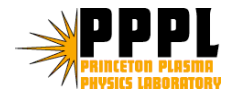


Integrated Gyrokinetic Particle Simulation of Fusion Plasmas

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Princeton Plasma Physics Laboratory
(July 2005)

In collaboration with
US DoE SciDAC Center for Gyrokinetic Particle Simulation
of Turbulent Transport in Burning Plasmas



Center for Gyrokinetic Particle Simulation of Turbulent Transport in Burning Plasmas

SciDAC - Advanced Simulation of Fusion Plasmas

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Scientific Application Partnership Program

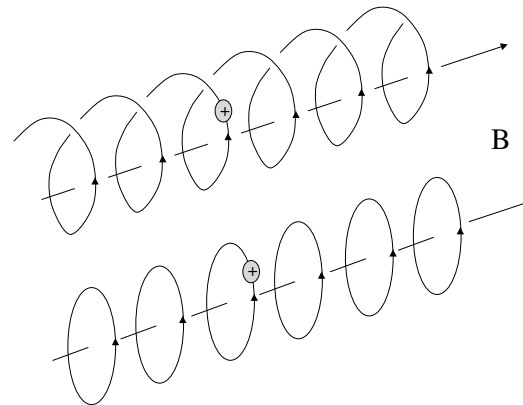
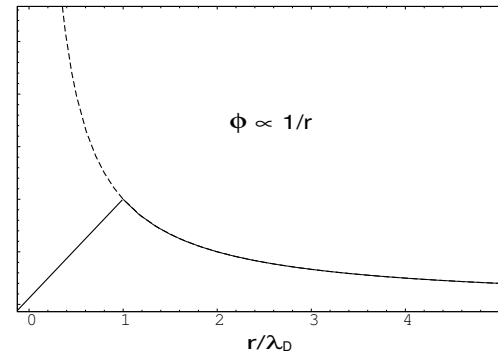
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Outline

- Basics of Gyrokinetic Particle Simulation
 - Finite size particles
 - Decoupling of gyromotion and polarization effects
 - Governing equations
- Gyrokinetic Particle Simulation of Microturbulence
 - Simulation using the Gyrokinetic Toroidal Code (GTC)
 - Scalability on MPP machines
 - Numerical Noise Issue
 - Influence of Parallel Velocity-Space Nonlinearity on Steady State Microturbulence
- Integrated Plasma Simulation for Burning Plasmas
 - Spatial Integration:
 - * Core-Edge Transport Simulations
 - Temporal Integration:
 - * Gyrokinetic-MHD
 - * Wave Heating
 - * Transport Time Scale Simulation

Basics of Gyrokinetic Particle Simulation

- Finite-size particles
[Dawson et al. '68; Birdsall et al. '68]
 - Coulomb interactions are collisionless
 - Collisional effects are subgrid phenomena
- Gyrokinetic particles
[Lee PF '83]
 - Gyromotion becomes motion of rotating charged rings
 - Polarization Effects in the field equations
- Efficient numerical methods to account for finite Larmor radius effects
[Lee JCP '87; Lee and Qin PP '03]



Gyrokinetic Vlasov-Maxwell Equations in Toroidal Geometry

- GK Vlasov equation - in gyrocenter coordinates

[Lee PF '83; Dubin et al. PF '83; Hahm et al. PF '88; Hahm PF '88; Brizard PF '88; Brizard J. Plas. Phys. '89; Qin et al. PoP '99; Qin et al. PoP '00; Qin et al., PoP '00; Lee and Qin PoP '03]

$$\frac{\partial F_{\alpha gc}}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial F_{\alpha gc}}{\partial \mathbf{R}} + \frac{dv_{\parallel}}{dt} \frac{\partial F_{\alpha gc}}{\partial v_{\parallel}} = 0,$$

$$\frac{d\mathbf{R}}{dt} = v_{\parallel} \mathbf{b}^* + \frac{v_{\perp}^2}{2\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times \nabla \ln B_0 - \frac{c}{B_0} \nabla \bar{\phi} \times \hat{\mathbf{b}}_0$$

$$\frac{dv_{\parallel}}{dt} = -\frac{v_{\perp}^2}{2} \mathbf{b}^* \cdot \nabla \ln B_0 - \frac{q_{\alpha}}{m_{\alpha}} \left(\mathbf{b}^* \cdot \nabla \bar{\phi} + \frac{1}{c} \frac{\partial \bar{A}_{\parallel}}{\partial t} \right) \quad \text{-- Velocity Nonlinearity}$$

$$\mu_B \equiv \frac{v_{\perp}^2}{2B_0} \left(1 - \frac{mc}{e} \frac{v_{\parallel}}{B_0} \hat{\mathbf{b}}_0 \cdot \nabla \times \hat{\mathbf{b}}_0 \right) \approx \text{cons.}$$

$$\mathbf{b}^* \equiv \mathbf{b} + \frac{v_{\parallel}}{\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla) \hat{\mathbf{b}}_0, \quad \mathbf{b} = \hat{\mathbf{b}}_0 + \frac{\nabla \times \bar{\mathbf{A}}}{B_0}$$

$$\begin{pmatrix} \bar{\phi} \\ \bar{\mathbf{A}} \end{pmatrix} (\mathbf{R}) = \left\langle \int \begin{pmatrix} \phi \\ \mathbf{A} \end{pmatrix} (\mathbf{x}) \delta(\mathbf{x} - \mathbf{R} - \rho) d\mathbf{x} \right\rangle_{\varphi}, \quad \text{-- Coordinate Transformation}$$

$$F_{\alpha gc} = \sum_{j=1}^{N_{\alpha}} \delta(\mathbf{R} - \mathbf{R}_{\alpha j}) \delta(\mu - \mu_{\alpha j}) \delta(v_{\parallel} - v_{\parallel \alpha j})$$

GK Equations in Toroidal Geometry (cont.)

- GK Poisson's equation - in laboratory coordinates [Lee JCP '87]

$$\nabla^2 \phi + \frac{\tau}{\lambda_D^2} [\phi(\mathbf{x}) - \tilde{\phi}(\mathbf{x})] = -4\pi \rho_{gc}(\mathbf{x}) \quad (k_{\perp} \rho_i)^2 \ll 1 \quad \Longrightarrow \quad \frac{\rho_s^2}{\lambda_D^2} \nabla_{\perp}^2 \phi(\mathbf{x}) = -4\pi \rho_{gc}(\mathbf{x})$$

$$\tilde{\phi}(\mathbf{x}) \equiv \langle \int \bar{\phi}(\mathbf{R}) F_i(\mathbf{R}, \mu, v_{\parallel}) \delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} d\mu dv_{\parallel} \rangle_{\varphi}$$

$$\rho_{gc}(\mathbf{x}) = \sum_{\alpha} q_{\alpha} \langle \int F_{\alpha gc}(\mathbf{R}) \delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} dv_{\parallel} d\mu \rangle_{\varphi}$$

- GK Ampere's law -- in laboratory coordinates [Qin et al. PP '99]

$$\nabla^2 \mathbf{A} - \frac{1}{v_A^2} \frac{\partial^2 \mathbf{A}_{\perp}}{\partial t^2} = -\frac{4\pi}{c} \mathbf{J}_{gc} \quad \omega^2 / k^2 v_A^2 \ll 1$$

$$\mathbf{J}_{gc}(\mathbf{x}) = \mathbf{J}_{\parallel gc}(\mathbf{x}) + \mathbf{J}_{\perp gc}^M(\mathbf{x}) + \mathbf{J}_{\perp gc}^d(\mathbf{x})$$

$$= \sum_{\alpha} q_{\alpha} \langle \int (\mathbf{v}_{\parallel} + \mathbf{v}_{\perp} + \mathbf{v}_d) F_{\alpha gc}(\mathbf{R}) \delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} dv_{\parallel} d\mu \rangle_{\varphi}$$

$$\mathbf{v}_d \equiv \frac{v_{\parallel}^2}{\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla) \hat{\mathbf{b}}_0 + \frac{v_{\perp}^2}{2\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times \nabla \ln B_0$$

GK Equations in Toroidal Geometry (cont.)

- Calculations of FLR effects for $k_{\perp}\rho_i \sim 1$ is only possible in the gyrocenter coordinates, **but not in the laboratory coordinates**

$$\begin{pmatrix} \bar{\phi} \\ \bar{\mathbf{A}} \end{pmatrix}(\mathbf{R}_{\alpha j}) = \left\langle \begin{pmatrix} \phi \\ \mathbf{A} \end{pmatrix}(\mathbf{x}_{\alpha j}) \right\rangle_{\varphi}$$

$$\rho_{gc}(\mathbf{x}) = \sum_{\alpha} q_{\alpha} \sum_{j=1}^N \langle \delta(\mathbf{x} - \mathbf{x}_{\alpha j}) \rangle_{\varphi}$$

$$\mathbf{J}_{\parallel gc}(\mathbf{x}) = \sum_{\alpha} q_{\alpha} \sum_{j=1}^N v_{\parallel \alpha j} \langle \delta(\mathbf{x} - \mathbf{x}_{\alpha j}) \rangle_{\varphi}$$

$$\mathbf{J}_{\perp gc}^M(\mathbf{x}) = \sum_{\alpha} q_{\alpha} \sum_{j=1}^N \langle \mathbf{v}_{\perp \alpha j} \delta(\mathbf{x} - \mathbf{x}_{\alpha j}) \rangle_{\varphi}$$

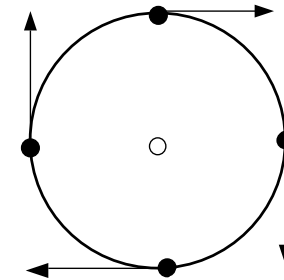
$$\mathbf{J}_{\perp gc}^d(\mathbf{x}) = \sum_{\alpha} q_{\alpha} \sum_{j=1}^N \langle \mathbf{v}_{d\alpha j} \delta(\mathbf{x} - \mathbf{x}_{\alpha j}) \rangle_{\varphi}$$

- Perpendicular current for $k_{\perp}\rho_i \ll 1$ [Qin et al. PP '00]

$$\mathbf{J}_{\perp gc} = \mathbf{J}_{\perp gc}^M + \mathbf{J}_{\perp gc}^d = \frac{c}{B_0} \sum_{\alpha} \left[\hat{\mathbf{b}}_0 \times \nabla p_{\alpha\perp} + (p_{\alpha\parallel} - p_{\alpha\perp})(\nabla \times \hat{\mathbf{b}}_0)_{\perp} \right]$$

- Pressure Balance: $p = p_{\alpha\parallel} = p_{\alpha\perp}$

$$\mathbf{J}_{\perp gc} = \frac{c}{B_0} \sum_{\alpha} \hat{\mathbf{b}}_0 \times \nabla p_{\alpha}$$



Coordinate Transformation

Perturbative Particle Simulation

- δf simulation schemes:

-- [Dimits and Lee, JCP '93; Parker and Lee, PF '93]

$$\text{Let } F = F_0 + \delta f \longrightarrow \frac{d\delta f}{dt} = -\frac{dF_0}{dt}$$

$$\text{Let } w \equiv \frac{\delta f}{F} \longrightarrow \delta f = \sum_{j=1}^N w_j \delta(\mathbf{R} - \mathbf{R}_j) \delta(\mu - \mu_j) \delta(v_{\parallel} - v_{\parallel j})$$

$$\text{Noise reduction: } |E|^2 \propto w^2 \quad [\text{Hu and Krommes, PoP '94}]$$

-- [Aydemir, PoP '94]

$$F = F_0 + \delta f \longrightarrow w \equiv \frac{\delta n}{n}$$

- Split-weight schemes: [Manuilskiy and Lee, PoP '00; Lee et al., PoP '01, Lewnadowski, PoP '03]

$$F = F_0 + \psi F_0 + \delta h \quad \psi = \phi + \frac{1}{c} \int \frac{\partial A_{\parallel}}{\partial t} dx_{\parallel 0}$$

- Hybrid Scheme [Lin and Chen, PoP '01] $\omega \ll k_{\parallel} v_{\parallel}$
- Time step is determined by zeroth order transit time of the electrons along the field line.

Fluctuation-Dissipation Theorem of Damped Modes for a⁹ Quiescent Plasmas in Particle Simulation

- Plasma Waves and Finite-Size Particles [Langdon and Birdsall, PF **13**, 2115 (1970)]

$$V \frac{|\mathbf{E}(\mathbf{k})|^2}{8\pi} = \frac{T/2}{1 + k^2 \lambda_D^2 / S^2} \rightarrow T/2, \quad V \text{ -- volume, } S \text{ -- particle shape}$$

- Gyrokinetic Particle Simulation [Krommes et al., PF '86; Lee, JCP '87]

$$V \frac{|\mathbf{E}(\mathbf{k})|^2}{8\pi} = \frac{\lambda_D^2}{\rho_s^2} (T/2) \quad \text{for } k\rho_i \ll 1$$

- Shear-Alfven Waves in Gyrokinetic Plasmas [Lee et al., PP '01]

$$V \frac{|\mathbf{E}(\mathbf{k})|^2}{8\pi} = \frac{\lambda_D^2}{\rho_s^2} \frac{T/2}{1 + \omega_{pe}^2 / c^2 k^2}, \quad \text{cold electrons}$$

$$V \frac{|\mathbf{E}(\mathbf{k})|^2}{8\pi} = k^2 \lambda_D^2 \frac{T/2}{1 + k^2 \rho_s^2}, \quad \text{warm electrons}$$

- Compressional-Alfven Waves in Gyrokinetic Plasmas

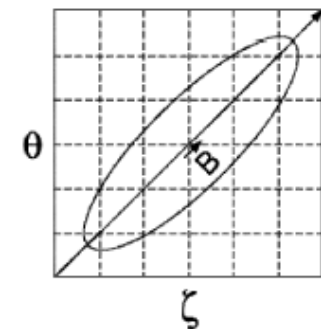
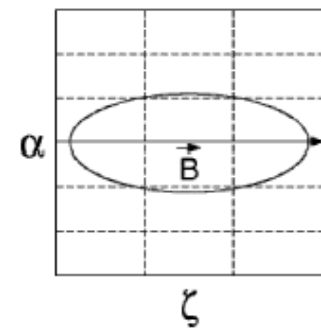
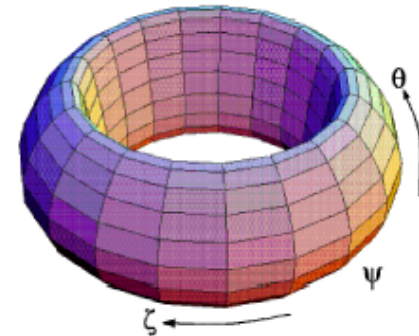
$$\frac{A_{\perp}}{A_{\parallel}} \sim \frac{\omega^2}{k^2 v_A^2} \ll 1$$

- FDT cannot be applied to an unstable system blindly.

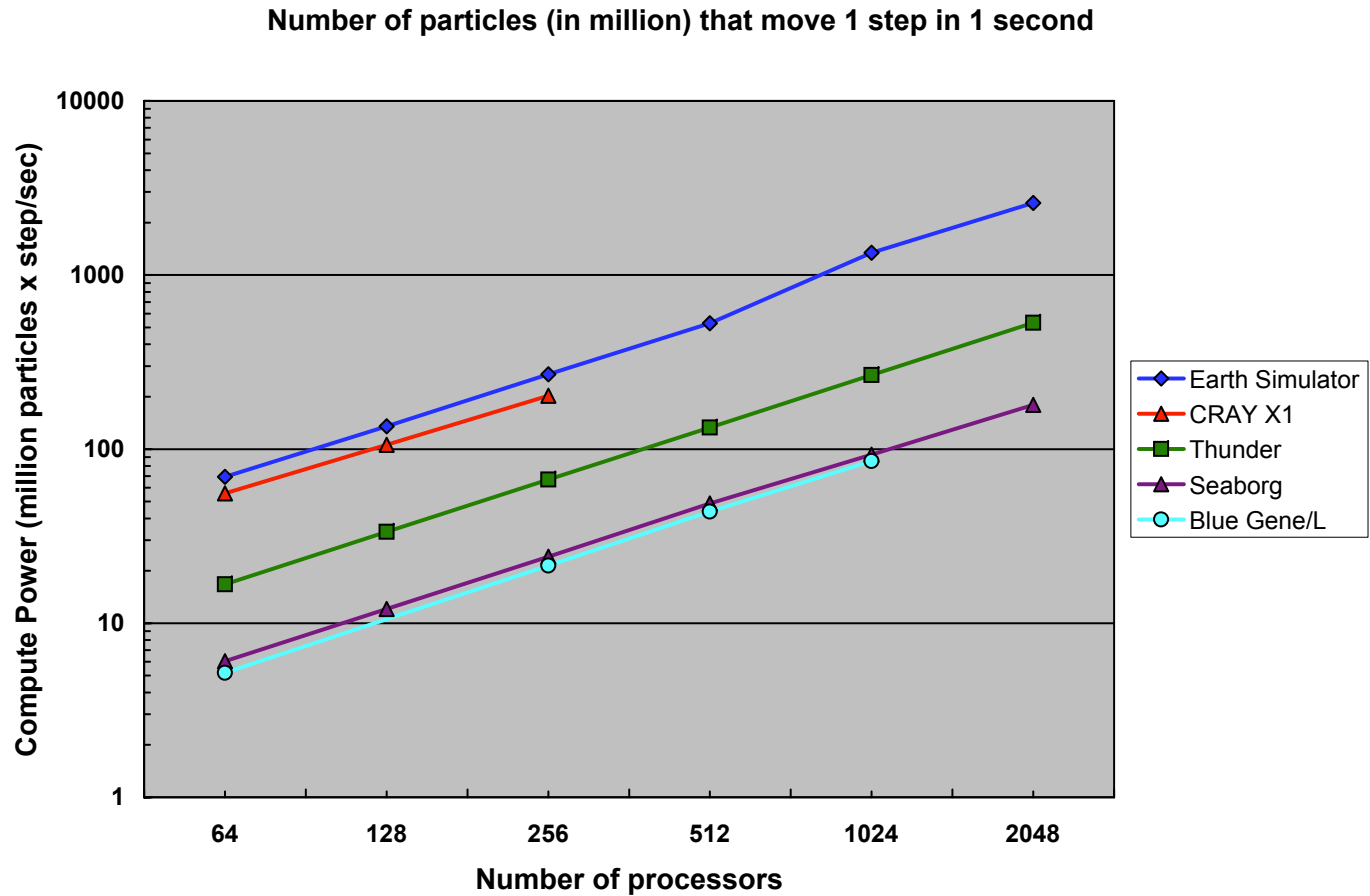
Global Gyrokinetic Toroidal Particle Simulation Code: GTC

[Z. Lin, T. S. Hahm, W. W. Lee, W. M. Tang and R. B. White, *Science* (1998)]

- Magnetic coordinates (ψ, θ, ζ) [Boozer, 1981]
- Guiding center Hamiltonian [Boozer, 1982; White and Chance, 1984]
- Non-spectral Poisson solver [Lin and Lee, 1995]
- Global field-line coordinates: (ψ, α, ζ) , $\alpha = \theta - \zeta/q$
 - Microinstability wavelength: $\lambda_{\perp} \propto \rho_i$, $\lambda_{\parallel} \propto qR$
 - With field-line coordinates: Grid # $N \propto a^2$, a : minor radius, $\Delta\zeta \propto R$
 - Without field-line coordinates: grid # $N \propto a^3$, $\Delta\zeta \propto \rho$
 - Larger time step: no high k_{\parallel} modes
- Collisions: e-i, i-i and e-e
- Neoclassical Transport Code: GTC-neo [W. X. Wang, 2004]



GTC performance on MPP platforms - S. Ethier



- Gyrokinetic particle codes are portable, scalable and efficient on both cache-based and vector-parallel MPP platforms

GTC performance

- S. Ethier

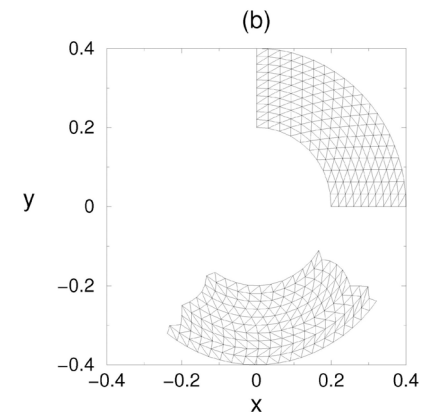
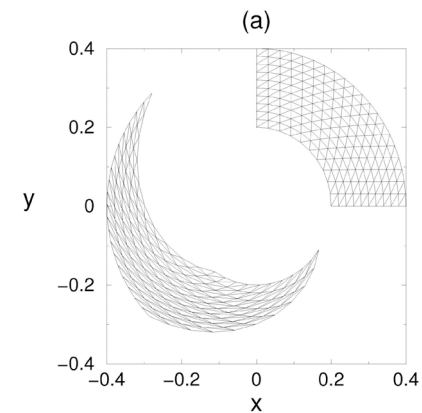
# of proc.	#part (Billion)	IBM SP 3 (Seaborg)		Itanium 2 + Quadrics		Opteron + Infiniband		CRAY X1		ES	
		Gflop	%Pk	Gflop	%Pk	Gflop	%Pk	Gflop	%Pk	Gflop	%Pk
64	0.207	9.0	9.3	25.0	6.9	37.8	13.3	82.6	10.1	102.4	20.0
128	0.414	17.9	9.3	49.9	6.9	75.5	13.3	156.2	9.6	199.7	19.5
256	0.828	35.8	9.3	97.3	6.9	145.9	13.1	299.5	9.1	396.8	19.4
512	1.657	71.7	9.4	194.6	6.8	261.1	11.6			783.4	19.1
1024	3.314	143.4	8.7	378.9	6.7					1,925	23.5
2048	6.627	266.2	8.4	757.8	6.7					3,727	22.7

3.7 Teraflops achieved on the Earth Simulator with 2,048 processors using 6.6 billion particles!!

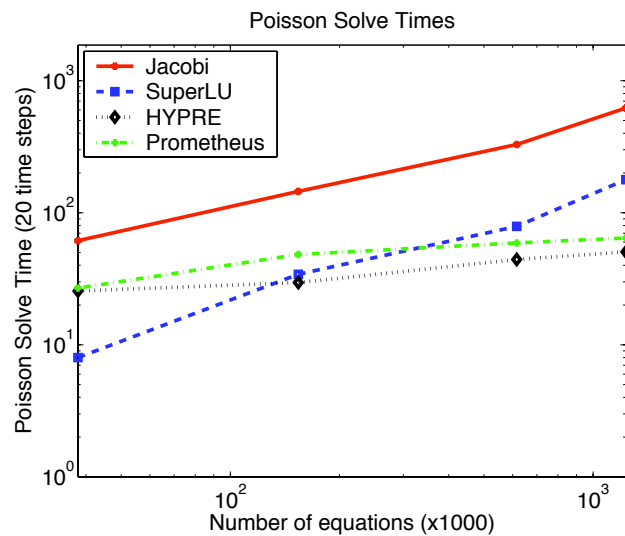
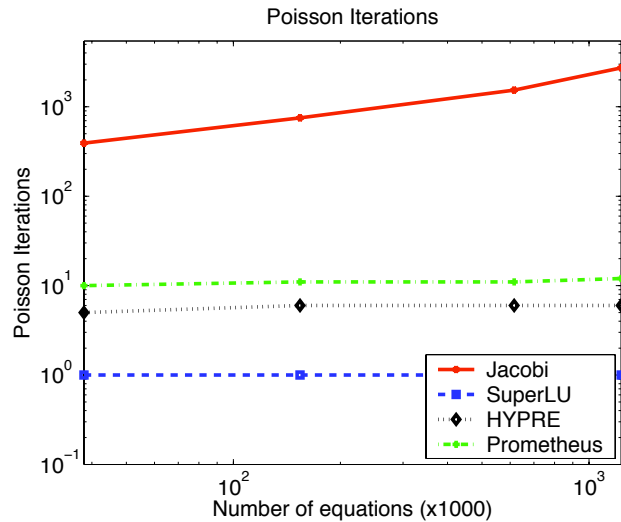
Finite Element (FEM) Elliptic Solver Developed for GTC Global Field Aligned Mesh

- FEM adapted for logically non-rectangular grids.
Need adjustments of elements at different toroidal angles.
- Linear sparse matrix solver
- PETSc (ANL)
- Enabled implementing split-weight (*Manuilskiy & Lee, POP2000*)
and hybrid electron models (*Lin & Chen, PoP2001*)
- Ongoing studies of kinetic electron effects on
ITG and TEM turbulence
- Ongoing studies of electromagnetic turbulences:
AITG/KBM & TAE/EPM

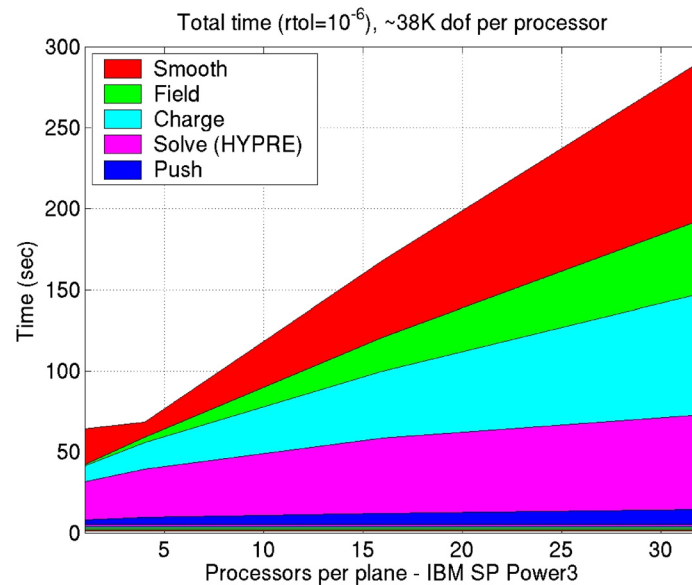
Nishimura, Ethier, Lewandowki & Lin, submitted to JCP, 2004



Poisson Solver Performance



- Multigrid preconditioned Krylov solver
 - Prometheus (Columbia) & HYPRE (LLNL)
- Scaled speedup
 - ~38K dof per processor
 - 1 to 32 processors/plane
 - 8 planes, 20 time steps, 4 particles per cell



Data Management challenges

- GTC is producing TBs of data
 - Data rates: 80Mbps now, 1.6Gbs 5 years.
 - Need quality of service (QOS) to stream data.
- This data needs to be post-processed
 - Essential to parallelize the post-processing routines to handle our larger datasets.
 - We need a cluster to post process this data.
 - > M (supercomputer processors) x N (cluster processors) problem.
 - > QOS becomes more important to sustain this post-processing.
 - > Workflow automation becomes essential to automate this process of moving data, analyzing data, and finally visualizing/publishing data.
- The post-processed data needs to be shared among collaborators
 - Different sections of the post-processed data may go to different users .
 - Post-processed data, along with other metadata should be archived into a relational database.
 - This technology is critical for data sharing of large datasets.

Beck, Bhat, Klasky, Ma, Parashar

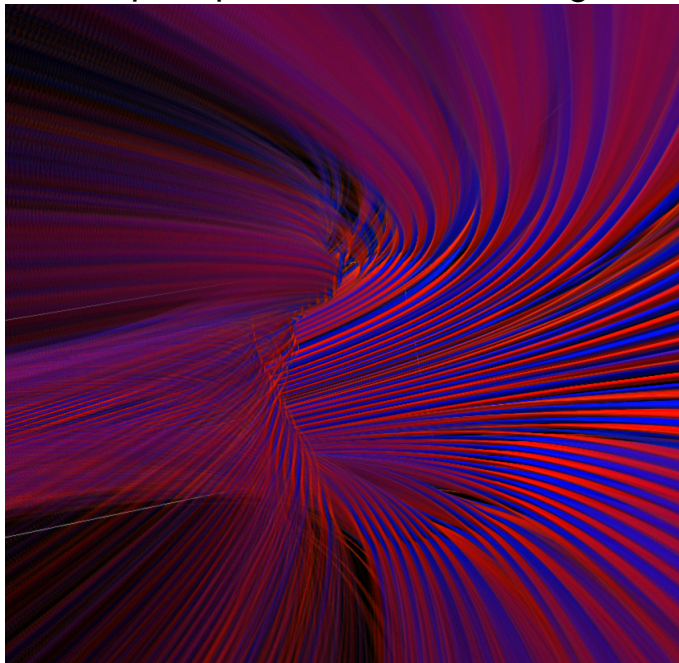
Post processing of GTC Data

- Particle Data

- No compression possible [already compressed by a factor of 10].
- Sent to 1 cluster for visualization/analysis.
- Work being done with K. Ma, U.C. Davis: Visualize a million particles.
- Gain new insights into the theory.

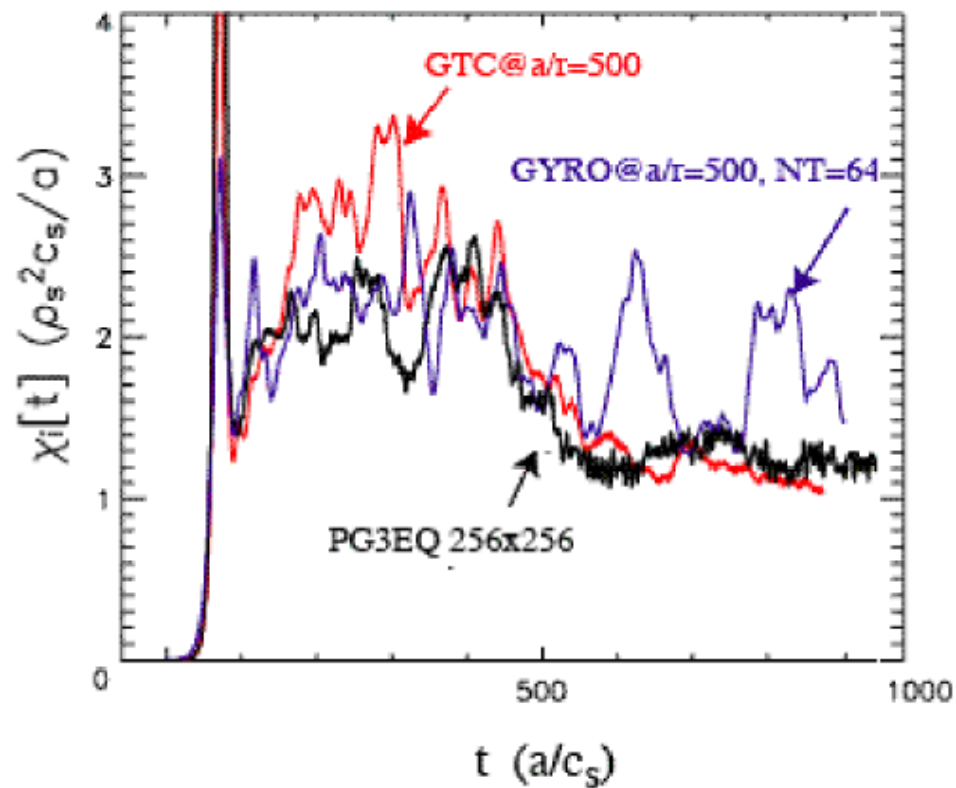
- Field Data

- Geometric/Temporal compression of the data is possible [a factor of 10 or more].
- Data needs to be **streamed** to a local cluster at PPPL.
- Reduced subset needs to be sent to PPPL + collaborators.
 - > Use Logistic Network. [Beck, UT-K]
 - > Data transfer needs to be automatic, and integrated into a dataflow/webflow for use with parallel analysis routines.
- We desire to see post-processed data during the simulation.



Recent PMP Code Comparisons and Controversies

(W. M. Nevins, 04)



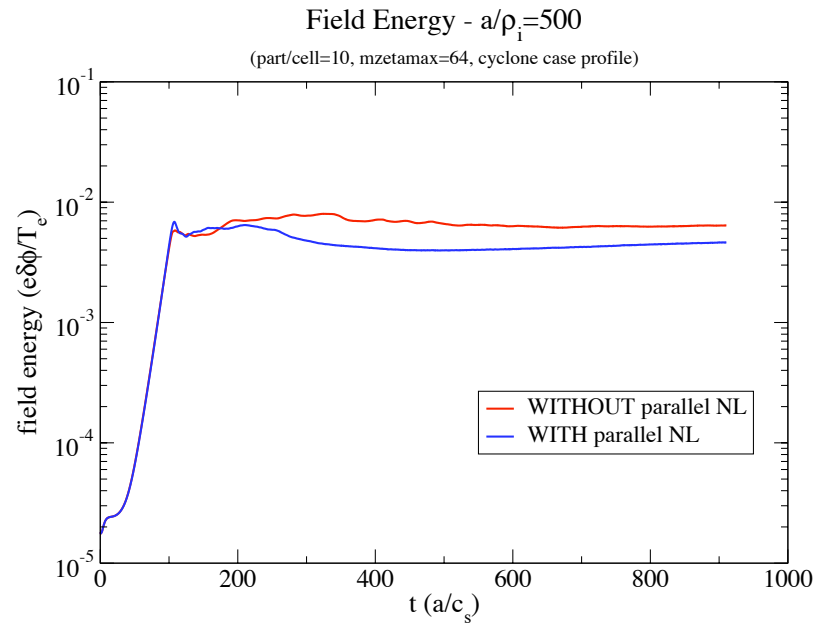
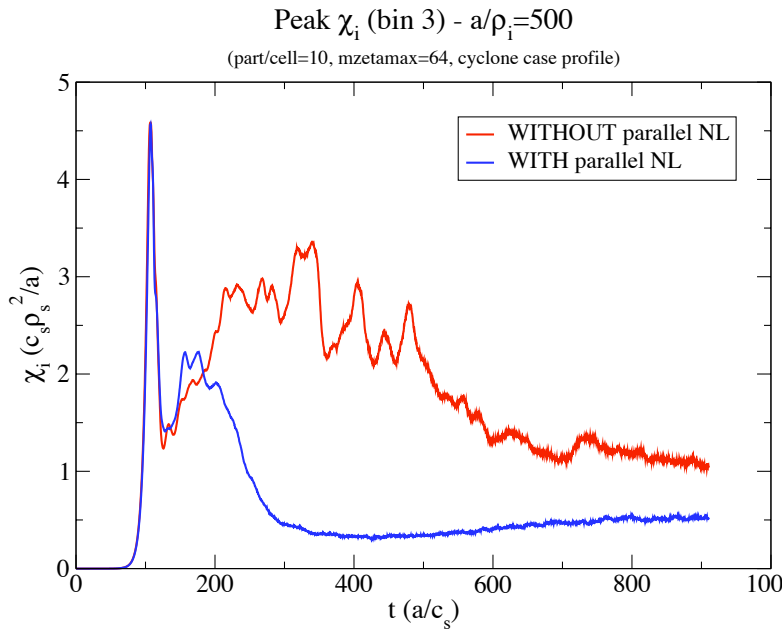
Code Comparisons:

GTC - Particle Code

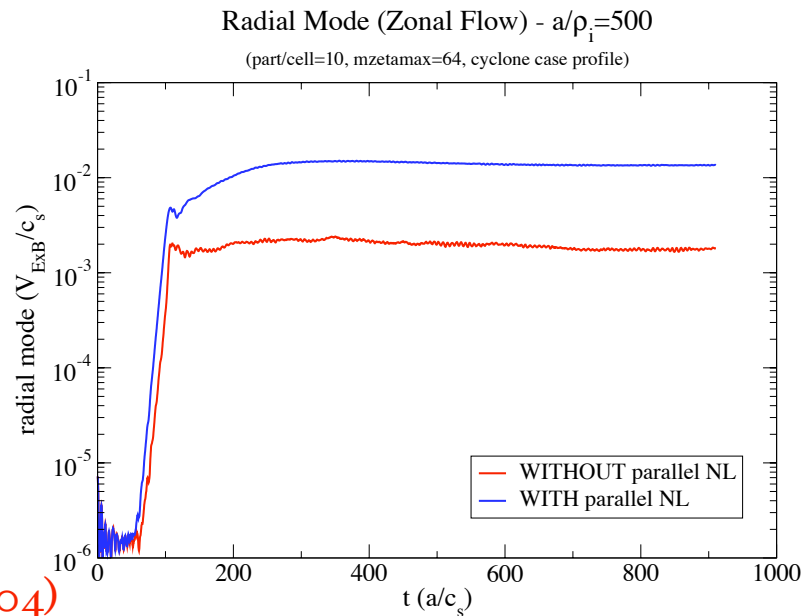
GYRO - Continuum Code

PG3EQ - Particle Code

Steady State ITG simulations with and without velocity-space nonlinearity

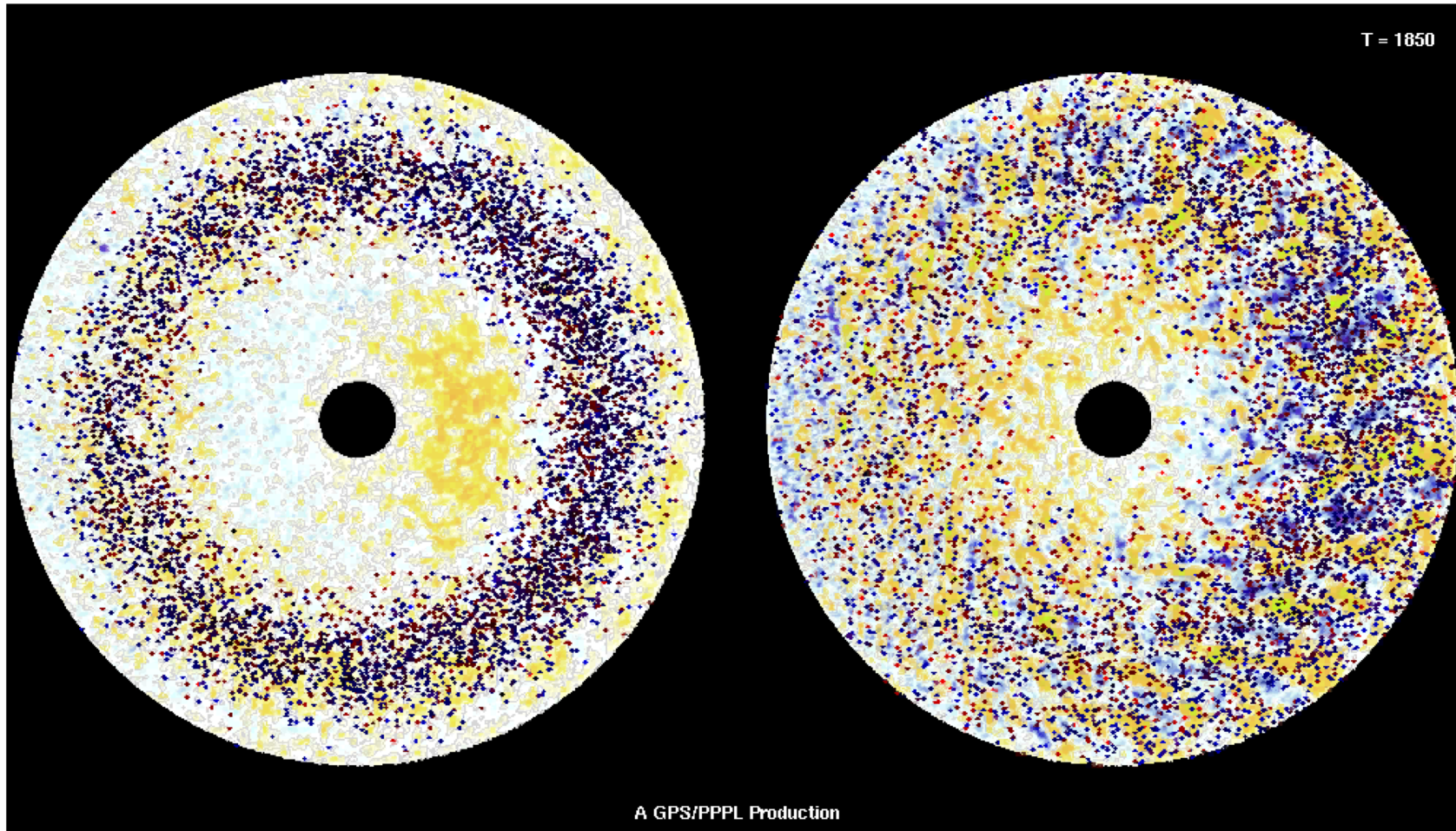


- Steady state turbulence can be achieved at much faster rate with velocity space nonlinearity, and with a lower level of thermal diffusivity and a much higher level of zonal flow



Particle Diffusion due to Toroidal ITG Modes

19



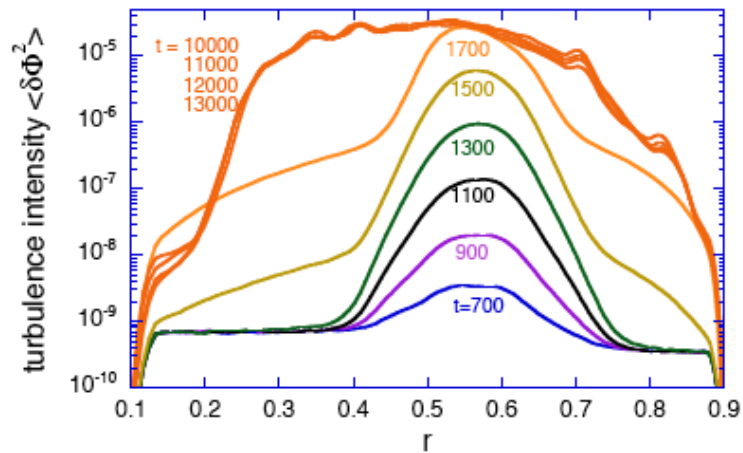
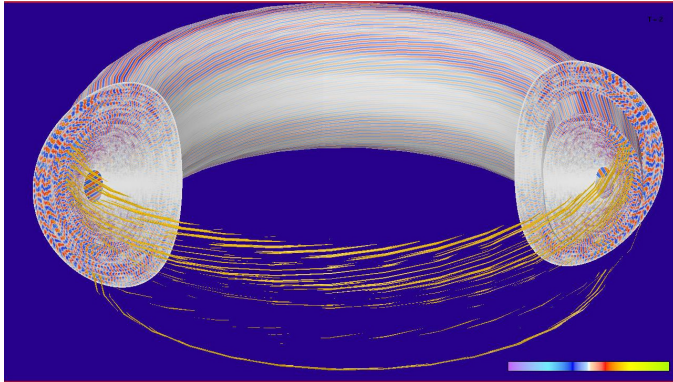
With
Parallel Velocity-Space Nonlinearity
GyroBohm?

Without
Bohm?

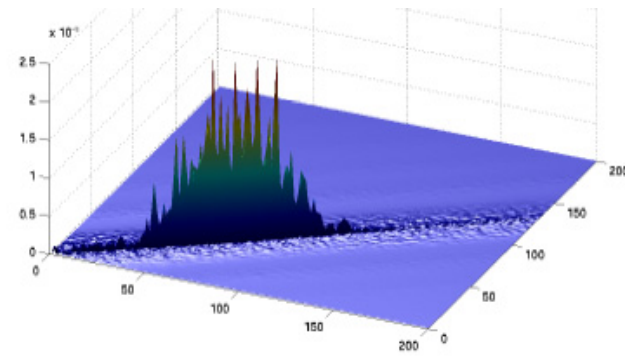
Data Management and Visualization
[Klasky, Ethier in collaboration with Beck, Ma]

GTC General Geometry

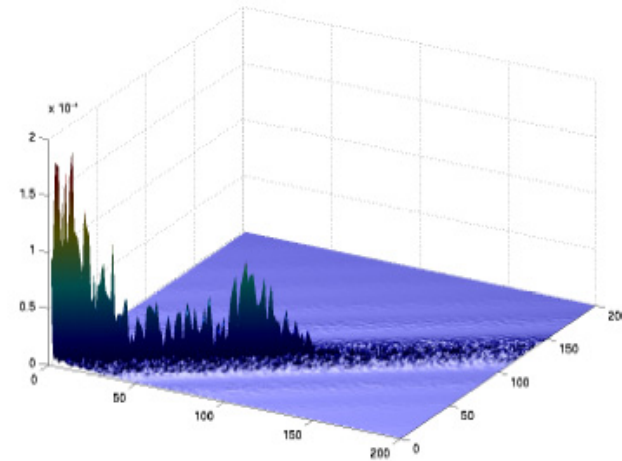
W. X. Wang



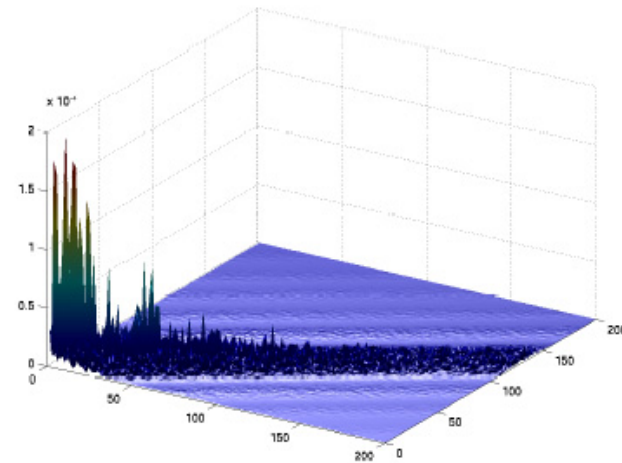
ITG Turbulence Spreading



$t=1200$



$t=2000$



$t=12400$

ITG Energy Cascade

Reduced MHD Equations vs. Gyrokinetic-MHD Equations²¹

- GK Three-field Equations for $k_{\perp} \rho_i \ll 1$ w/o geometric simplification [Lee and Qin PP '03]

$$\frac{d}{dt} \nabla_{\perp}^2 \phi + \frac{v_A^2}{c} (\hat{\mathbf{b}} \cdot \nabla) \nabla_{\perp}^2 A_{\parallel} - 4\pi \frac{v_A^2}{c^2} \nabla_{\perp} \cdot \mathbf{J}_{\perp gc}^d = 0$$

$$\frac{1}{c} \frac{\partial A_{\parallel}}{\partial t} + \mathbf{b} \cdot \nabla \phi = 0.$$

$$\frac{dp_{\alpha}}{dt} = 0$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B} \nabla \bar{\phi} \times \hat{\mathbf{b}}_0 \cdot \nabla$$

$$\mathbf{J}_{\perp gc}^d = \frac{c}{B_0} \sum_{\alpha} \left[p_{\alpha} (\nabla \times \hat{\mathbf{b}}_0)_{\perp} + p_{\alpha} \hat{\mathbf{b}}_0 \times (\nabla \ln B_0) \right]$$

- Reduced High- β Three-Field MHD Equations [Strauss PF '78]

$$\frac{d \nabla_{\perp}^2 \phi}{dt} + \frac{v_A^2}{c} (\hat{\mathbf{b}} \cdot \nabla) \nabla_{\perp}^2 A_{\parallel} - \frac{2}{R_0} \frac{\partial p}{\partial y} = 0$$

$$\frac{1}{c} \frac{\partial A_{\parallel}}{\partial t} + \mathbf{b} \cdot \nabla \phi = 0.$$

$$\frac{dp}{dt} = 0$$

Magnetic Field Calculations & GK-MHD

- For given zeroth-order field and density, parallel current and temperature profiles

$$\mathbf{J}_{\parallel gc}(\mathbf{x}) = \sum_{\alpha} q_{\alpha} \sum_{j=1}^N v_{\parallel \alpha j} \langle \delta(\mathbf{x} - \mathbf{x}_{\alpha j}) \rangle_{\varphi}$$

$$\mathbf{J}_{\perp gc}^M(\mathbf{x}) = \sum_{\alpha} q_{\alpha} \sum_{j=1}^N \langle \mathbf{v}_{\perp \alpha j} \delta(\mathbf{x} - \mathbf{x}_{\alpha j}) \rangle_{\varphi}$$

$$\mathbf{J}_{\perp gc}^d(\mathbf{x}) = \sum_{\alpha} q_{\alpha} \sum_{j=1}^N \mathbf{v}_{d\alpha j} \langle \delta(\mathbf{x} - \mathbf{x}_{\alpha j}) \rangle_{\varphi}$$

- Ampere's law

$$\nabla^2 \mathbf{A} = -\frac{4\pi}{c} \mathbf{J}_{gc}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

- FLR effects are taken into account in gyrocenter-space
- Implement these calculations in GTC, we will have temporal integration between turbulence and gyrokinetic-MHD

High-Frequency Gyrokinetics

- Gyrokinetic equation in slab geometry ($\rho/L \rightarrow 0$) as

$$\frac{\partial F}{\partial t} + (U\hat{\mathbf{b}} + \frac{c\mathbf{E} \times \hat{\mathbf{b}}}{B}) \cdot \frac{\partial F}{\partial \mathbf{R}} + \frac{q}{m}\mathbf{E} \cdot \hat{\mathbf{b}} \frac{\partial F}{\partial U} - \Omega \frac{\partial}{\partial \varphi} (F - \frac{q}{mB} \frac{\Phi \partial F}{\partial \mu}) - \Omega \frac{\partial}{\partial \mu} (\frac{q}{mB} \frac{\Phi \partial F}{\partial \varphi}) = 0.$$

- For $F = F_0 + \delta f$ and $\partial F_0 / \partial \varphi = 0$,

$$\frac{\partial \delta f}{\partial t} + (U\hat{\mathbf{b}} + \frac{c\mathbf{E} \times \hat{\mathbf{b}}}{B}) \cdot \frac{\partial (F_0 + \delta f)}{\partial \mathbf{R}} + \frac{q}{m}\mathbf{E} \cdot \hat{\mathbf{b}} \frac{\partial (F_0 + \delta f)}{\partial U} - \Omega \frac{\partial}{\partial \varphi} (\delta f - \frac{q}{mB} \frac{\Phi \partial F_0}{\partial \mu}) = 0,$$

- Separate gyrocenter motion from gyromotion: $\delta f = \delta f_g(\mathbf{R}) + g(\varphi)$,

$$\frac{dg}{dt} \equiv \frac{\partial g}{\partial t} - \Omega \frac{\partial g}{\partial \varphi} = -\Omega \frac{\partial}{\partial \varphi} (\frac{q}{mB} \frac{\Phi \partial F_0}{\partial \mu}) \rightarrow \frac{dg}{dt} = -\mathbf{v}_\perp \cdot \mathbf{E} \frac{q}{mB} \frac{\partial F_0}{\partial \mu}.$$

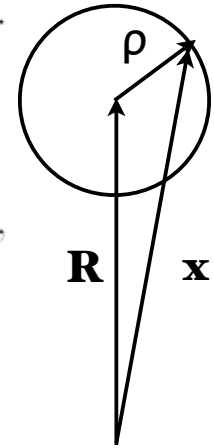
- To the next order, we have

$$\frac{df_g}{dt} \equiv \frac{\partial \delta f_g}{\partial t} + (U\hat{\mathbf{b}} + \frac{c\mathbf{E} \times \hat{\mathbf{b}}}{B}) \cdot \frac{\partial \delta f_g}{\partial \mathbf{R}} + \frac{q}{m}\mathbf{E} \cdot \hat{\mathbf{b}} \frac{\partial \delta f_g}{\partial U} = -\frac{c\mathbf{E} \times \hat{\mathbf{b}}}{B} \cdot \frac{\partial F_0}{\partial \mathbf{R}} - \frac{q}{m}\mathbf{E} \cdot \hat{\mathbf{b}} \frac{\partial F_0}{\partial U},$$

- Dispersion Relation:
[Qin et al. PoP, 1999.]

$$1 = \sum_j \frac{1}{k^2 \lambda_D^2} \sum_{n=1}^{\infty} \frac{2n^2}{(\omega/\Omega)^2 - n^2} \exp(-\frac{k^2 T}{\Omega^2 m}) I_n(\frac{k^2 T}{\Omega^2 m})$$

- Need efficient numerical schemes for wave heating.



Integrated Gyrokinetic Particle Simulation of Burning Plasmas in Time and Space

- Microturbulence simulation including electron dynamics and finite- β physics:
 - Time step set by electron zeroth-order orbit, e.g., $0.1 \mu\text{sec}$.
 - FL-coord. + Split-weight + Adiabatic field pusher
 - Simulation time to reach steady-state turbulence: e.g. 1 msec .
 - Transport time scale: e.g. 1 sec .
- Gyrokinetic MHD
 - MHD physics and equilibrium with FLR effects and compressional Alfvén effects
- Wave Heating
 - Time step to resolve Bernstein harmonics: $0.01 \mu\text{sec}$
- Disparate spatial scales: $c/\omega_{pe} \approx 0.05 \text{ cm}$, $\rho_i = 0.5 \text{ cm}$, $a = 1 \text{ m}$.
- Core-Edge Integration

Core-Edge Simulations via GTC

- Basic requirements for the validity of gyrokinetic Vlasov-Maxwell equations are:

$$\rho/L_B \sim o(\epsilon),$$

$$\partial F/\partial \phi = 0,$$

$$d\mu_B/dt = 0.$$

- GTC already has Lorentz collision operators for e-i, and momentum and energy conserving collision operators for like species.

- The core uses the δf scheme of

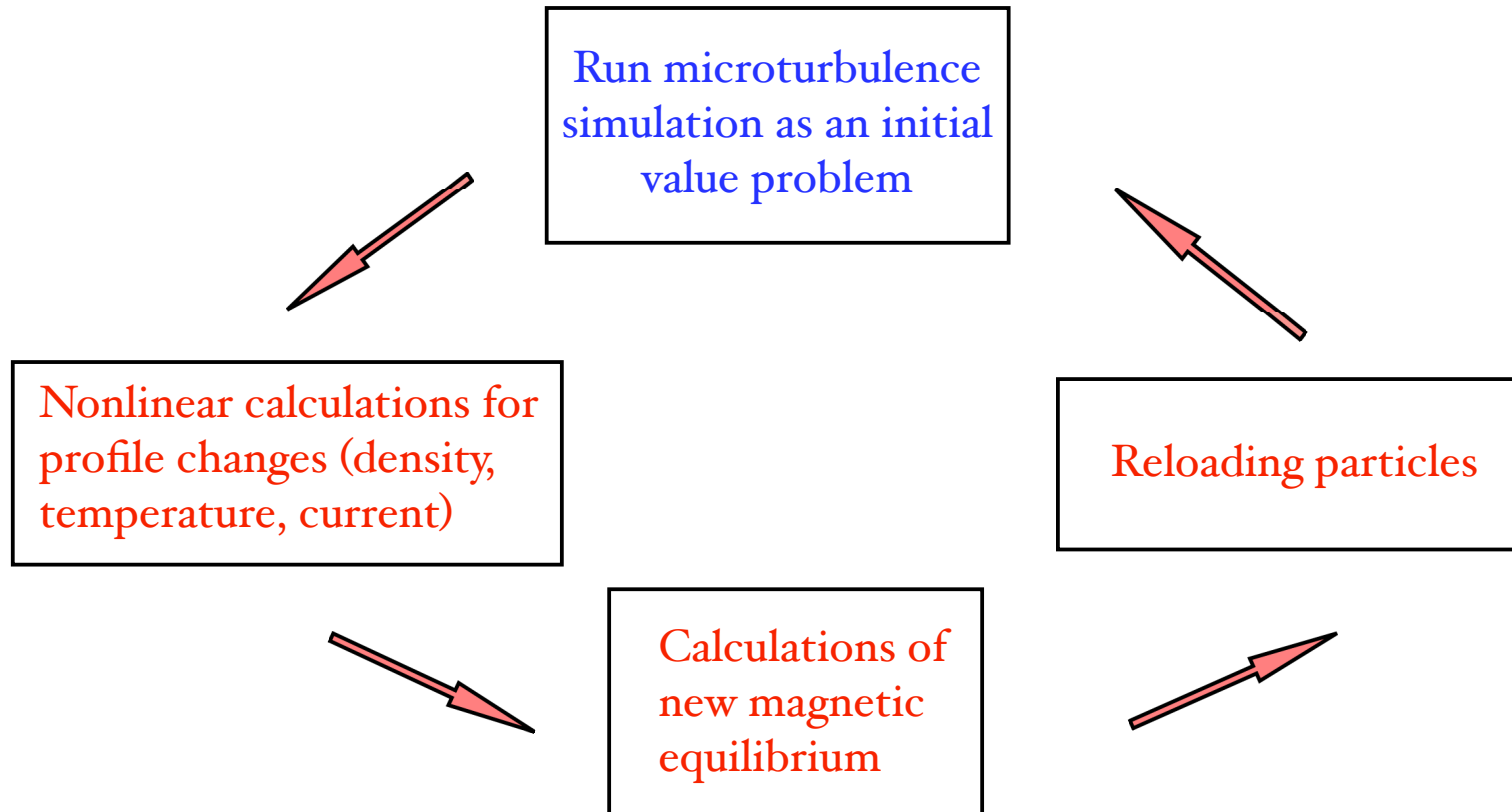
$$\frac{D\delta f}{DT} = -\frac{DF_0}{Dt}.$$

- The edge uses the δf scheme of

$$\delta f = F - F_0.$$

- Core-edge simulation inside the separatrix using GTC with electrons and multi-species ions is feasible.

Turbulence Simulation in Transport Time Scale



Conclusions

- Gyrokinetic Particle Simulation is a vital tool for fusion research
- Gyrokinetic formalism is most suitable for tokamak and stellarator physics when FLR effects, inertial effects and linear and nonlinear wave-particle interactions are important
- Gyrokinetic PIC toroidal simulation is an international effort:
GTC, GEM, PG₃EQ, GT₃D (JAERI), ORB₅(CRPP)
- SciDAC GPS Center : Finite- β physics, Poisson solver, Turbulent Transport for ITER Plasmas
- Team coding: version control, developer's manual, OO programming
- Parallelization, Optimization, Visualization and Data Management.
- Also involved with Fusion Simulation Prototype Center for Plasma Edge Simulation (CPES) and the proposal for Multiscale Mathematics to DoE.