PROBLEM SET 8
NE 290 H BARNARD & LUND

(1)

20 POINTS

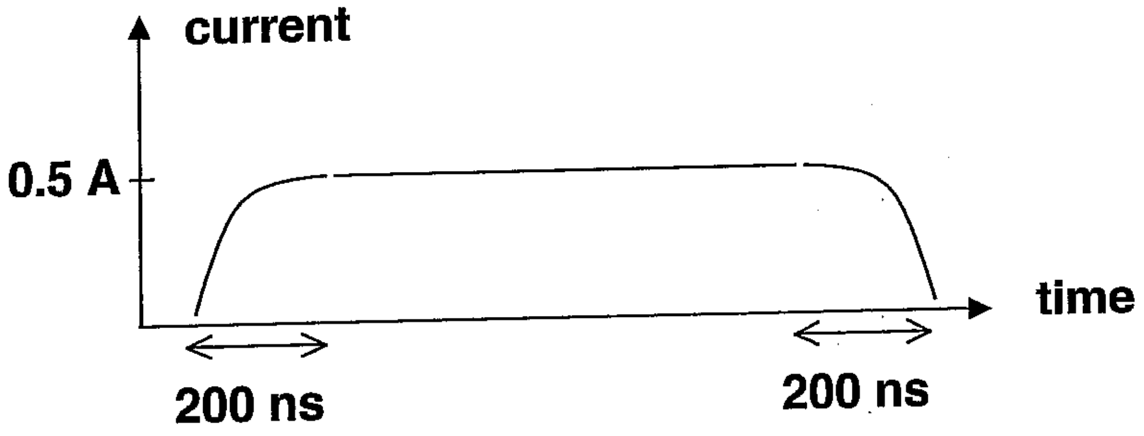
A 0.5 A, 2 MeV potassium$^+$ (A=39) beam is injected into a transport section with a 25 cm half-lattice period L. There are 10 half-lattice periods in the transport section. The beam has a flattop of 1 μs. Assume $g = 2\ln\frac{r_p}{a} = 1.0$. ($r_p/a \cong 1.65$).

a). What is the beam velocity?

b). What is the space charge wave speed in the comoving beam frame?

c). What will the duration of the beam flattop be at the end of the transport section? (Assume no ear fields are applied, and assume a square pulse [with instantaneous rise and fall] for this calculation, at the beginning of the transport section.)

d). For a 200 ns long head and tail, with parabolic fall off (see figure), how large an "ear" field is required to keep the beam from spreading longitudinally?



(2)

~20 POINTS

A velocity perturbation $\overline{z_1'}$ on a long coasting beam with center position $s = s_0$ has the initial form:

$$\overline{z_1'} = \delta \exp\left[\frac{-z^2}{\Delta^2}\right]$$

There is no initial density perturbation ($\lambda_1(z) = 0$). The space charge wave velocity of the beam is c_s and the beam velocity is v_0.

What is the density of the perturbation after the beam center propagates a distance s-s_0?
What is the velocity perturbation $\overline{z_1'}$ for the same location of the beam center?

Sketch λ_1 and $\overline{z_1'}$ vs. z at a point when $s - s_0 > v_0 \Delta / c_s$.

20 POINTS

(3)

IN CLASS, IT WAS ASSERTED THAT THE CHILD LANGMUIR SOLUTION BREAKS DOWN WHEN THE BEAM VELOCITY V_z IS OF THE ORDER OF THE THERMAL VELOCITY $V_{th} = \sqrt{\frac{kT}{m}}$, WHERE kT IS THE SOURCE temperature.

a) EXPRESS THIS DISTANCE z_0 IN TERMS OF kT, beam charge q, diode voltage V_0, and diode gap distance d.

b). Calculate the Child-Langmuir charge density ρ at this distance, and the plasma frequency $\omega_p \equiv \left(\frac{q\rho}{\varepsilon_0 m}\right)^{1/2}$.

c) Calculate the Debye length $\lambda_D = V_{th}/\omega_p$ using the density from part b and at temperature kT. What is the ratio of the distance found in part a) to the Debye length? (z_0/λ_D)

d). For a source temperature $kT = 0.1\,eV$ and a diode voltage of $100\,kV$ what is the ratio of the z_0/d? (ASSUME $qV_0 = 100\,keV$).