PROBLEM SET 10
NE 290H
BUKANAN & LUND

DUE 4/15/2009

40 POINTS

PROBLEM (1)

Let the equation of motion for a test particle oscillating in a mismatched beam be modeled by:

$$x'' + k_p^2 x = -\frac{Q}{r_{b0}^6} x^5 + \frac{2\epsilon Q}{r_{b0}^2} x \cos k_B s$$

Let $\frac{x}{r_{b0}} = A \sin\psi$ & $\frac{x'}{r_{b0}} = k_p A \cos\psi$

where $\psi \equiv k_p s + \alpha$

a) CALCULATE A' & α' IN TERMS OF $A, \psi,$ & $k_B s$ (and parameters).

Let $\Psi_r' = 2k_p - k_B + 2\alpha_r'$

b). AVERAGE OVER RAPID VARIATIONS TO OBTAIN EQUATIONS FOR THE AMPLITUDE AND PHASE OF THE NEAR RESONANT PARTICLES. (i.e. find $A_r'(\Psi_r, A_r)$ & $\Psi_r'(\Psi_r, A_r)$.

c). DEFINE $w = A_r^2$. FIND $w'(\Psi_r, w)$ & $\Psi_r'(\Psi_r, w)$.

d). FIND THE HAMILTONIAN H, such that

$$w' = \frac{\partial H(w, \Psi_r)}{\partial \Psi_r} \quad \text{and} \quad \Psi_r' = -\frac{\partial H(w, \Psi_r)}{\partial w}$$

e). Verify that $H(w, \Psi_r)$ is a constant.

HINT FOR PART (b): $\sin^6 x = \frac{1}{32}(10 - 15\cos 2x + 6\cos 4x - \cos 6x)$

$\cos x \sin^5 x = \frac{1}{32}(5\sin 2x - 4\sin 4x + \sin 6x)$

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PROBLEM 2 [20 POINTS]

1. SUPPOSE THE SPACE CHARGE ELECTRIC FIELD EXTERIOR TO THE BEAM VARIES AS:

$E_r \sim \frac{\lambda}{4\pi\epsilon_0} \frac{r_{b0}^2}{r^3}$ INSTEAD OF $E_r \sim \frac{\lambda}{4\pi\epsilon_0 r}$.

WOULD YOU EXPECT THE RADIAL EXTENT OF A BEAM HALO DUE TO MISMATCH TO BE LARGER OR SMALLER THAN THE ONE OBEYING THE $1/r$ DEPENDENCE?

HINT: CONSIDER THE LOCATION OF THE RADIUS WHERE THE PARTICLE - ENVELOPE RESONANCE IS STRONGEST. (ASSUME HALO EXTENT IS PROPORTIONAL TO THIS RADIUS).

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