac{xr[[i]]^2 \, M[En, i] \, W[[i]] \, Sens[En, i]^2}{(M_{core}[[i]])^2}, \{i, 1, nbands\}\right]$$

Plot[-Mix[x, gain], {x, 0, 60}]
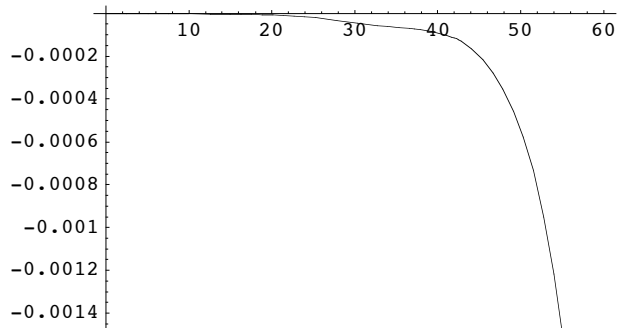
- Graphics -

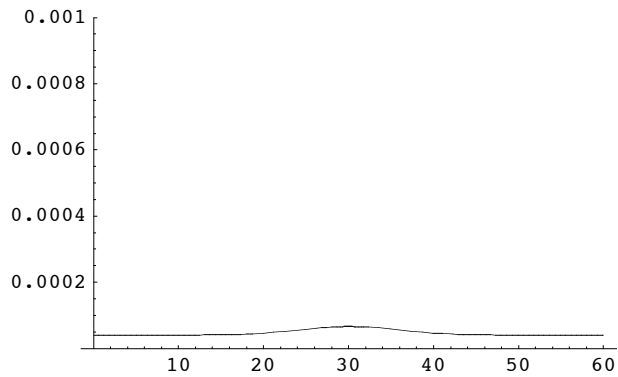the noise term, pure heating. Energy dependence in kicker sensitivity? .. doesn't enter squared since it is squared above!

$$\text{Noise}[\text{En\_, xr\_}] := \text{Sum}\left[\frac{xr[[i]]^2 \text{ Sens}[\text{En, i}] \text{ U}_0[[i]] \text{ W}[[i]]}{(\text{M}_{\text{core}}[[i]])^2}, \{i, 1, \text{nbands}\}\right]$$

Noise[10, gain]

0.0000408927

Plot[Noise[x, gain], {x, 0, 60}, PlotRange → {0, 0.001}]

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net cooling, ignoring noise

```
Plot[Cool[x, gain] - Mix[x, gain], {x, 0, 60}]
```
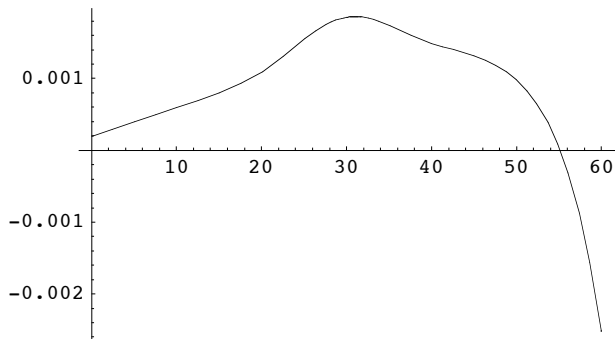

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integral to calculate how emittance varies with time through the stacktail and core momentum cooling process

```
IP[t_, xr_] := NIntegrate[Cool[Energy[τ], xr] - Mix[Energy[τ], xr], {τ, 0, t}]
```

```
Plot[IP[x, gain], {x, 0, 1800}]
```
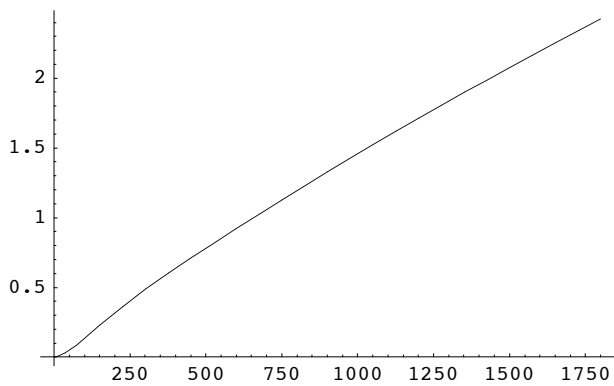

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emittance versus time

```
ε[t_, xr_] := (ε₀ Exp[-IP[t, xr]] +
    Exp[-IP[t, xr]] NIntegrate[Exp[IP[y, xr]] Noise[Energy[y], xr], {y, 0, t}])
```

```
ε[1, gain]
```

4.99882

```
ε[60, gain]
```

4.69522

```
ε[120, gain]
```

4.2256

```
ε[240, gain]
```

```
3.41465
```

```
ε[1800, gain]
```

```
0.473936
```

```
plot3bands = Table[{{0.1, ε[6, gain]}, {0.3, ε[18, gain]}, {0.5, ε[30, gain]},
    {0.75, ε[40, gain]}, {1, ε[60, gain]}, {5, ε[300, gain]}, {10, ε[600, gain]},
    {15, ε[900, gain]}, {20, ε[1200, gain]}, {30, ε[1800, gain]}, {40, ε[2400, gain]}}]
```

```
{{0.1, 4.9877}, {0.3, 4.9445}, {0.5, 4.88736}, {0.75, 4.83038}, {1, 4.69522}, {5, 3.09882},
 {10, 2.00959}, {15, 1.3501}, {20, 0.929132}, {30, 0.473936}, {40, 0.269677}}
```

```
dataplot =
 ListPlot[plot3bands, PlotRange -> {0, 5}, PlotStyle -> {Hue[0], PointSize[0.0175]}]
```
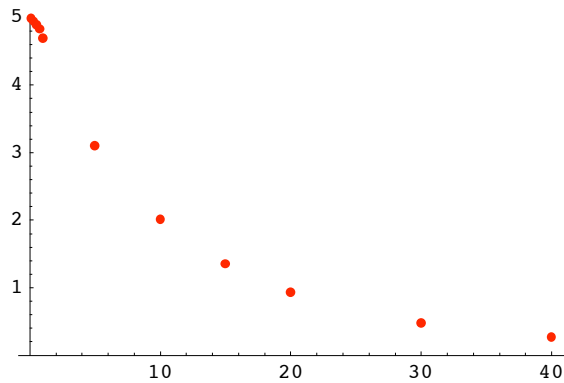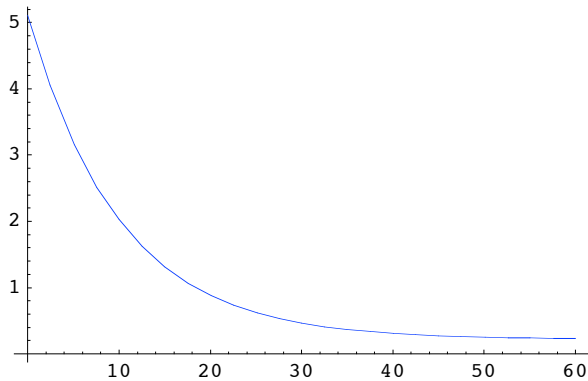


```
- Graphics -
```

```
NonlinearFit[plot3bands, A Exp[-x / t] + Af, x, {A, Af, t}]
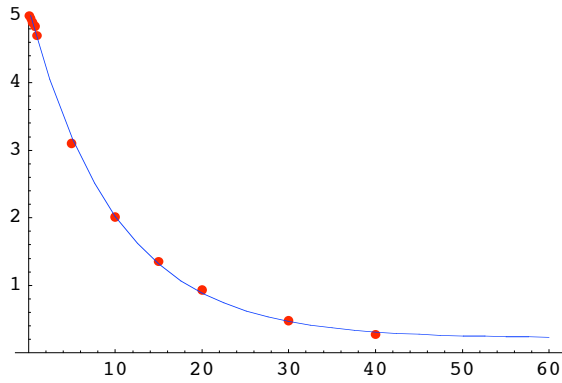```

$$0.220888 + 4.88654\, e^{-0.0997955\, x}$$

```
fitplot = Plot[%, {x, 0, 60}, PlotStyle → {Hue[0.65]}]
```
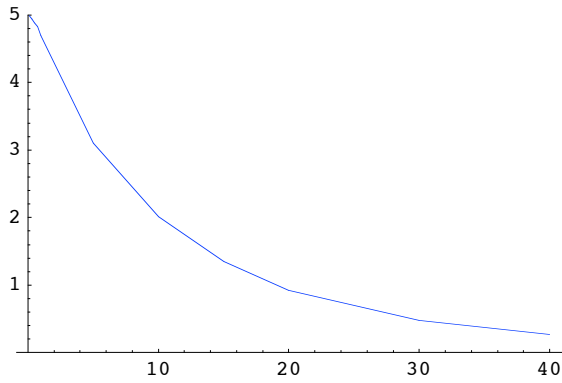


```
- Graphics -
```

Show[dataplot, fitplot]

- Graphics -

ListPlot[plot3bands, PlotRange → {0, 5},
 PlotStyle -> {Hue[0.65], PointSize[0.001]}, PlotJoined → True]

- Graphics -