Dynamics of Open-field-line MHD configurations

P. M. Bellan

A. L. Moser, E. V. Stenson

Caltech

Recall slide from Shibata-san's MR2010 presentation:

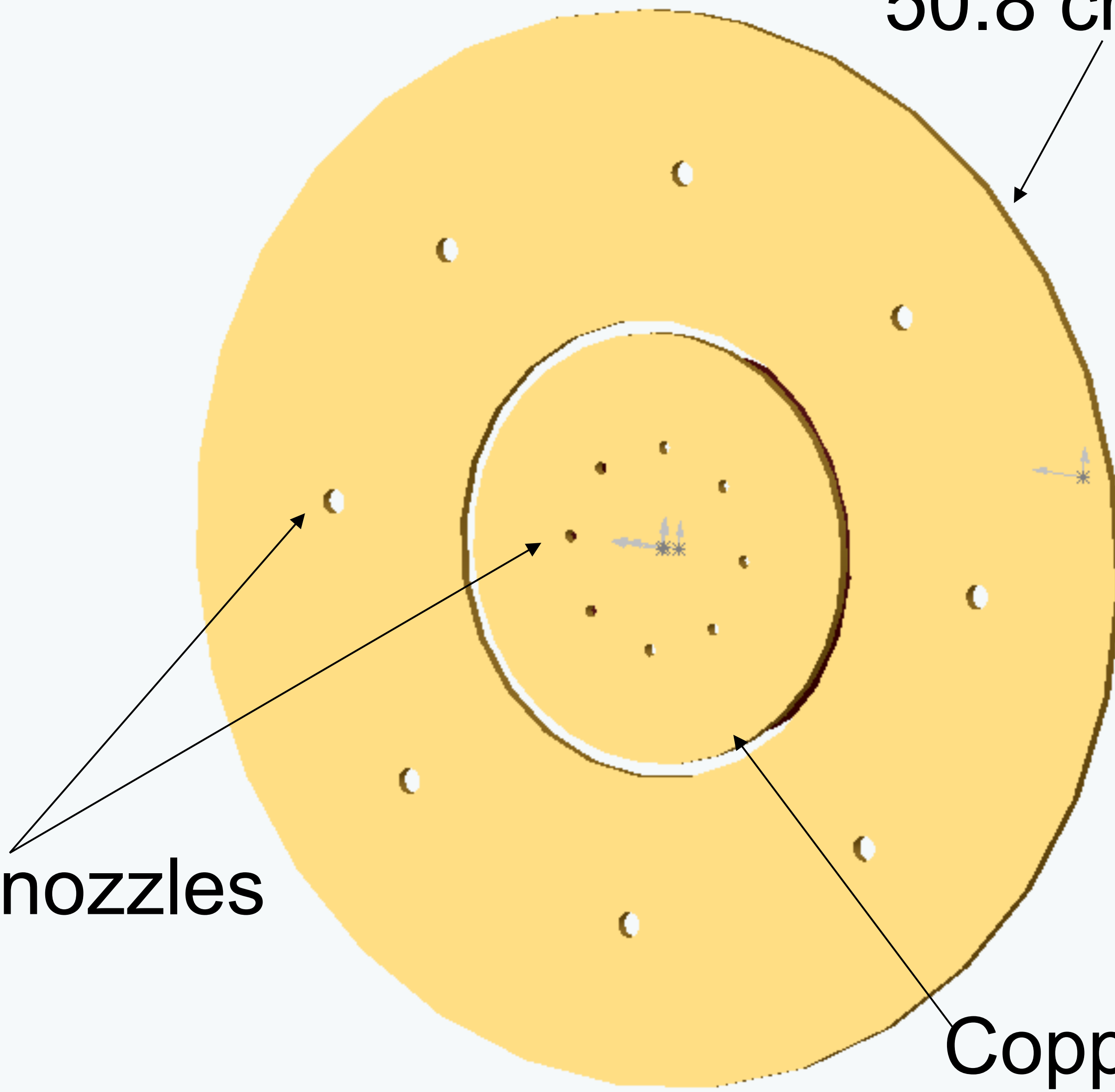
- Fundamental puzzle inherent to solar reconnection
- Microscopic plasma scale (ion Larmor radius or ion inertial length = 10 –100 cm) is much smaller than the size of a flare (= 10^9 cm)
- So even if micro-scale plasma physics is solved, there remains fundamental puzzle how to connect micro and macro scale physics to explain solar flares

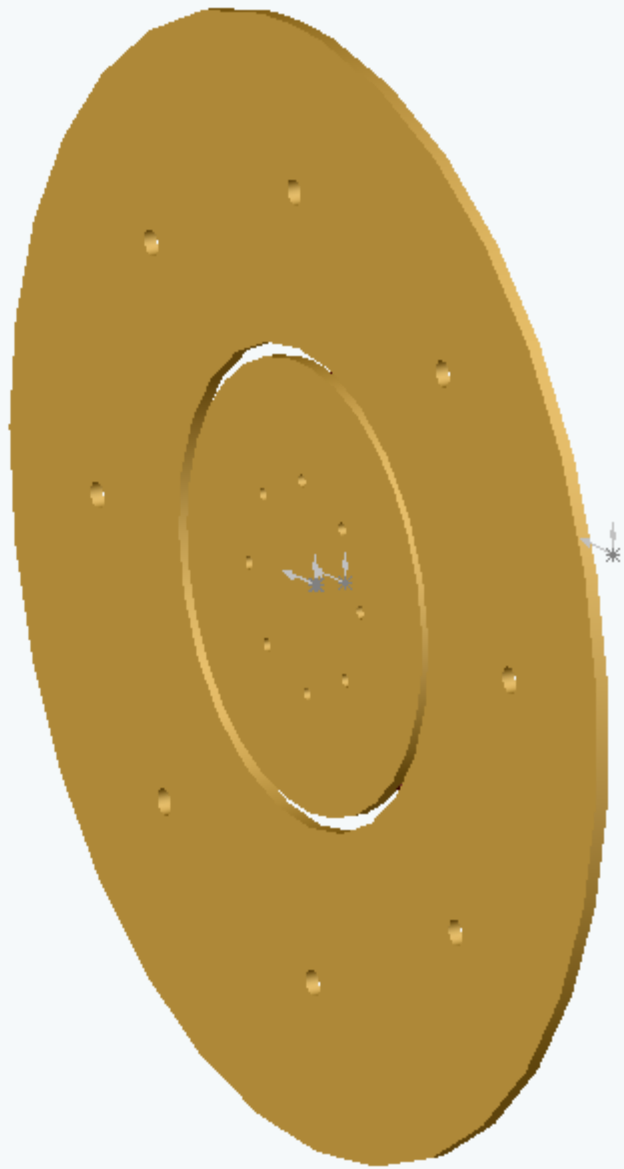
Provide here one solution
to this fundamental puzzle

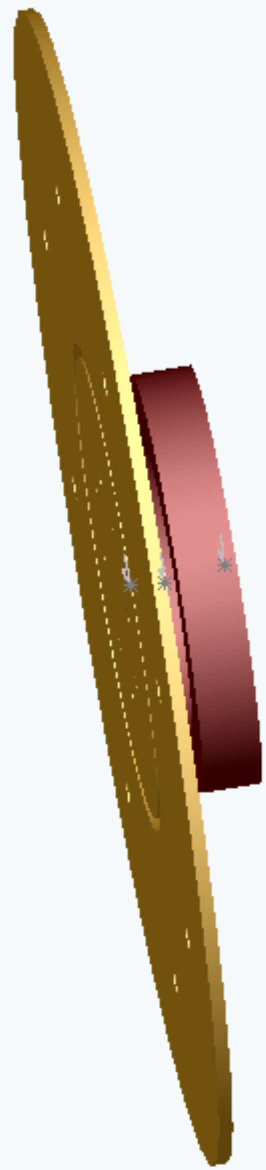
experimentally observed

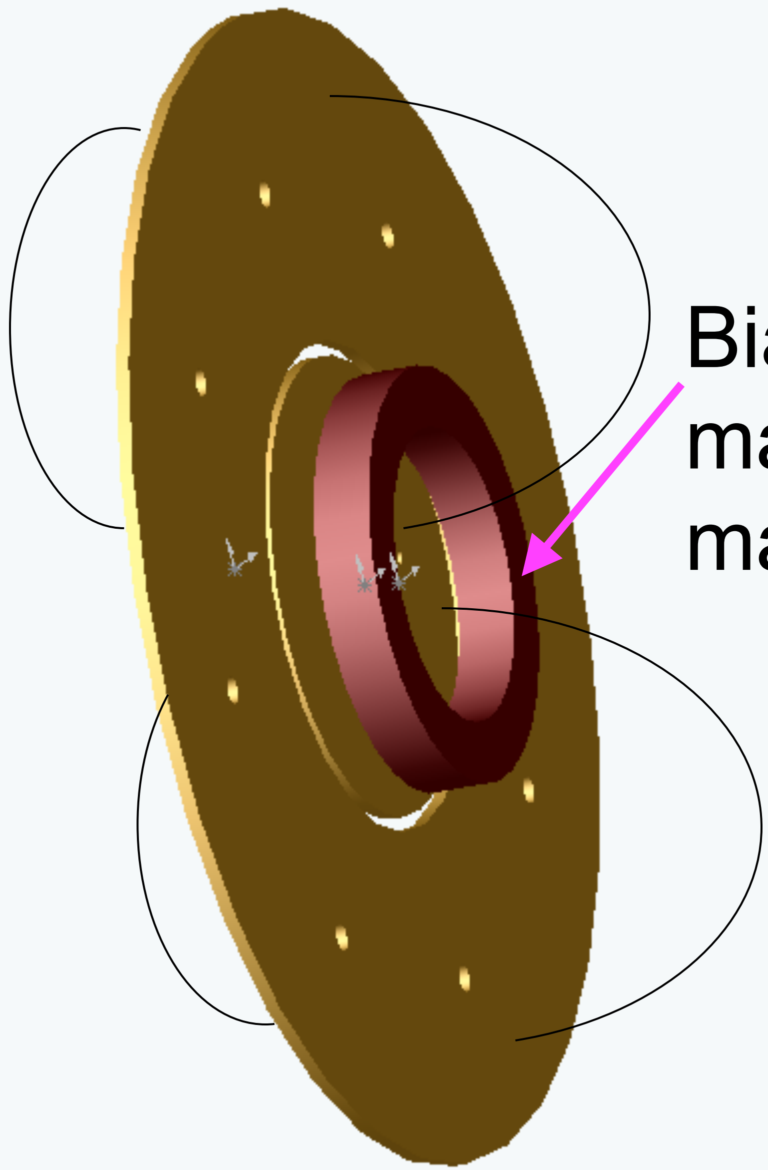
Experimental Setup

Coaxial, co-planar electrodes

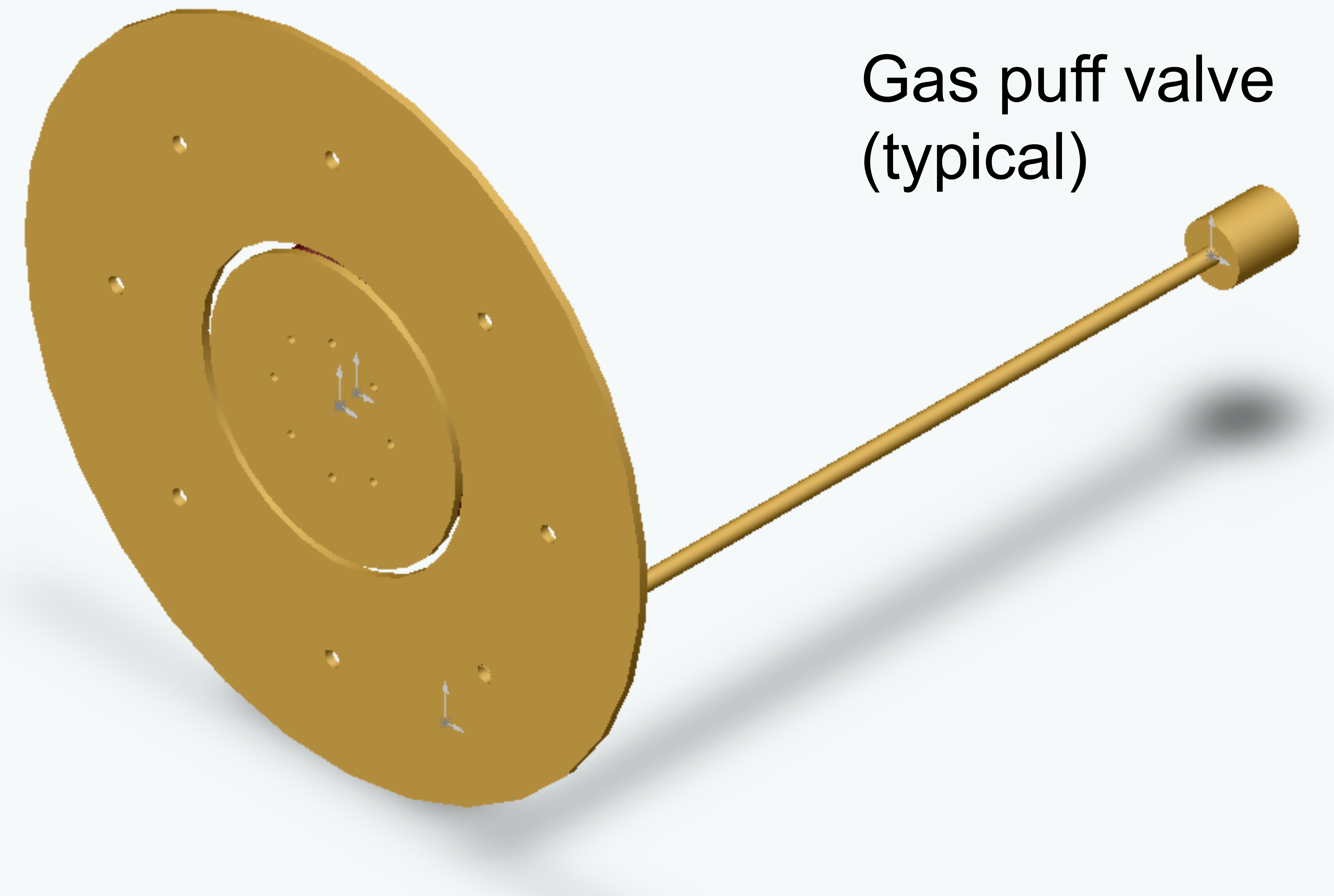


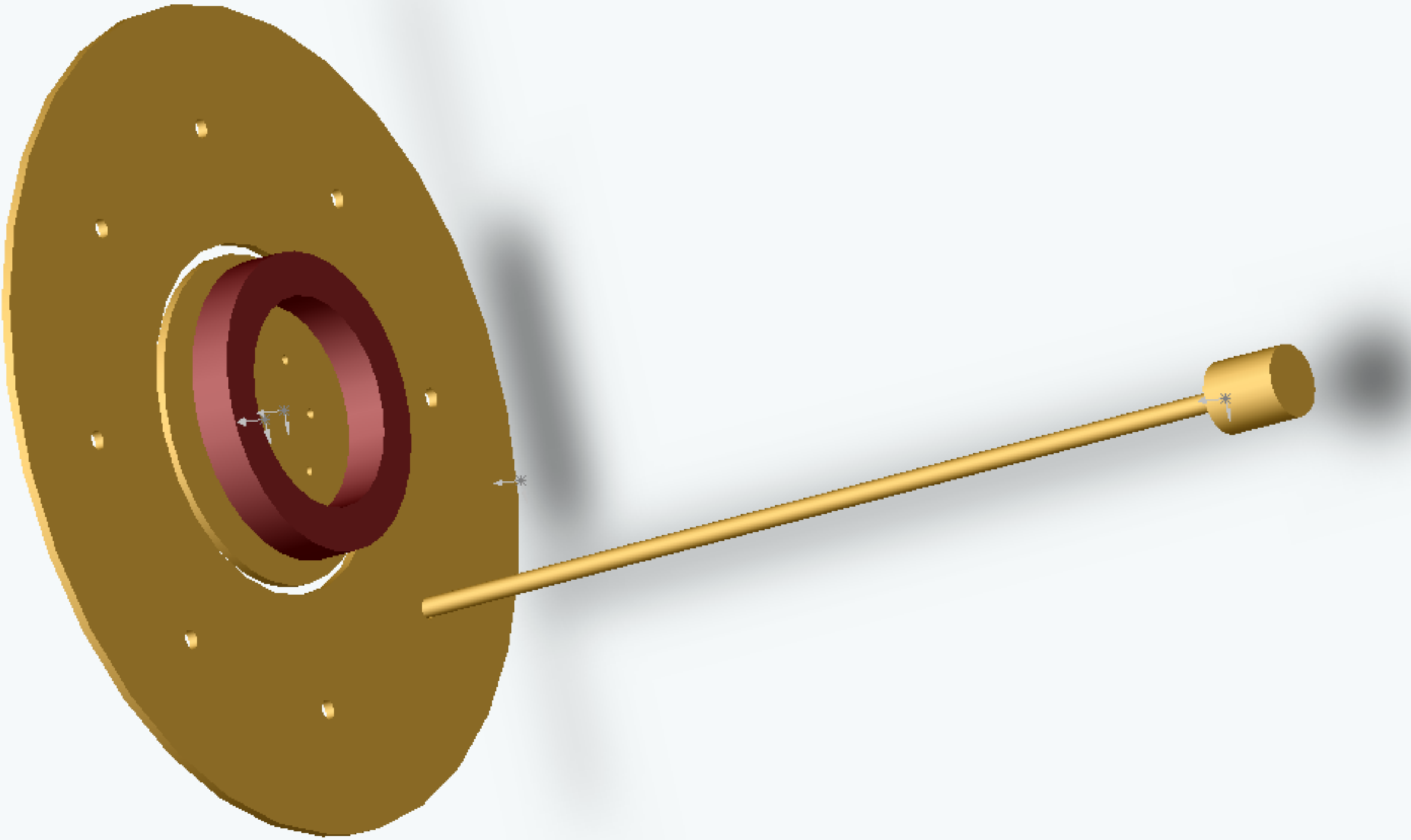


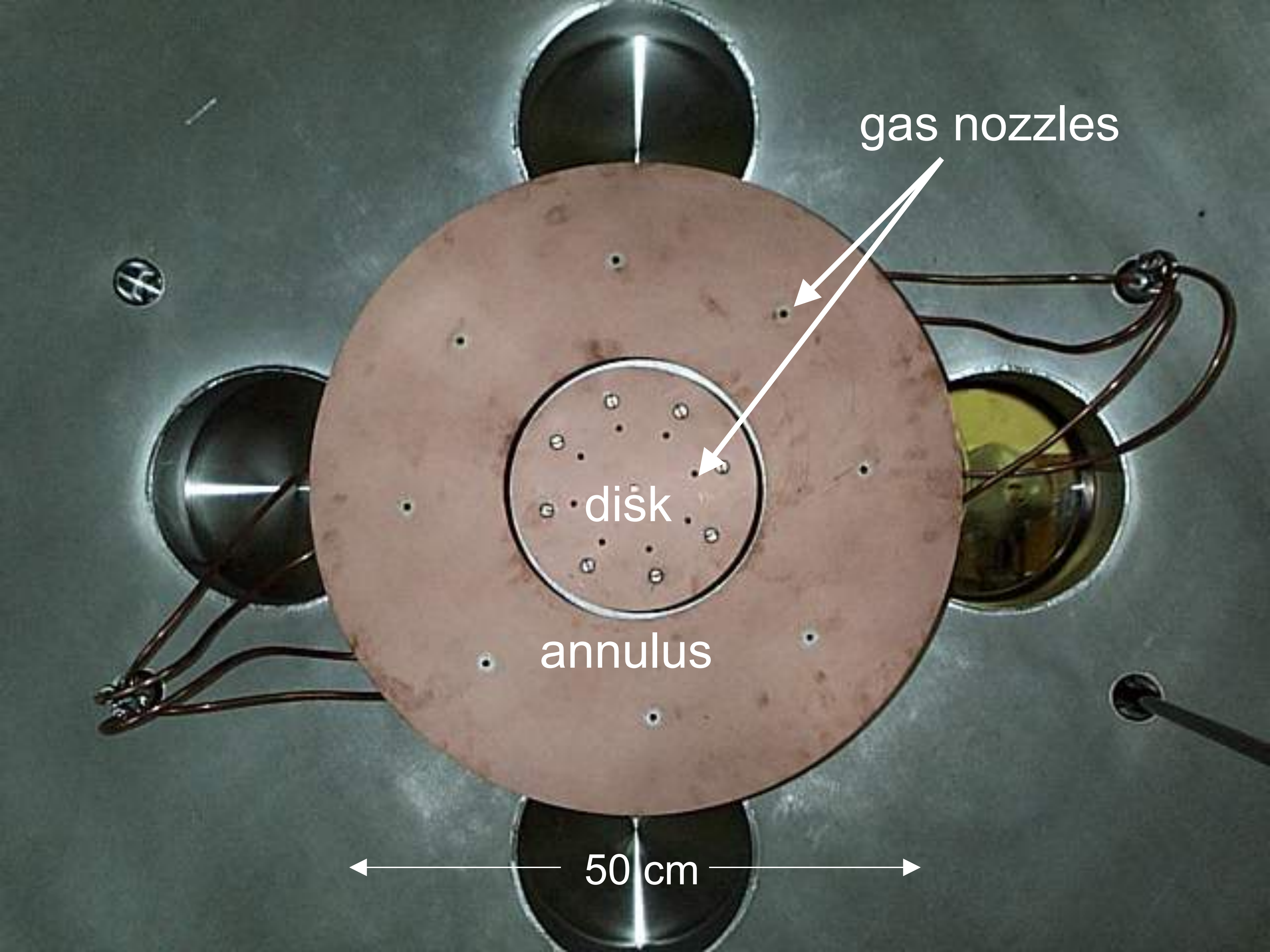




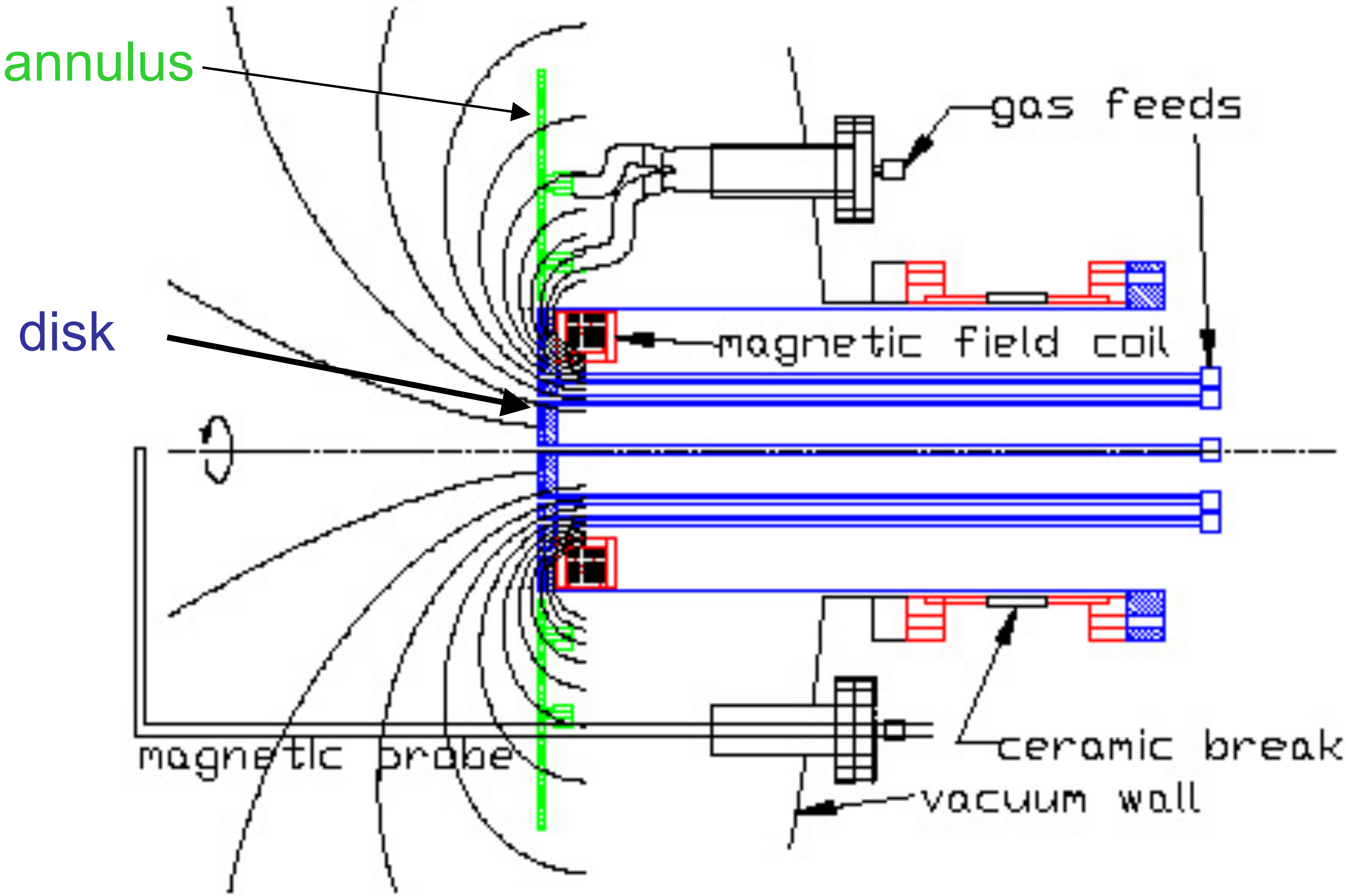
Bias field coil makes linked magnetic flux


Gas puff valve
(typical)

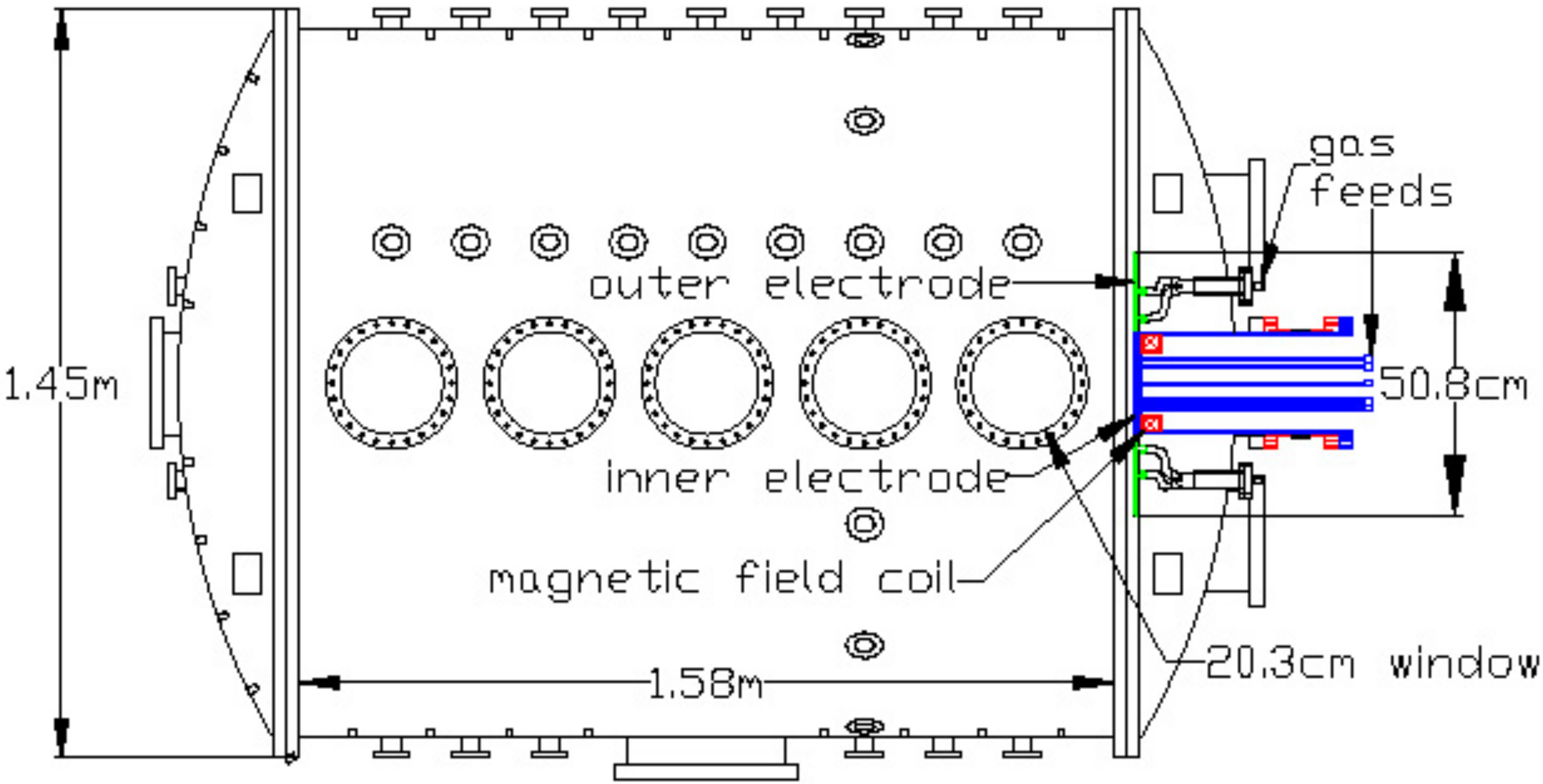


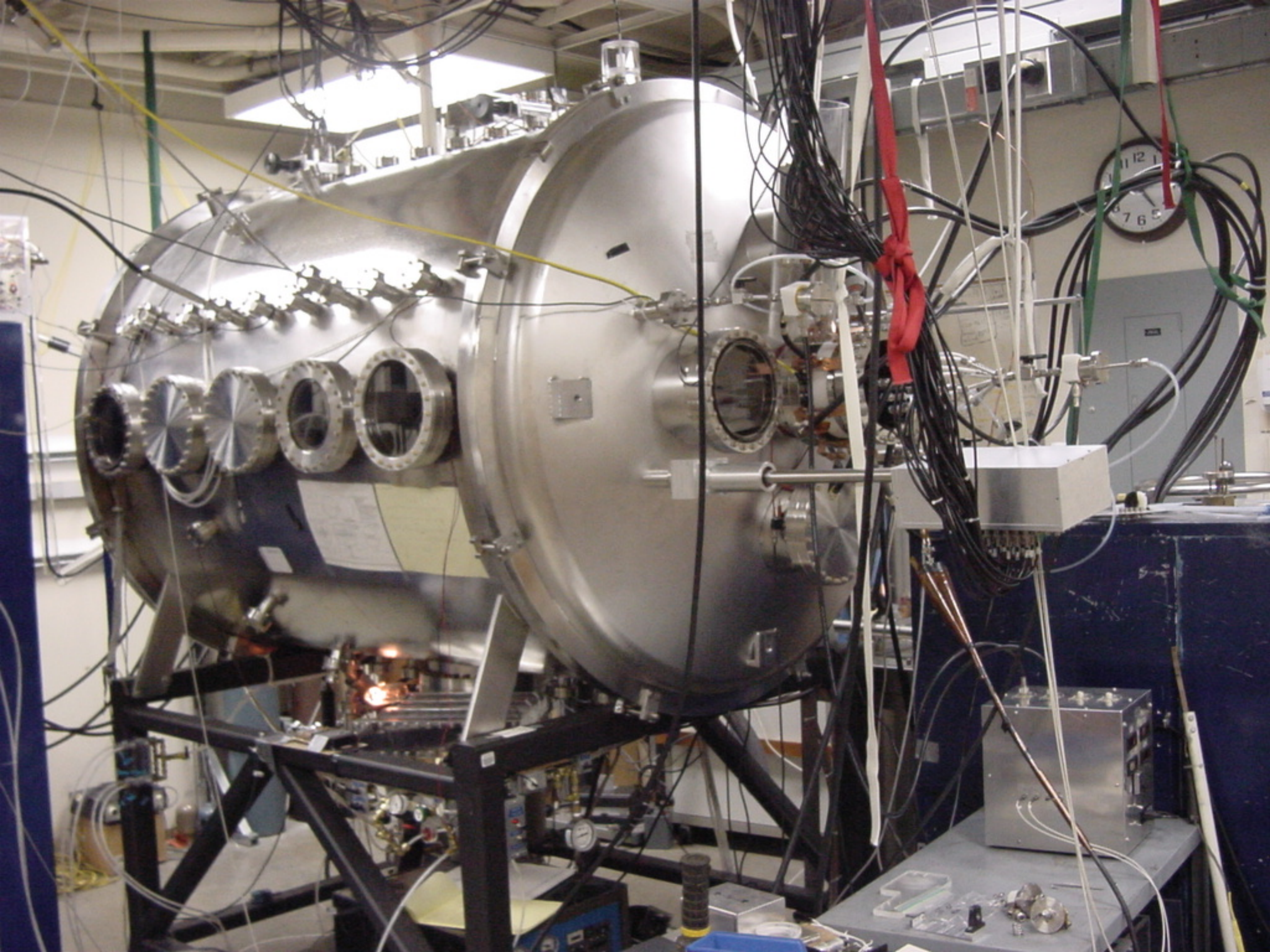

gas nozzles
disk
annulus
50 cm

Side View


annulus
disk
gas feeds
magnetic field coil
magnetic probe
ceramic break
vacuum wall

Installation

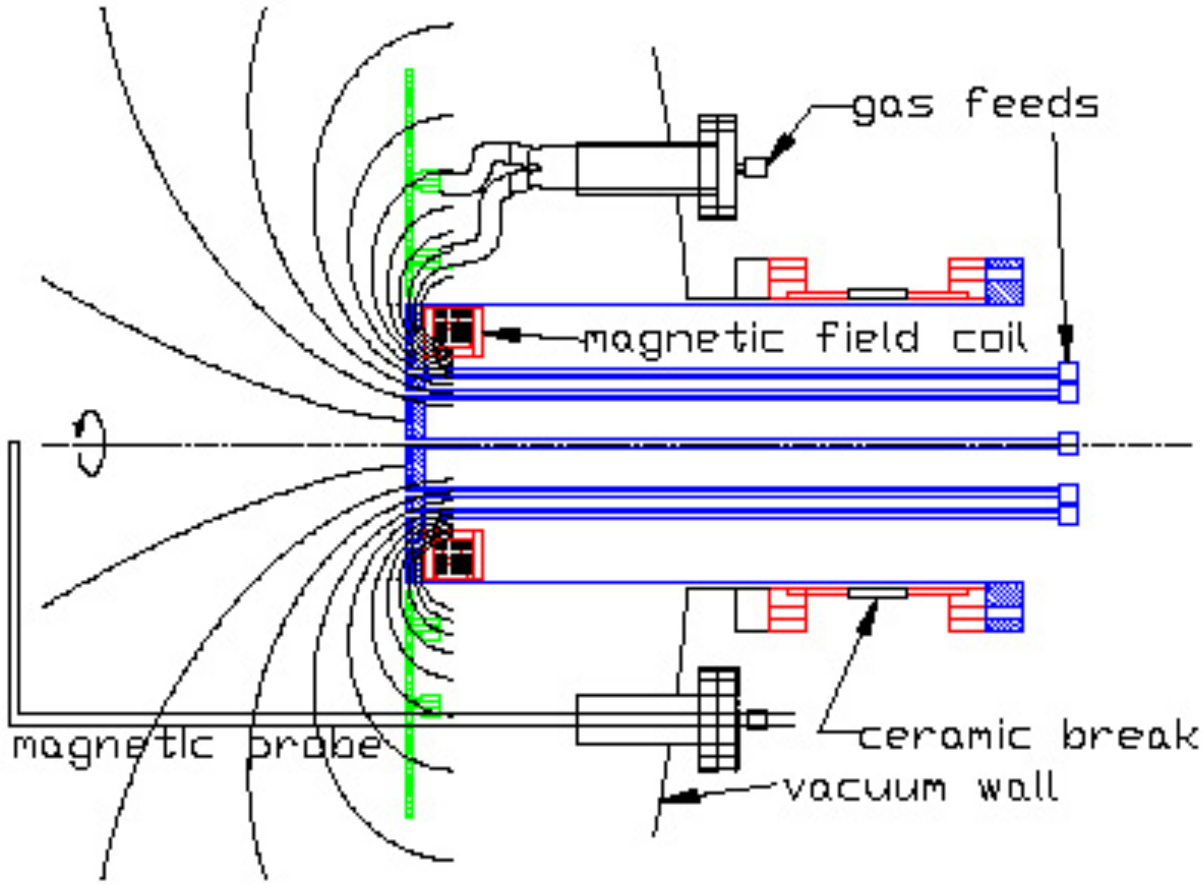




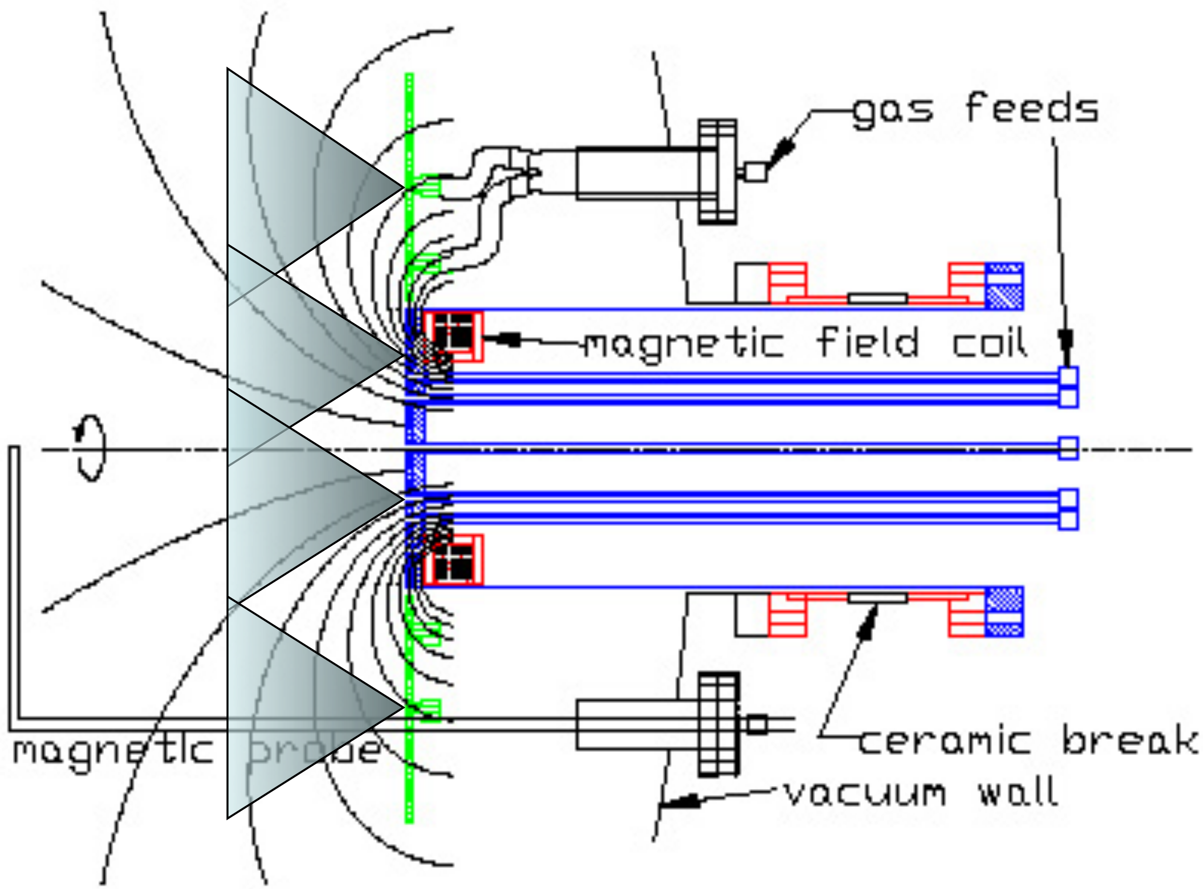
Sequence

Bias field

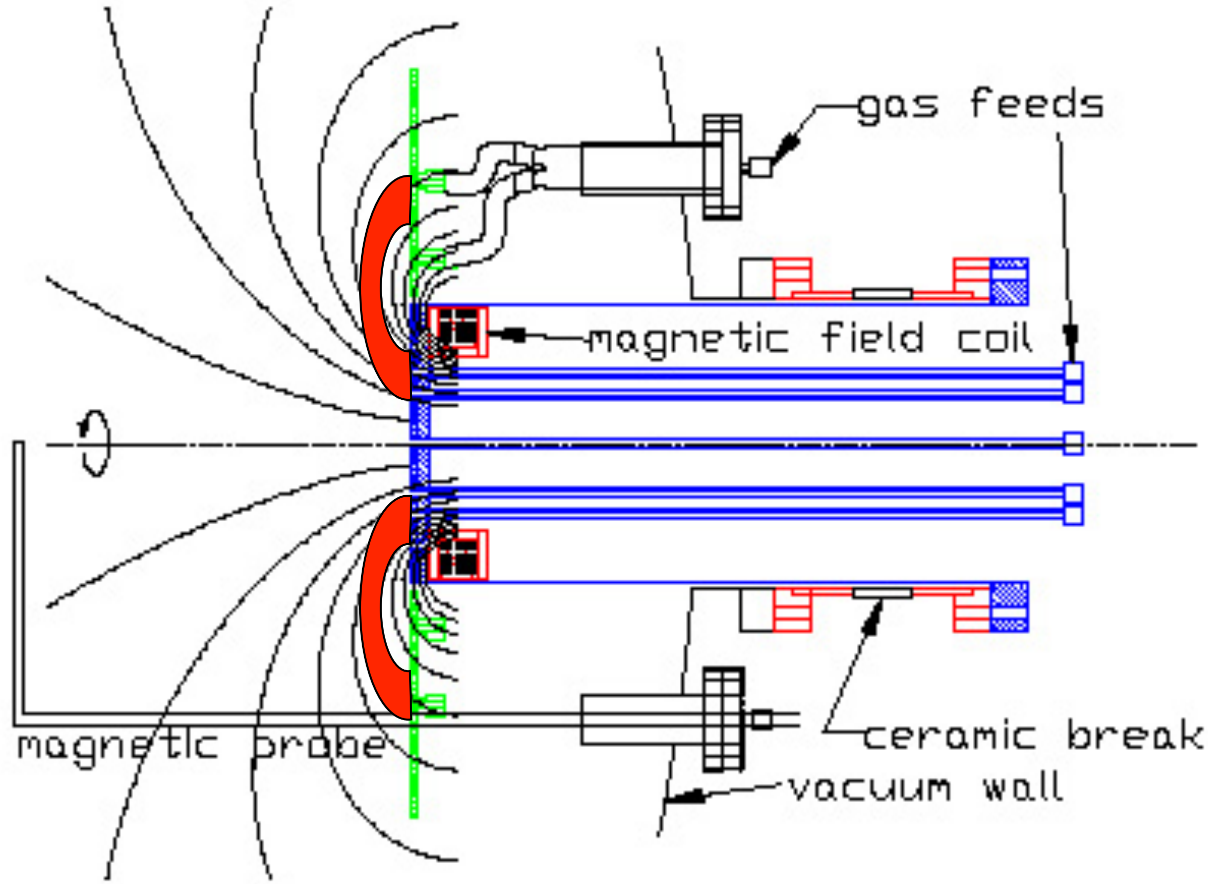
high vacuum



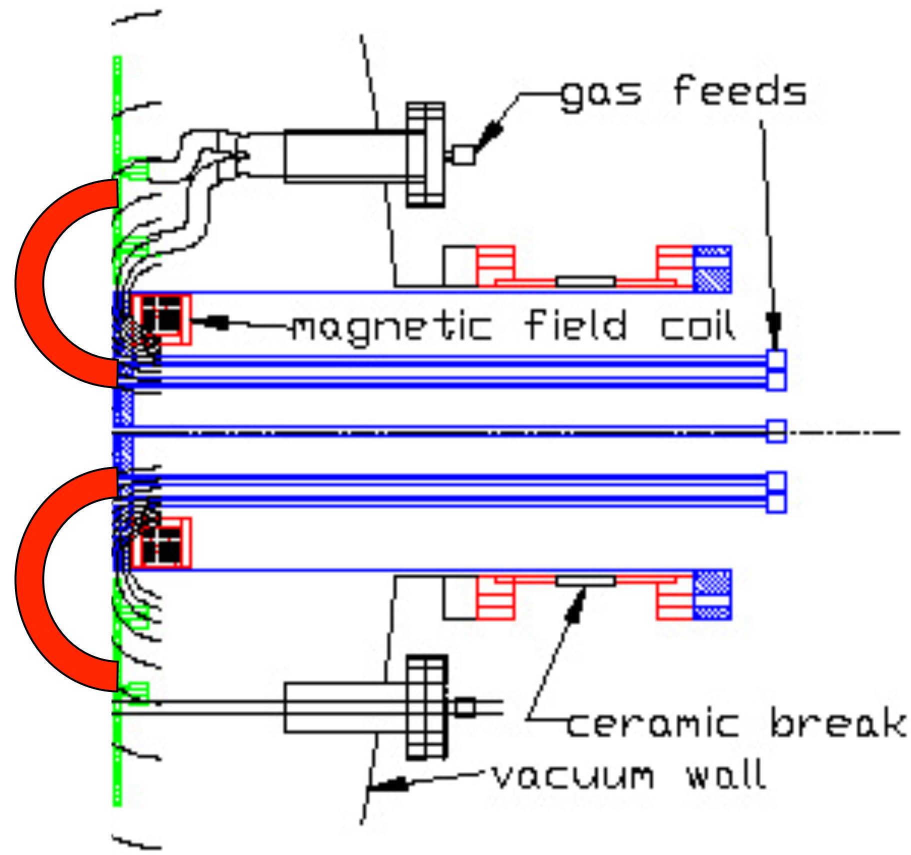
Puff in neutral gas
Neutral spatial/temporal profile measured using fast ion gauge probe

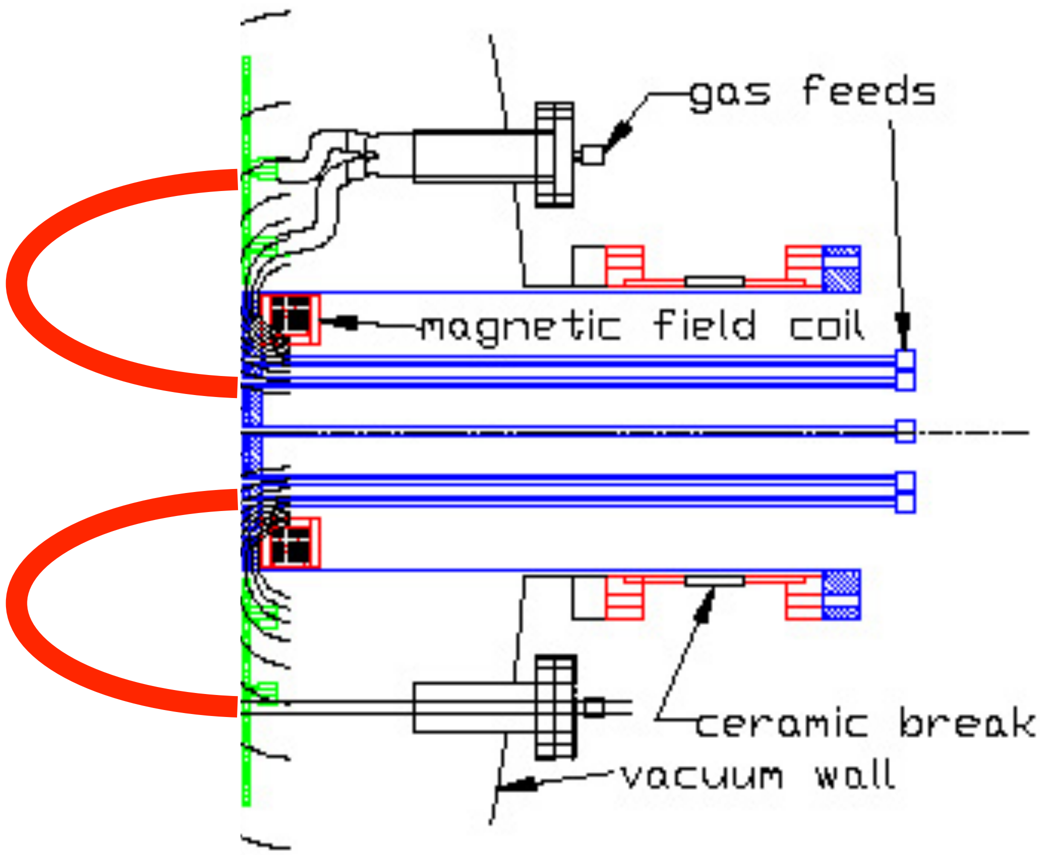


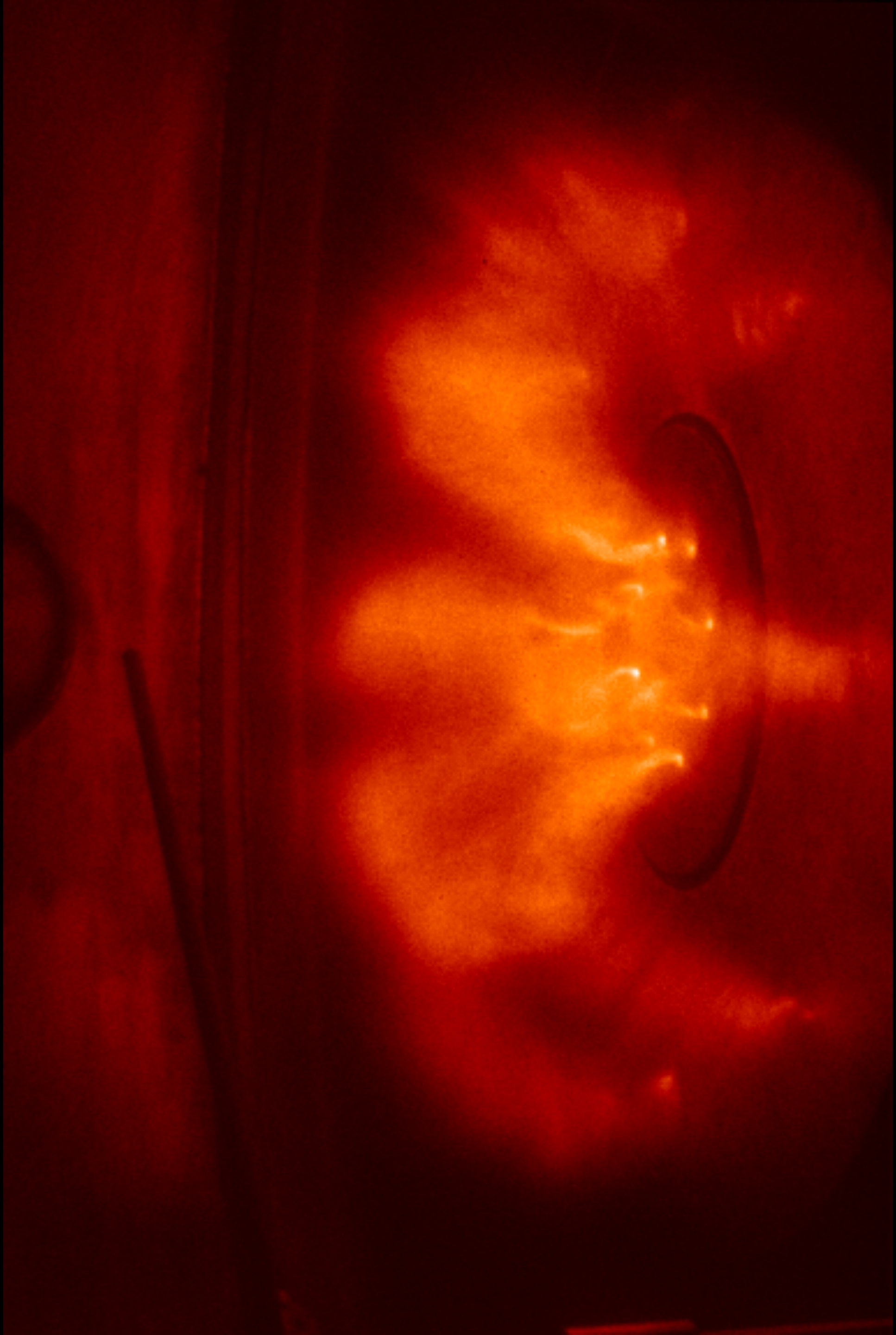
Breakdown, “spider leg formation”



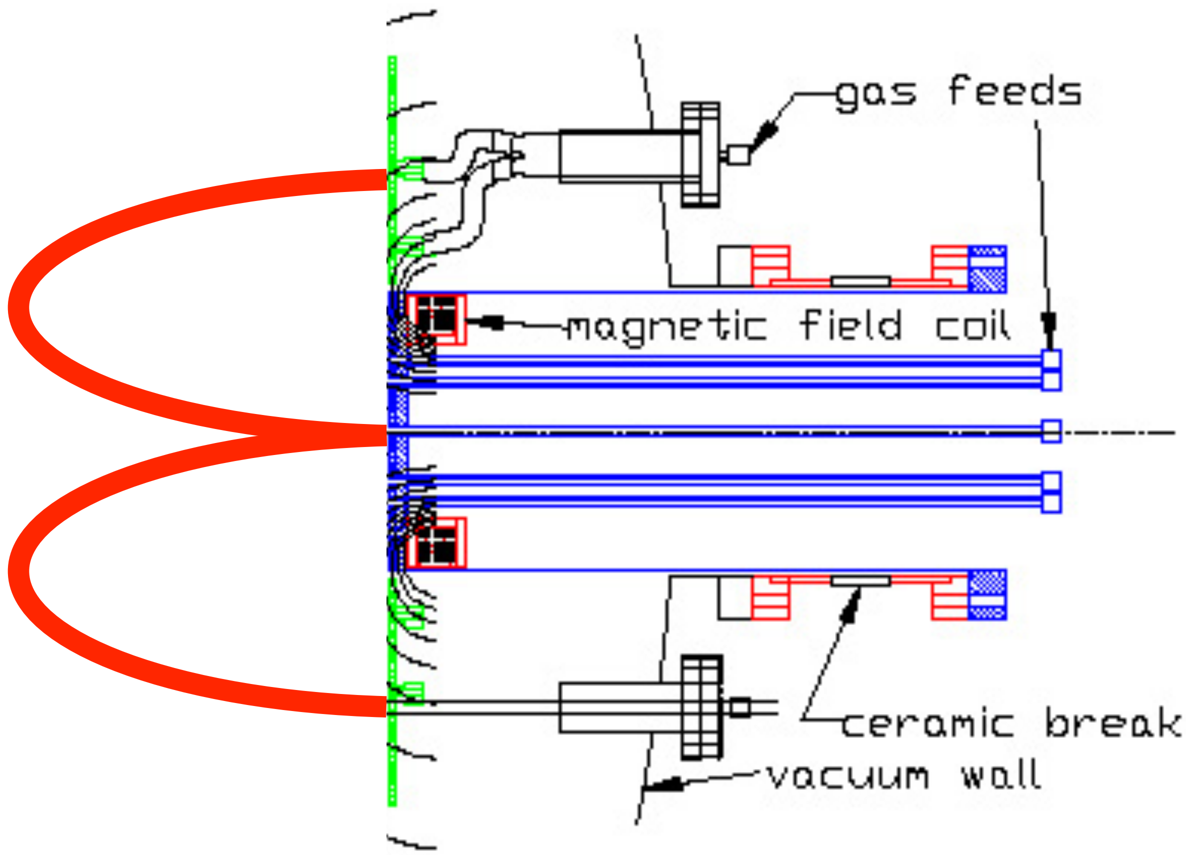
Spider legs get bigger

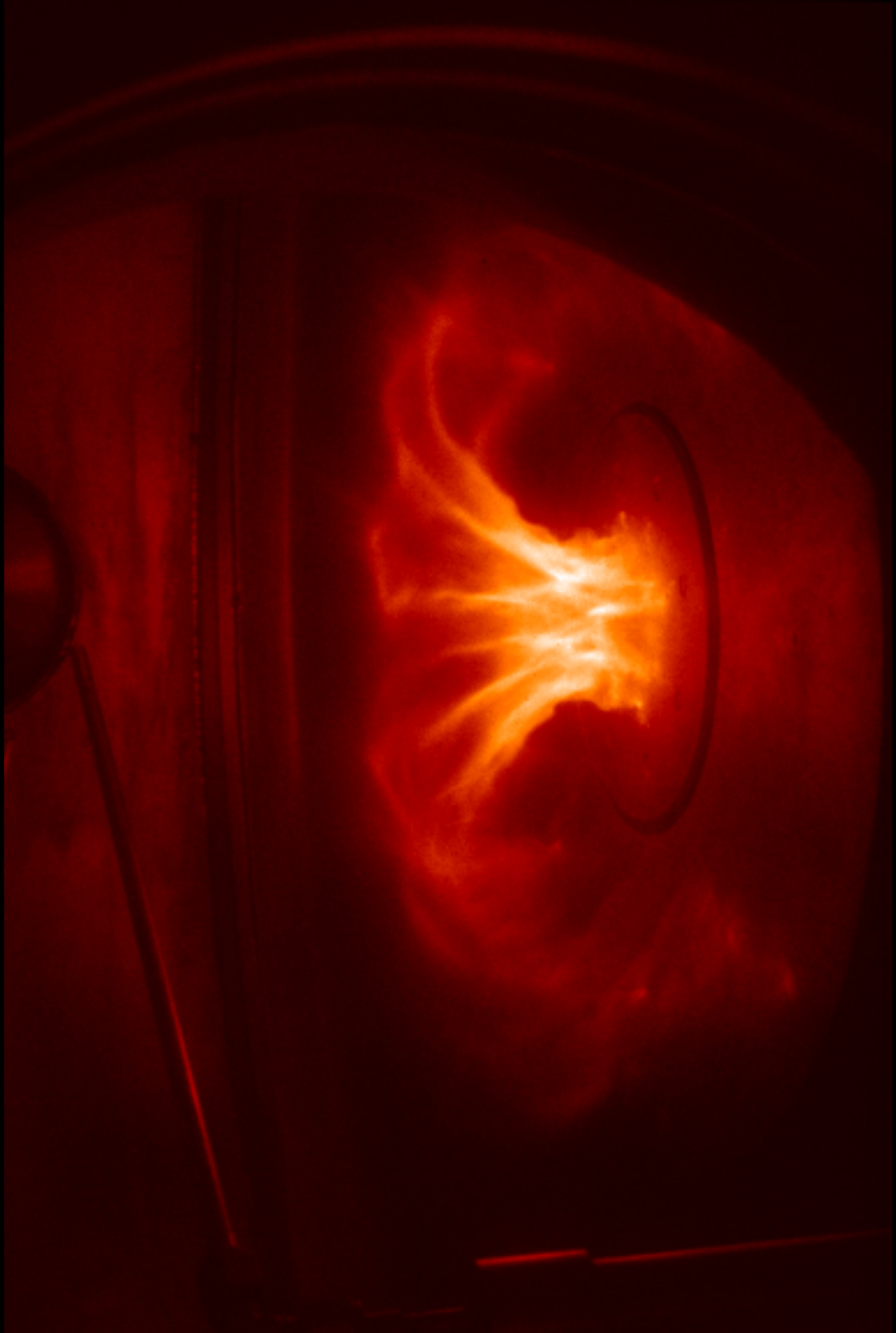



gas feeds
magnetic field coil
ceramic break
vacuum wall

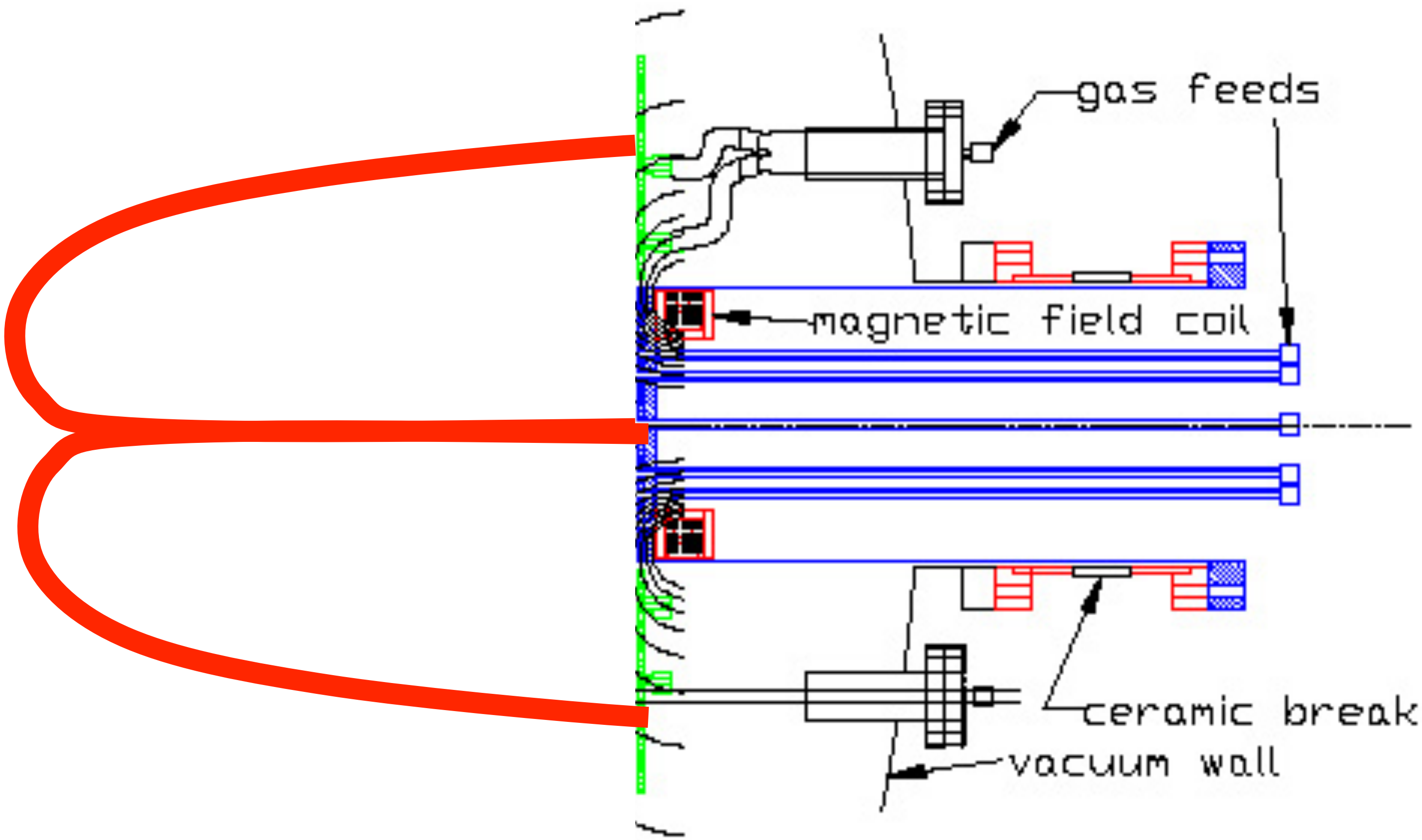


Spider legs merge to form central column

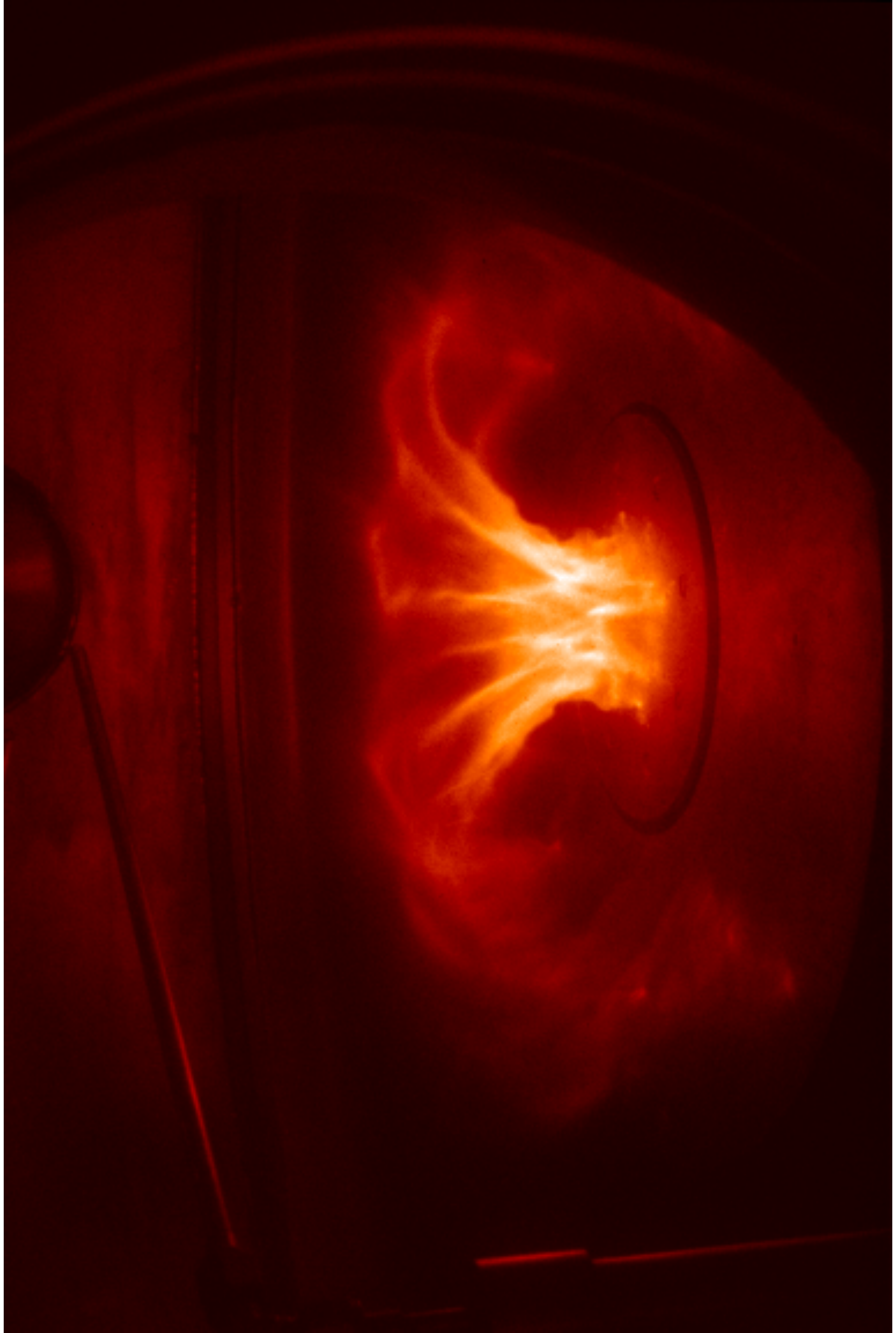


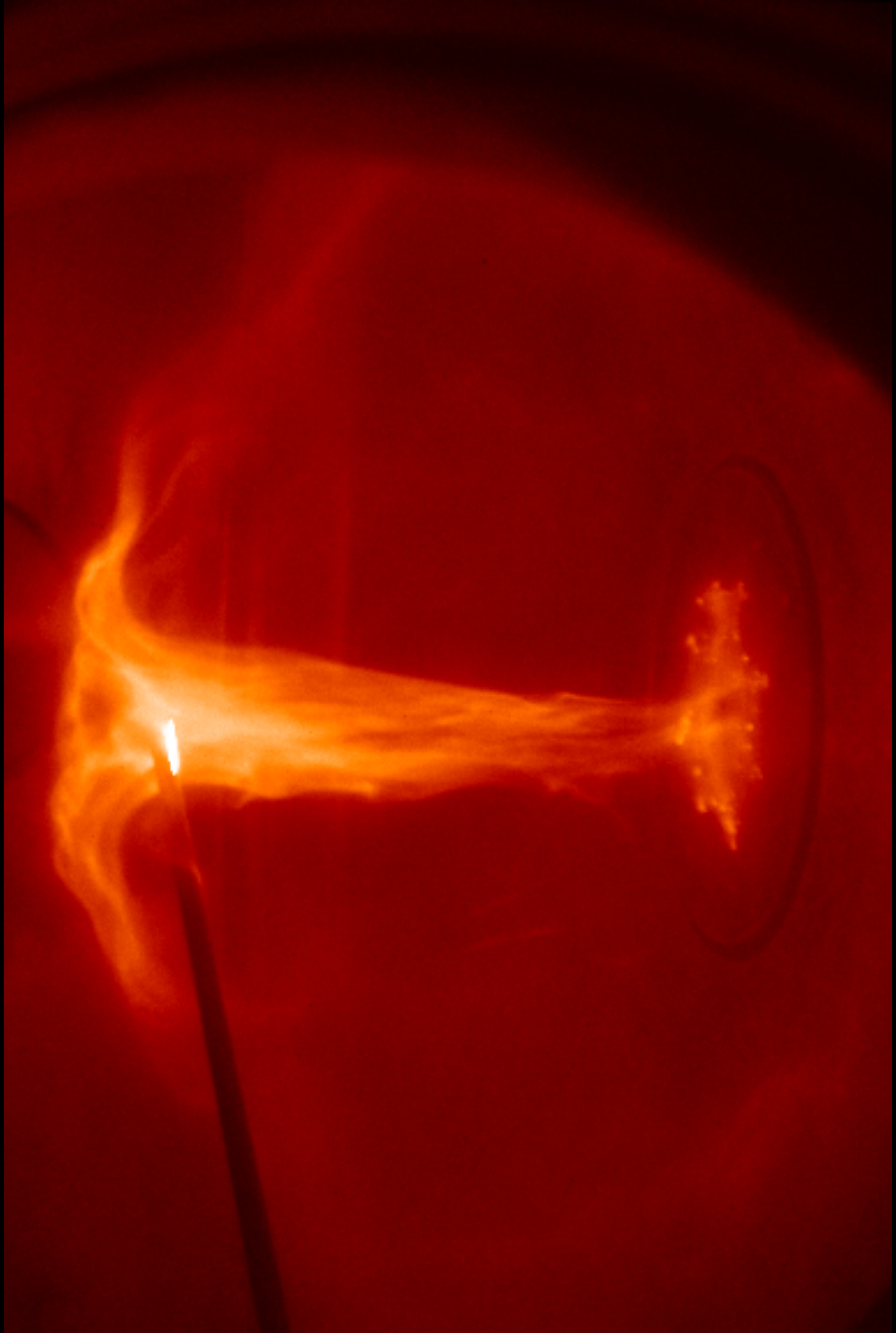


Central column lengthens



I ~ 100 kA





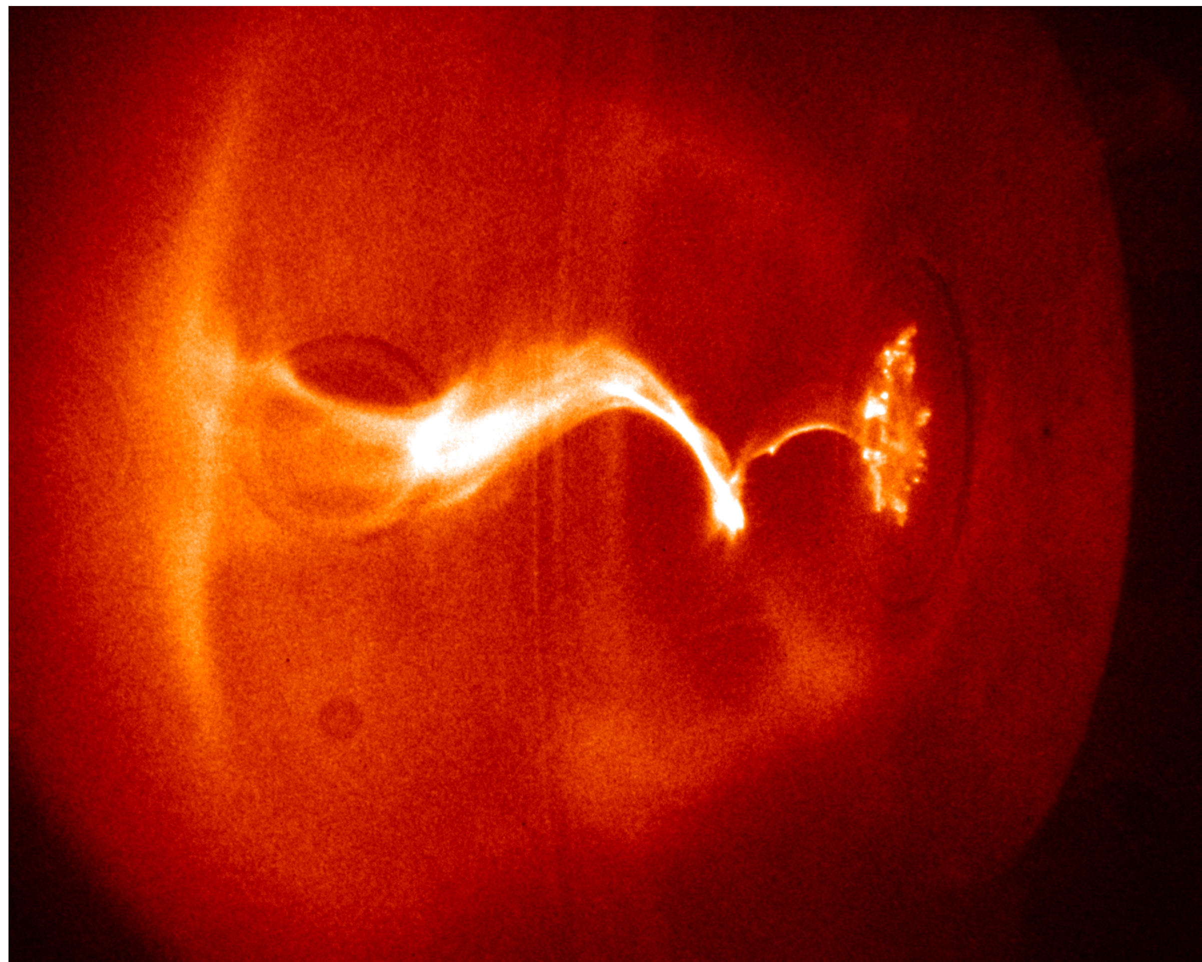
Kink instability of central column, cont'd

Kink occurs when central column becomes sufficiently long to satisfy Kruskal-Shafranov instability condition

$$q \equiv \frac{2\pi R}{L}\frac{B_\phi}{B_z}$$

S. C. Hsu & P. M. Bellan
MNRAS 334, 257 (2002)

← L →



New result

Instability of an instability leading to magnetic reconnection

secondary instability,
macro to micro scale cascade

Rayleigh Taylor instability

Interchange instability

Heavy fluid on top of light fluid

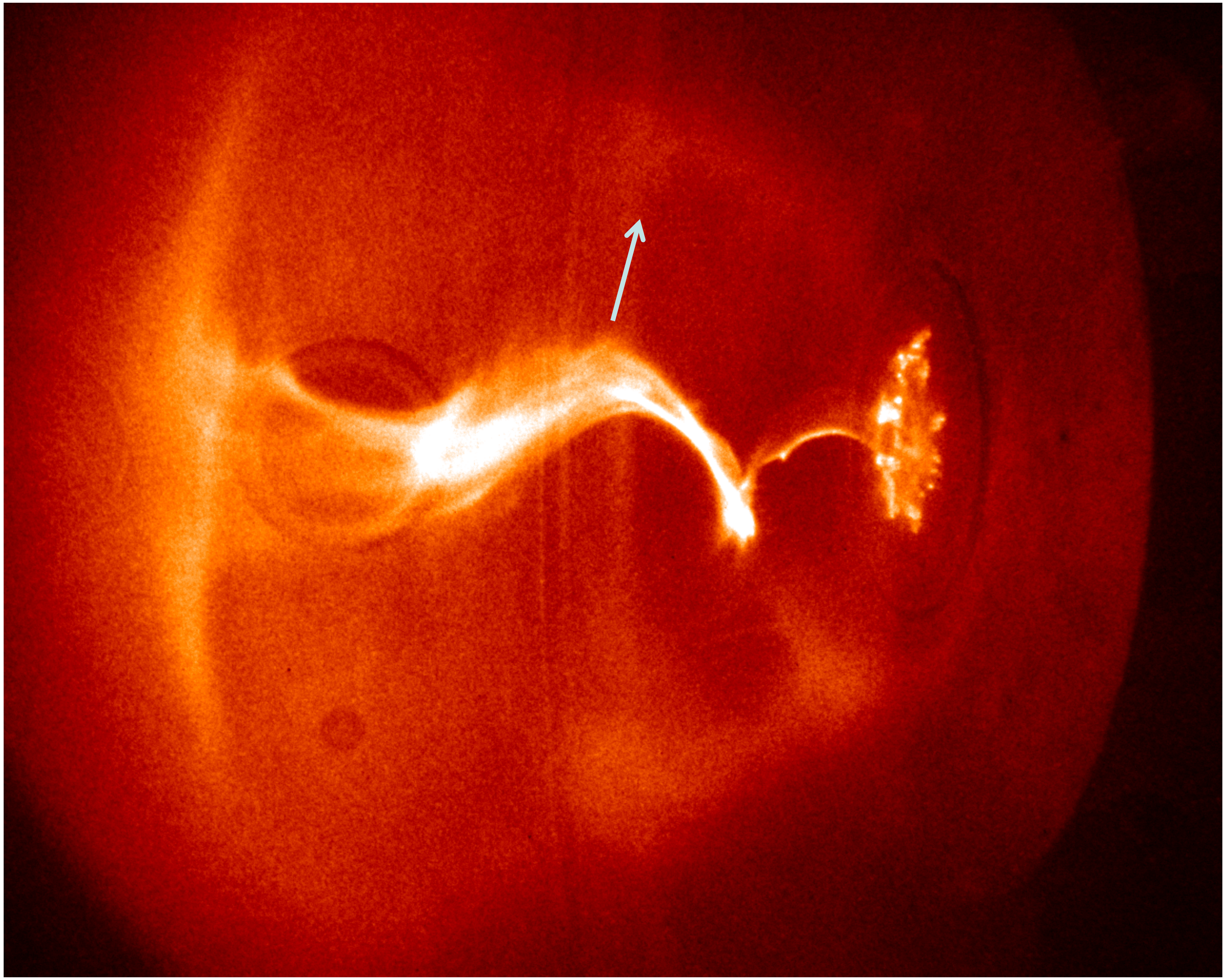
Ripples develop

Heavy fluid goes down

Light fluid goes up (exchange places)

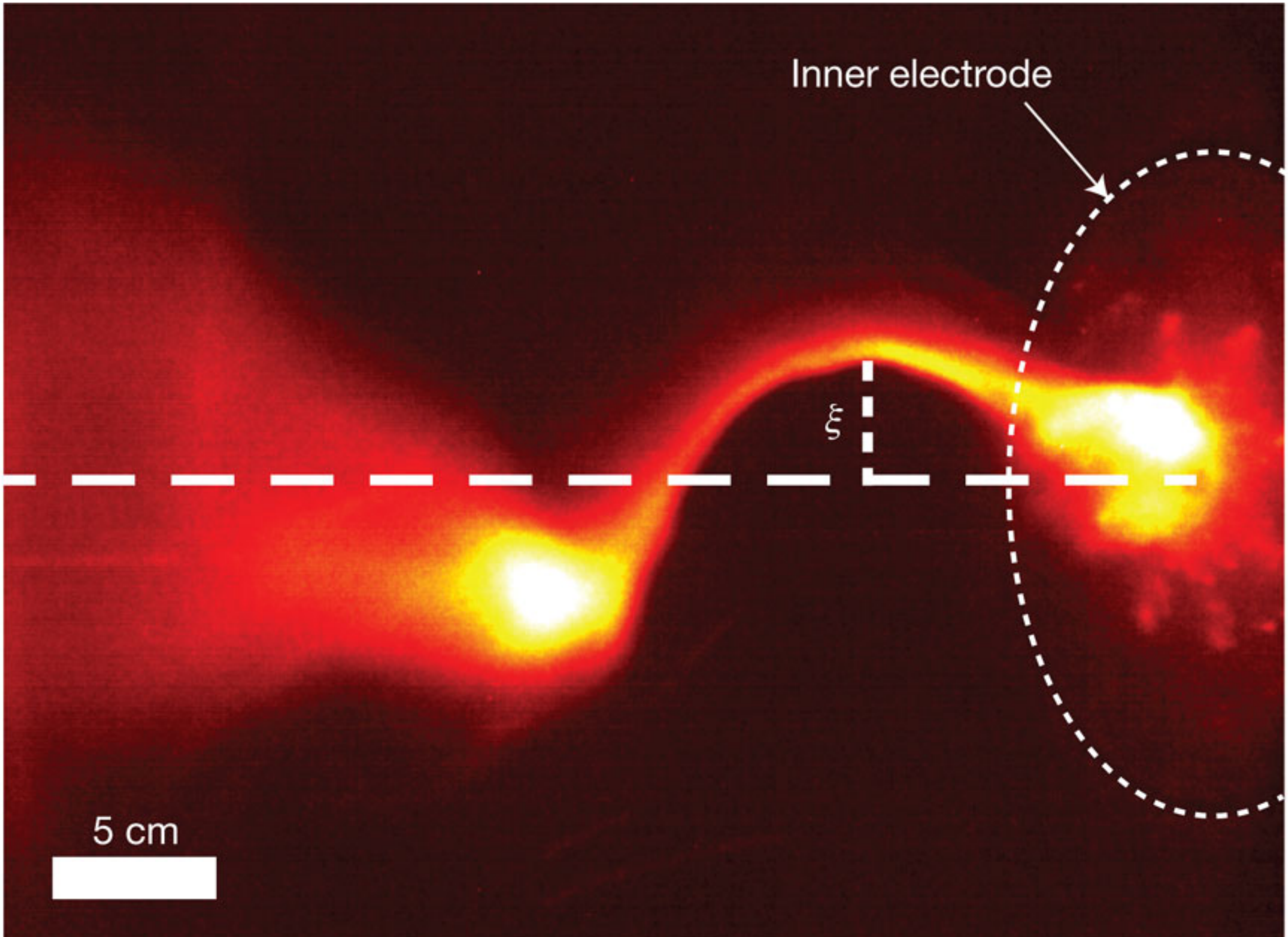
Reduction of gravitational potential energy

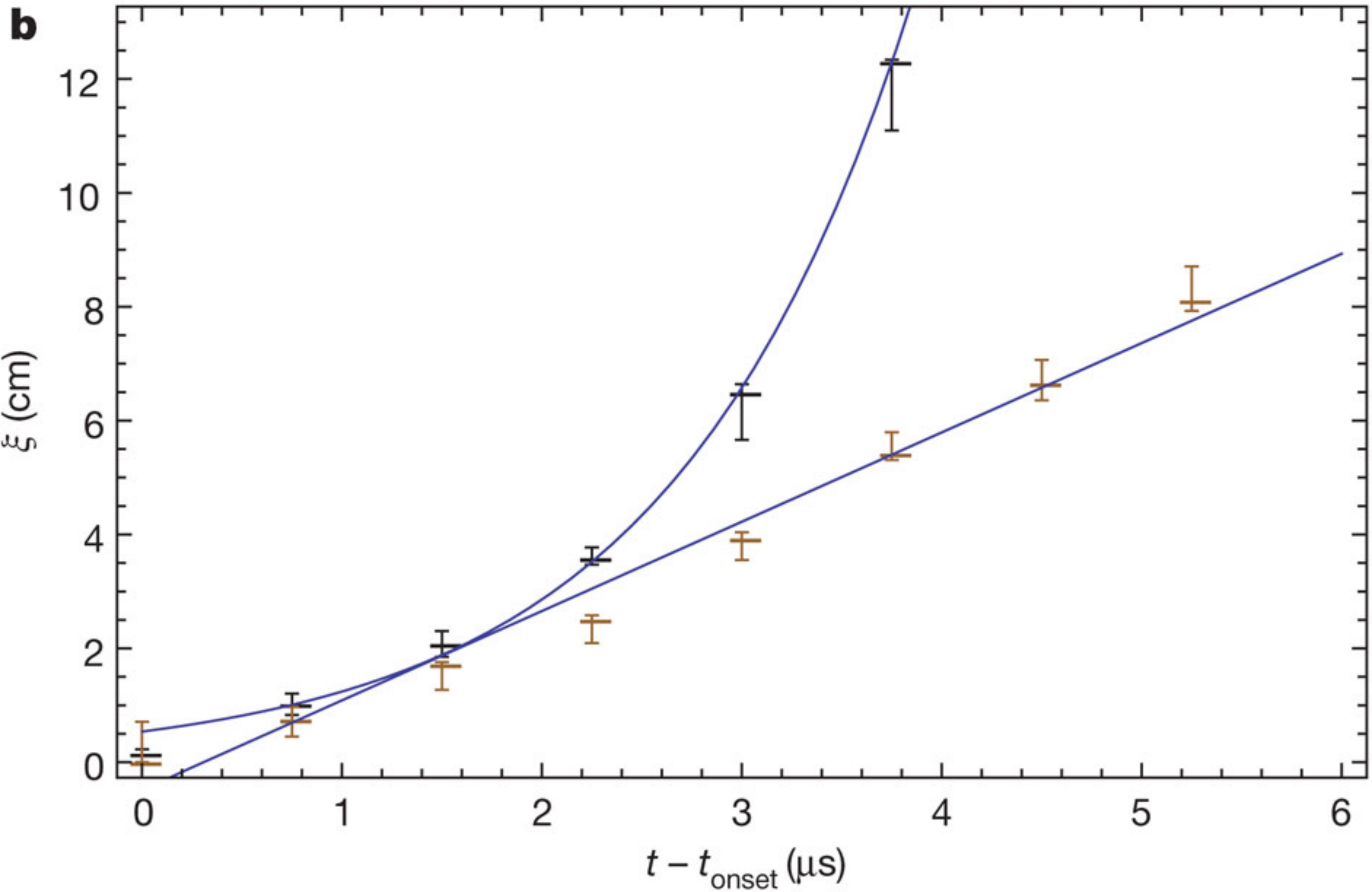
Kinking plasma provides effective gravity, heavy fluid on top of light fluid
Get Rayleigh-Taylor instability

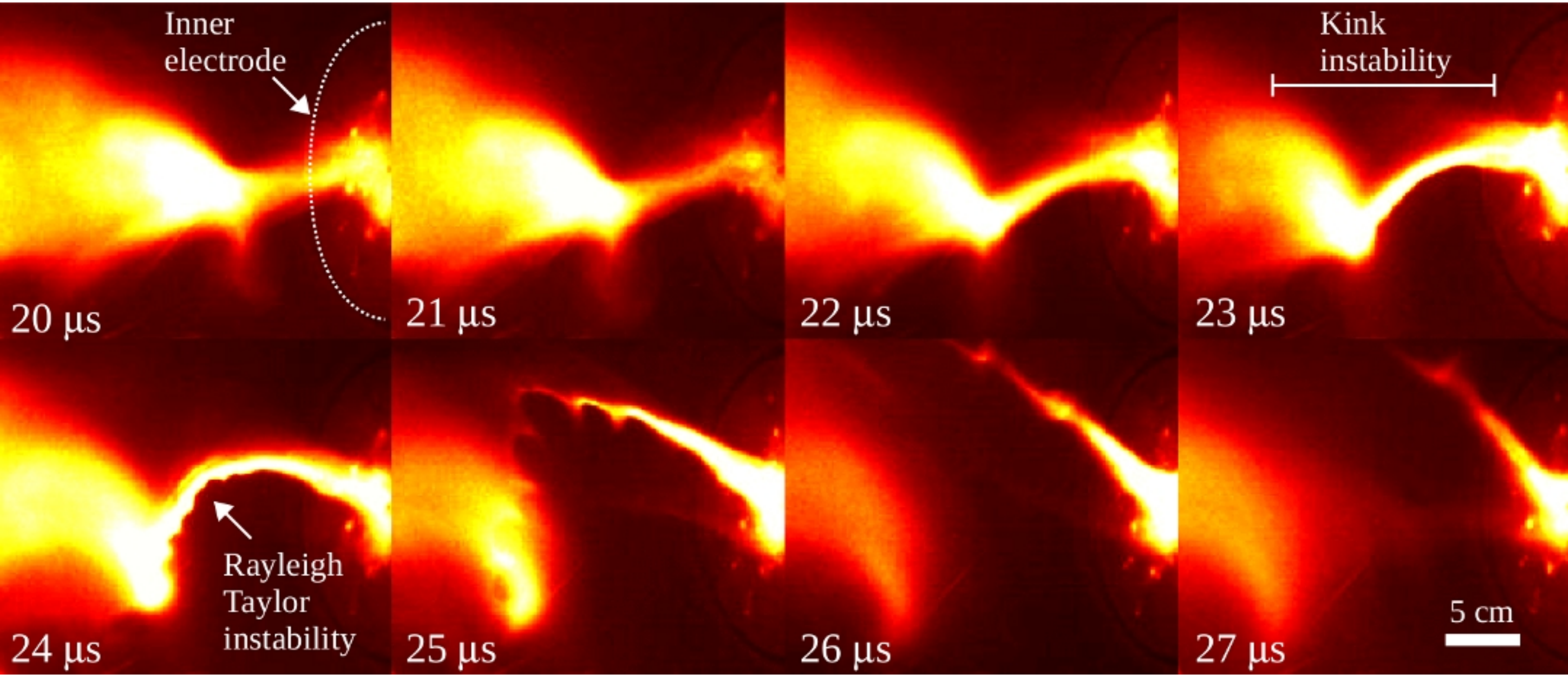


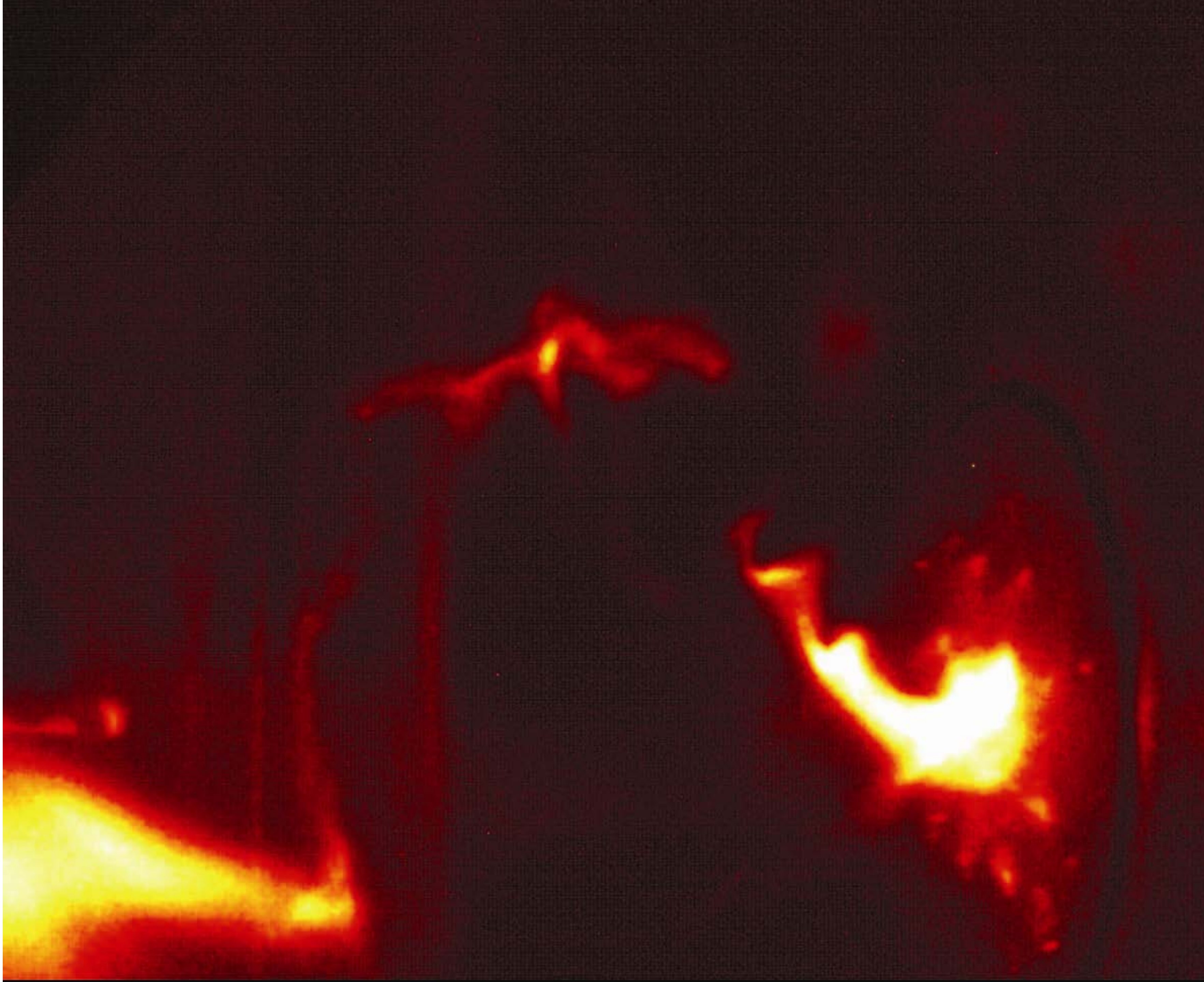

FunnyChill.com

a




b
0
2
4
6
8
10
12
ξ (cm)
0
1
2
3
4
5
6
t − t_onset (µs)


Inner electrode
Kink instability
20 μs
21 μs
22 μs
23 μs
Rayleigh Taylor instability
24 μs
25 μs
26 μs
27 μs
5 cm



Rayleigh-Taylor dispersion serves as diagnostic for macro to micro coupling:

RT growth rate $\gamma^2 = kg - \frac{(\mathbf{k}\cdot\mathbf{B})^2}{\mu_0 \rho}$

Fastest growing mode has $\mathbf{k}\cdot\mathbf{B} = 0$

Assume $\mathbf{k} \square \mathbf{B} \; \square \; 0$ so $\frac{m}{r} B_{\square} \square k_z B_z \; \square \; 0$

and assume $m \; \square \; \square 1$ based on images so $B_{\square} \; \square \; k_z r B_z$.

The axial current is $\mu_0 J_z = \frac{1}{r}\frac{\partial}{\partial r}\left(r B_\theta\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(k_z r^2 B_z\right) = 2 k_z B_z$

The electron drift velocity is $v_d \approx J_z / ne$.

MHD assumes v_d is negligible compared to the Alfven velocity v_A.

Consider then $\frac{v_d}{v_A} = \frac{J_z}{nev_A} = \frac{2k_z B_z}{W_0} \frac{\sqrt{W_0 n m_i}}{B_z} = 2k_z \frac{c}{g_{pi}} = \frac{4}{L_{RT}} \frac{c}{g_{pi}}$

It is observed that $\frac{v_d}{v_A}$ is order unity and

that reconnection occurswhenRT chokes jet diameter to be smaller than c/g_{pi}

Hence macro to micro coupling has been demonstrated.

Jet reconnects when choked to diameter smaller than c/g_{pi}

Summary macro to micro scale coupling via “instability of an instability”
(answer to Shibata-san question)

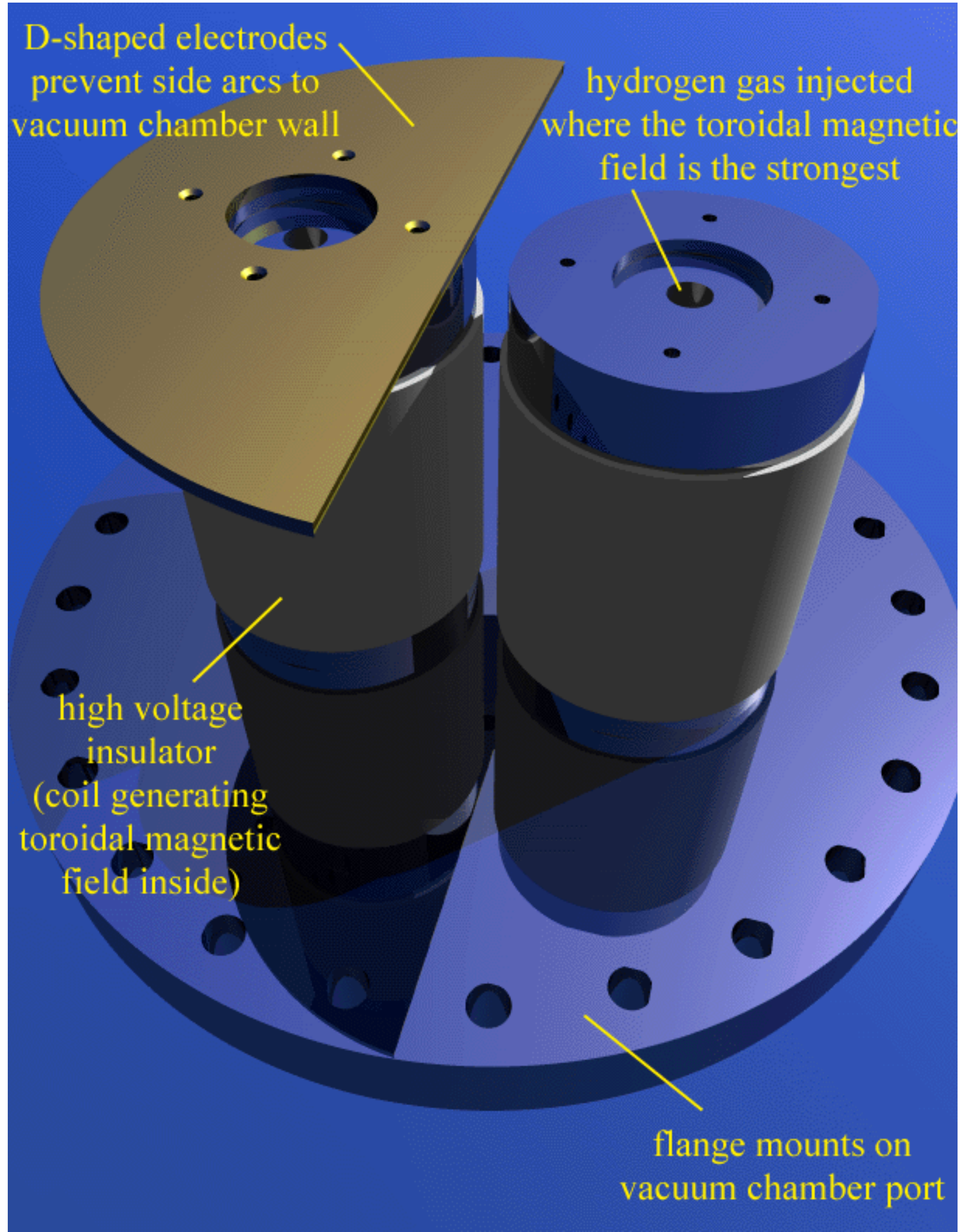
1. Current drives MHD kink (ideal macroscopic instability)
2. Kink has enormous lateral acceleration
3. Gives large effective gravity in accelerating frame
4. Jet is heavy fluid on top of light in accelerating frame
5. Rayleigh-Taylor (RT) driven by heavy fluid on top of light
6. RT occurs on trailing edge of laterally accelerating jet
7. RT erodes jet diameter (axial periodicity)
8. Jet gets choked to ion skin depth scale
9. Effectively have ion beam moving axially at Alfven velocity through electrons
10. MHD description collapses when such a beam exists
11. Get reconnection since MHD violated, system accesses ion skin depth scale

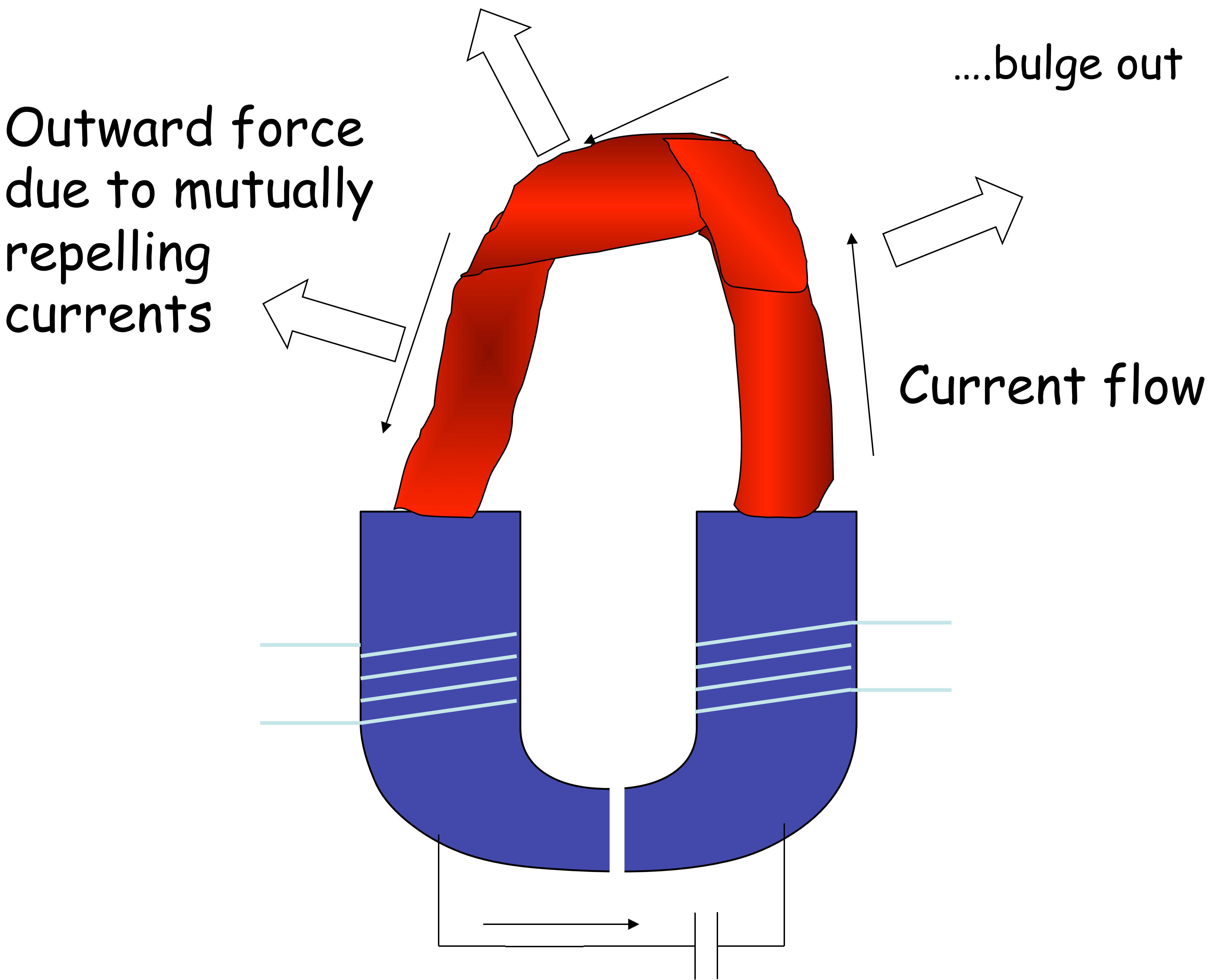
Bi-directional jets and their relation to solar corona loops

Setup

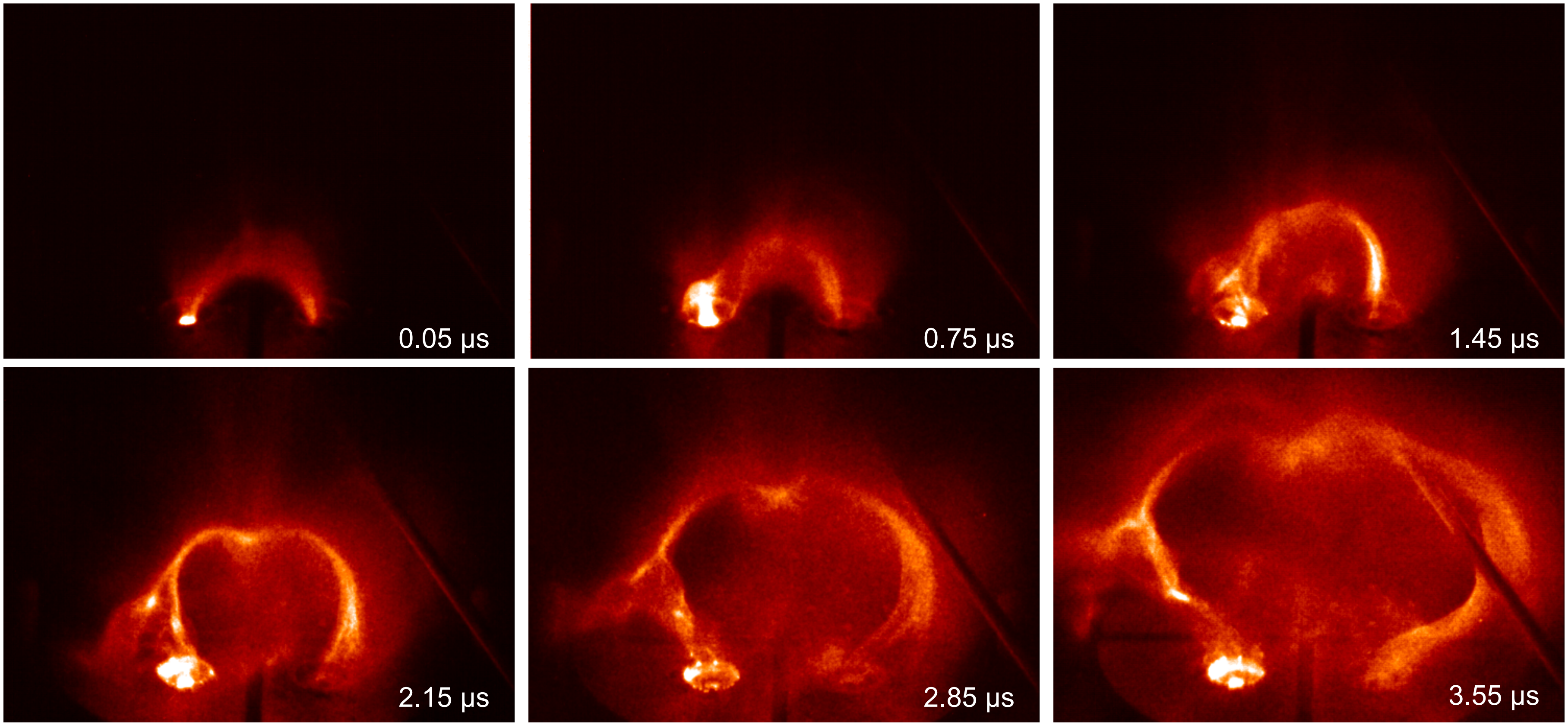
to simulate

solar loops




Outward force due to mutually repelling currents
....bulge out
Current flow

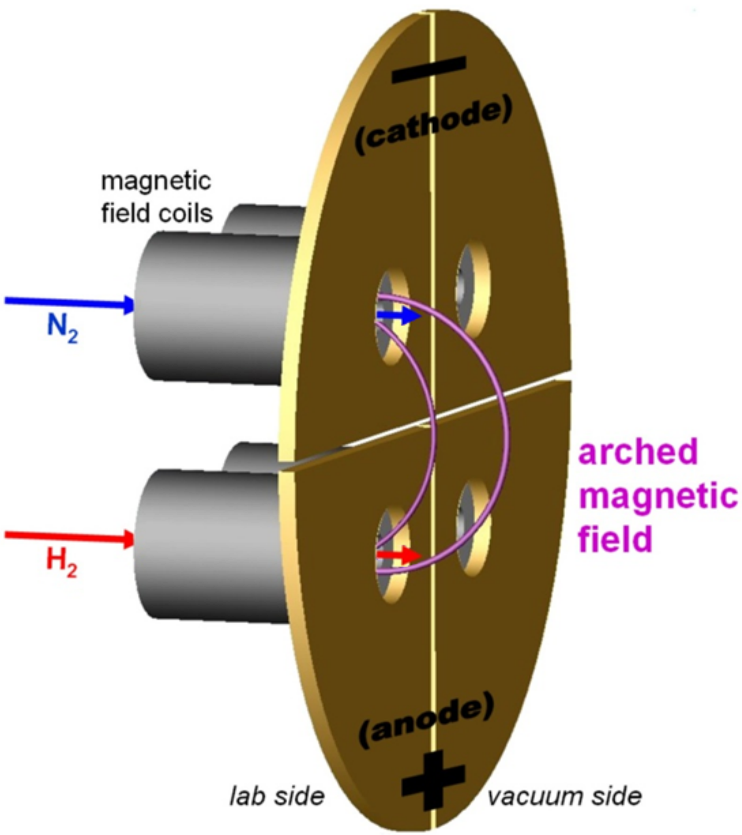
Evolution of a plasma loop

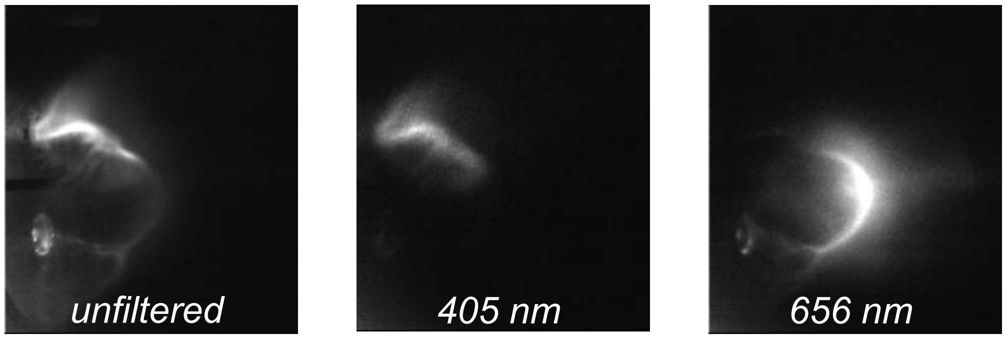


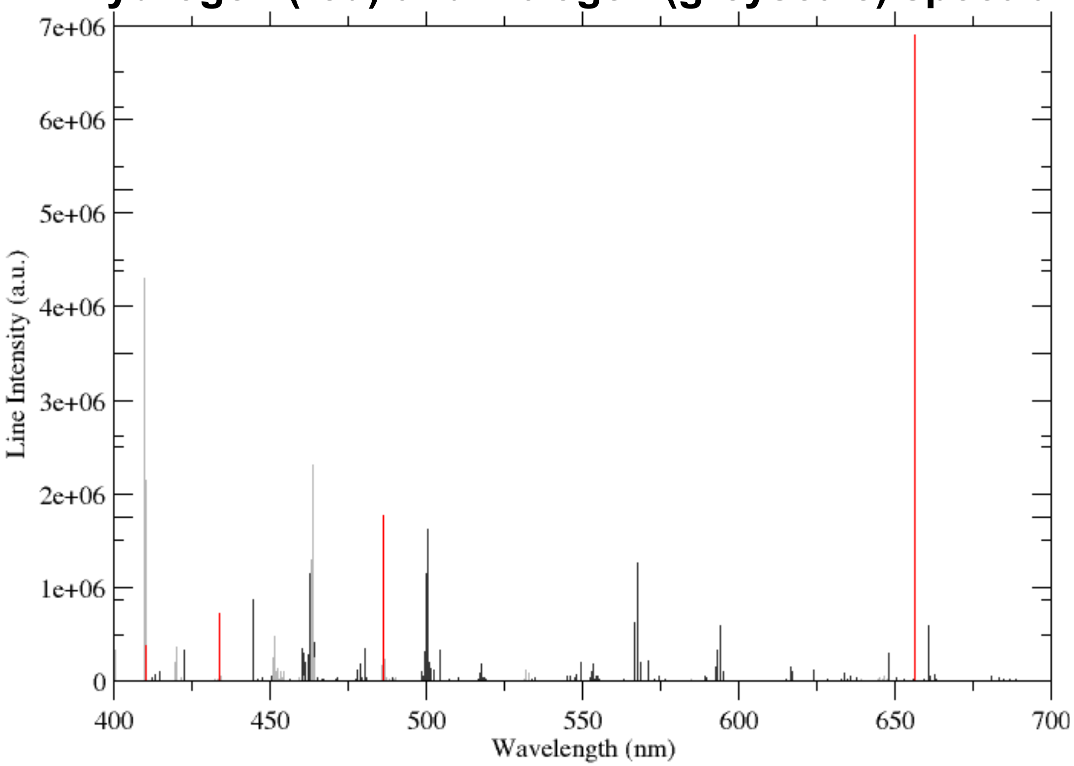
What force or forces are behind this behavior?

Where is the material coming from?

Use dual-species plasmas to track material



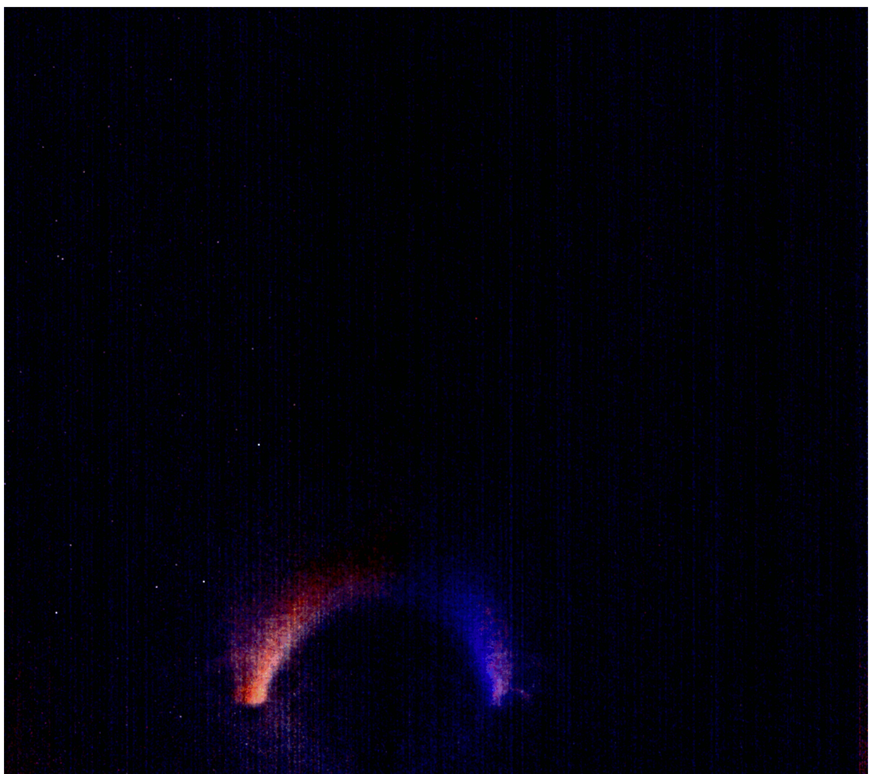


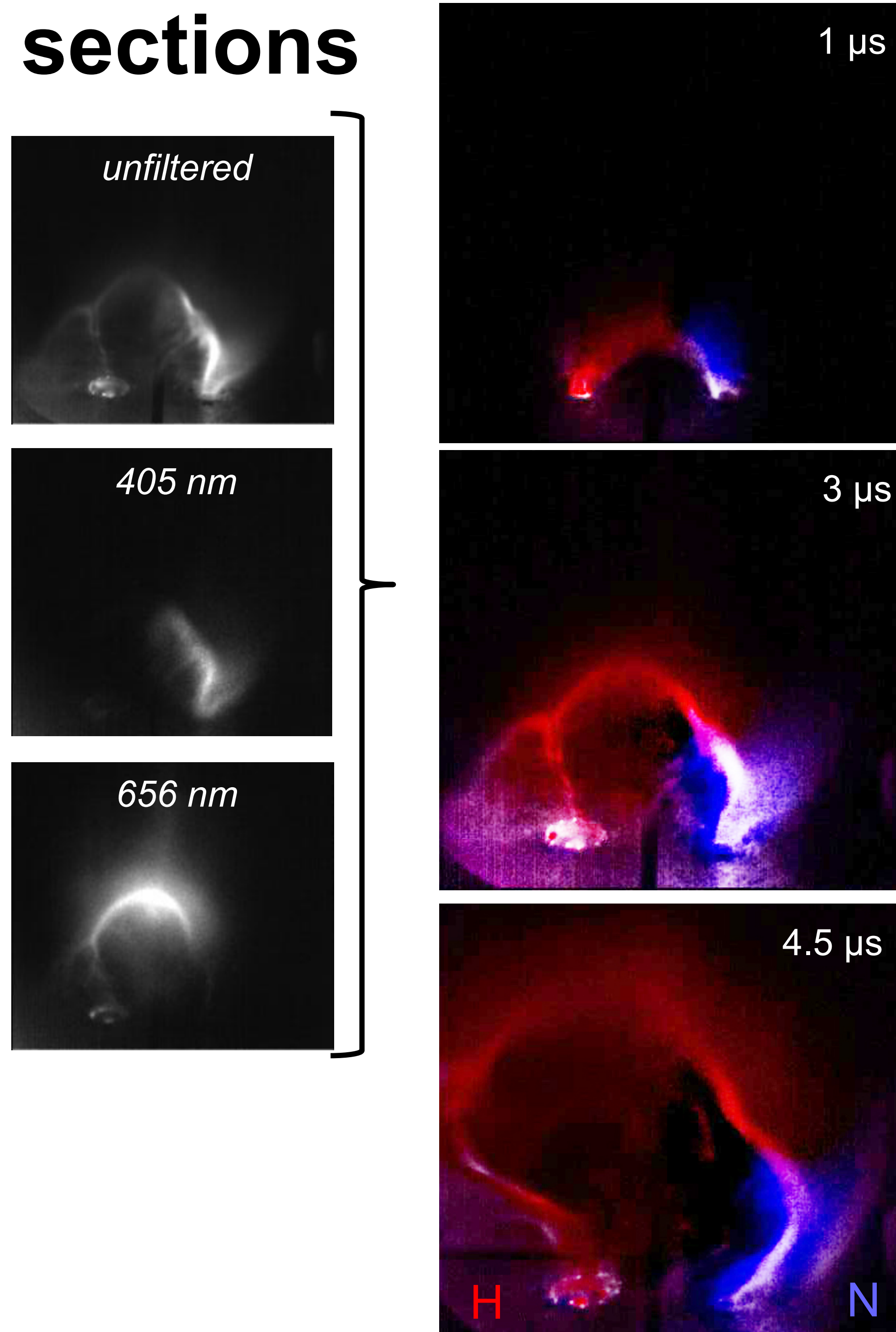


1. **Highly reproducible dual-gas plasma**
2. **Standard optical filters**

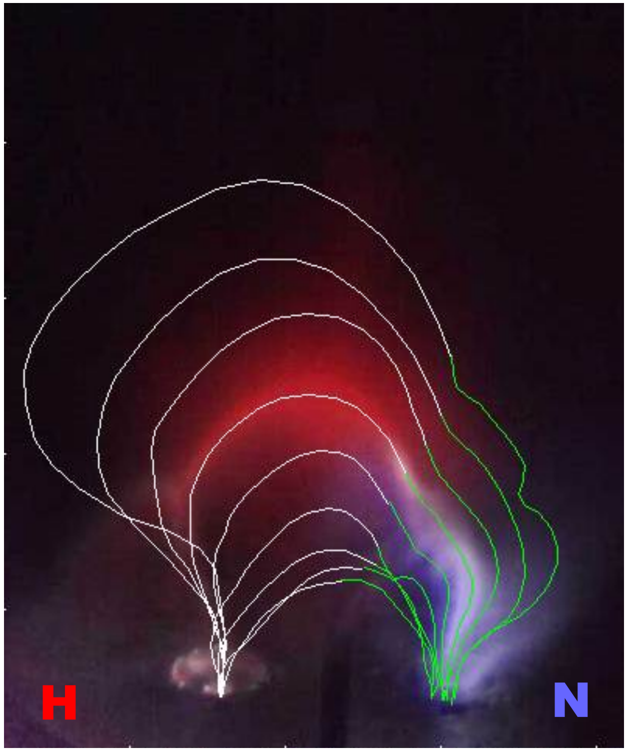
“Color-coded” plasma sections

Supply different species to the two foot points of the plasma to see from where the plasma enters the arched flux tube.

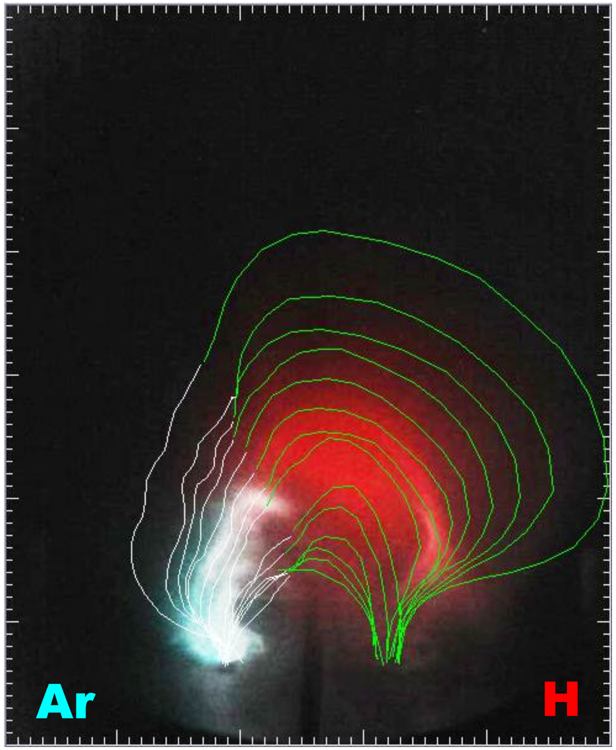




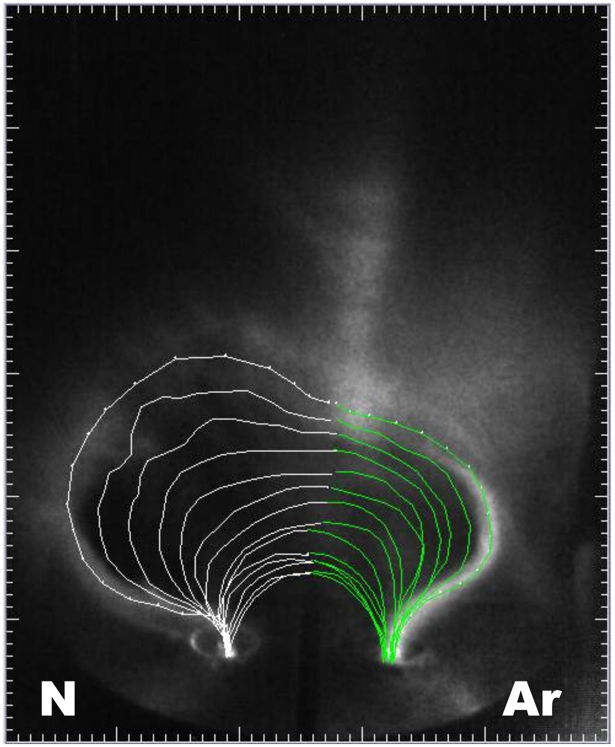
Quantify and repeat



curve traces overlaying 6th frame

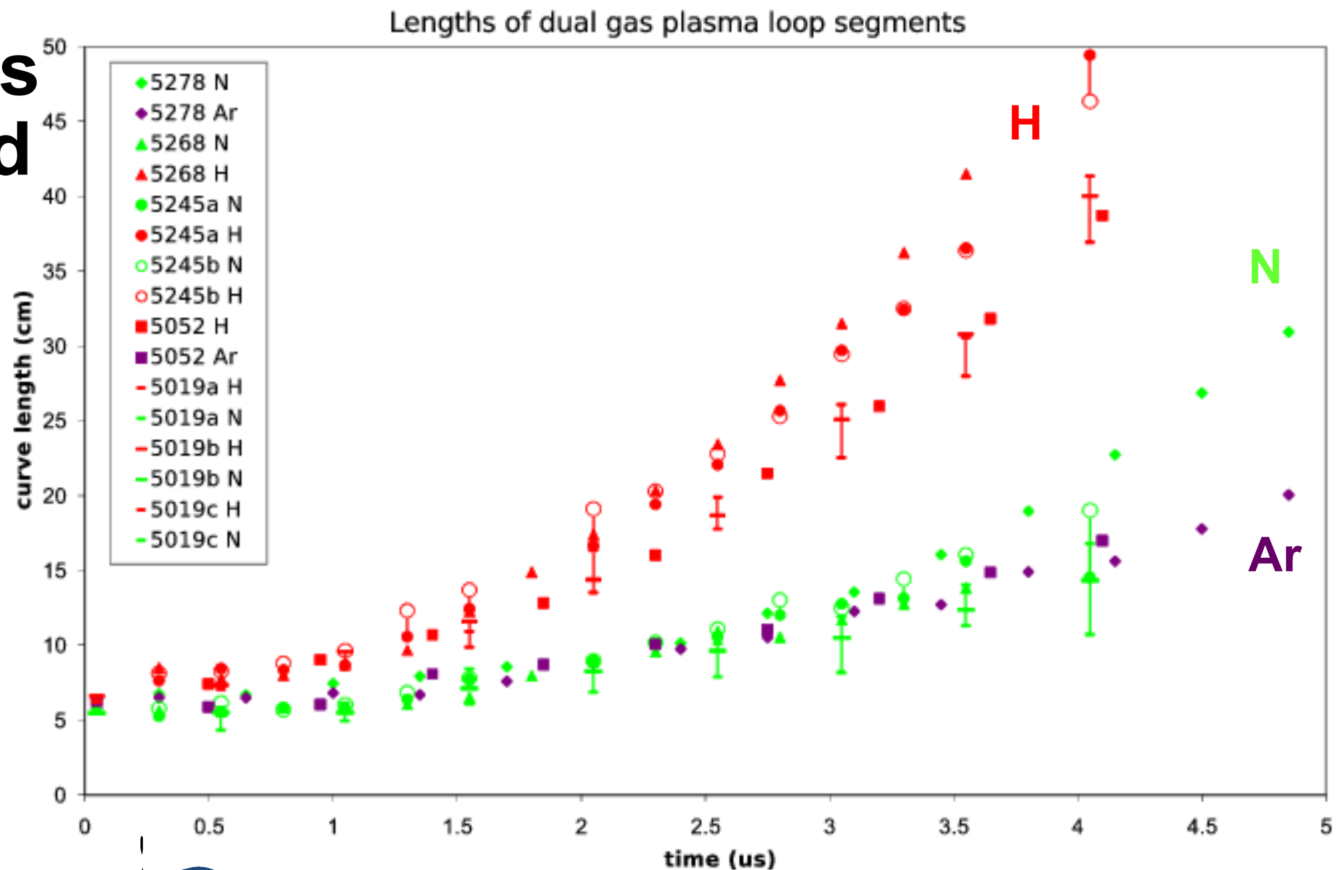


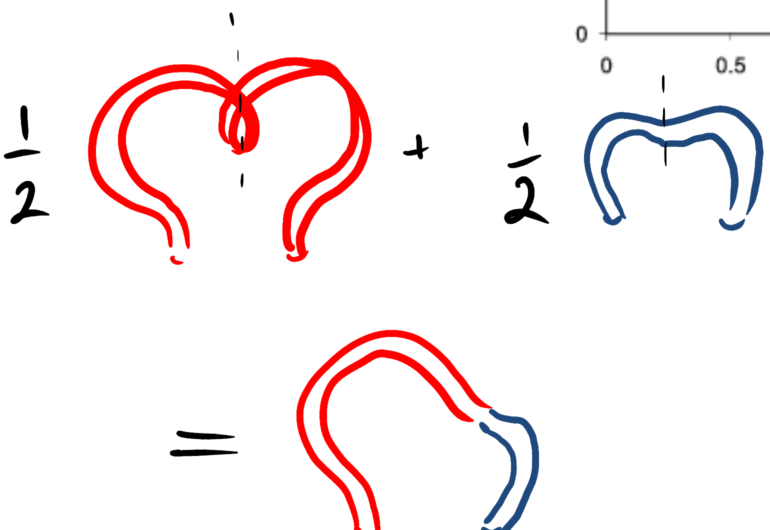
curve traces overlaying 8th frame



curve traces overlaying 14th frame

Each plasma loop comprises two high speed jets



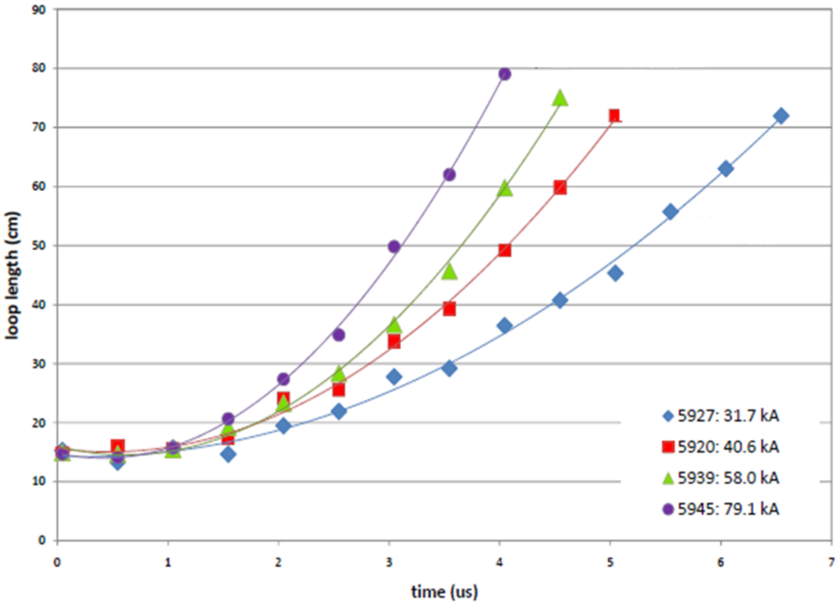


Important features:

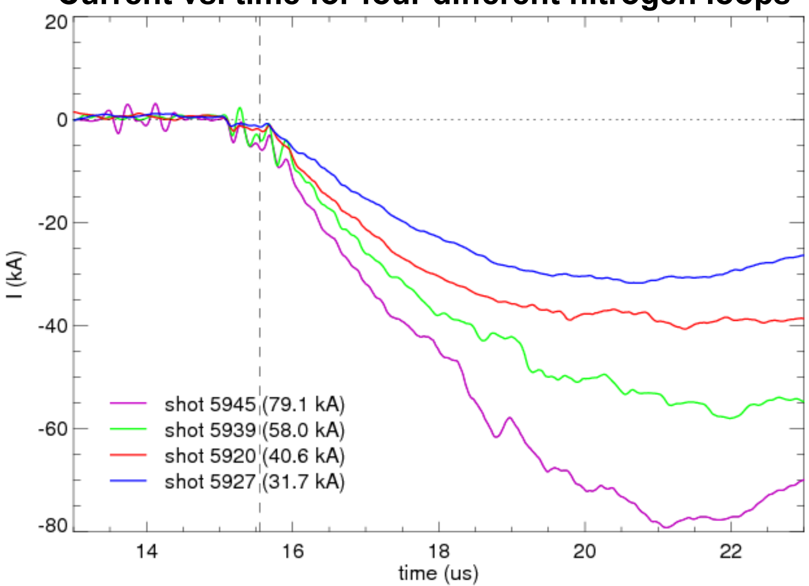
- no dependence on origin of flow (cathode or anode), or with which other gas the species is paired
- strong dependence on mass

Speed and current are proportional at early times

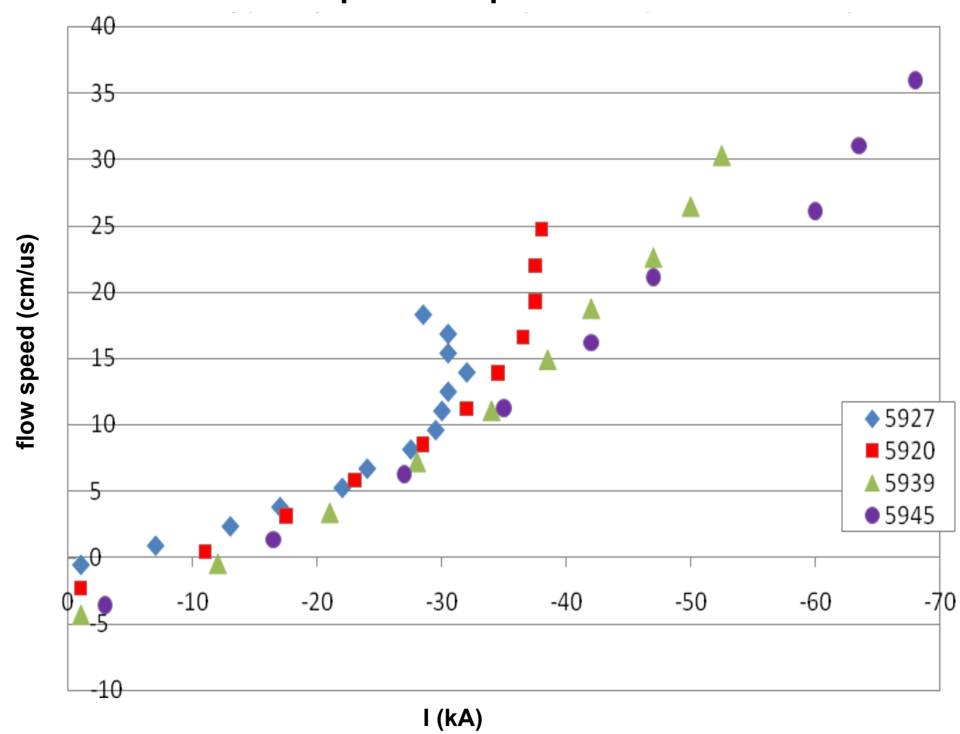
Loop length vs. time for four different nitrogen loops



Current vs. time for four different nitrogen loops



Expansion speed vs. time

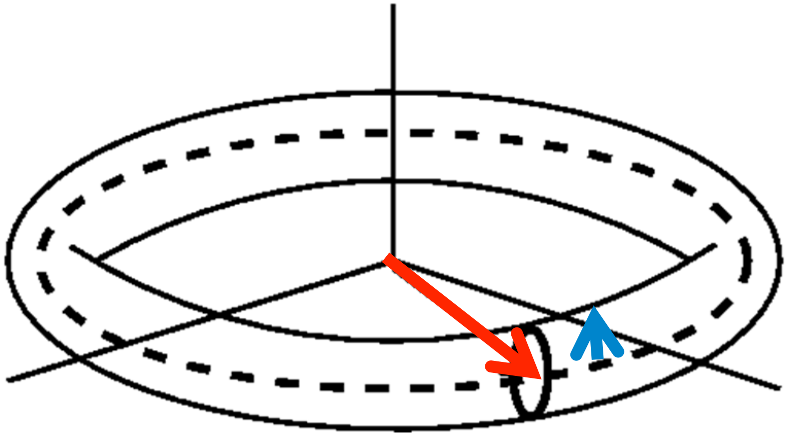


Model 1: Expansion via the hoop force

Force per unit length, calculated from E&M equations (Shafranov 1966)**:**

$$F_{hoop} = \frac{\mu_o I^2}{4\pi r}\left[\ln\left(\frac{r}{a}\right) + 1.08 + \frac{l_i}{2}\right]$$

r = major radius
a = minor radius
l_i = internal inductance
s = length of plasma loop



with simplifying assumptions *(I=kt* ; logarithmic term = a)**:**

$$\ddot{r}(t) = \alpha\left(\frac{1}{2\pi}\right)^2 \frac{\mu_o k^2}{m_i n a^2}\frac{t^2}{r(t)}$$

$$r(t) = \frac{1}{2\pi}\sqrt{\frac{\mu_o \alpha}{2 m_i n}}\frac{k}{a}t^2$$

This predicts that the electric current and the rate of loop length increase should be proportional:

$$\frac{\dot{s}(t)}{I(t)} = \frac{\pi \dot{r}(t)}{kt} = \sqrt{\frac{\mu_o \alpha}{2 m_i n}}\frac{1}{a}$$

Thus: $\dot{s}(t) = const \cdot I(t)$

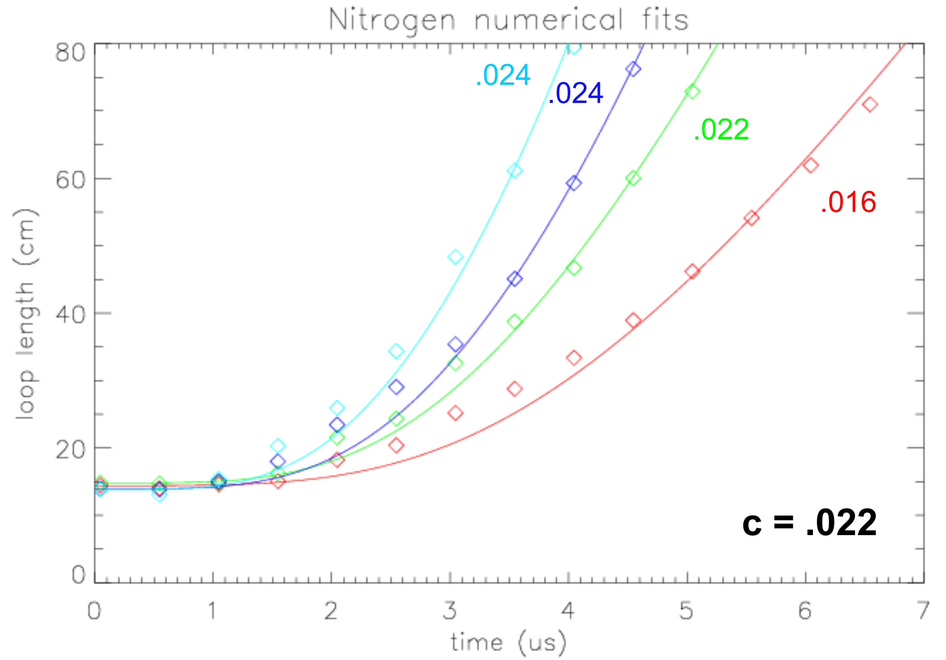
torus graphic adapted from: http://www.math.rutgers.edu/~greenfie/vnx/math251/diary3.html

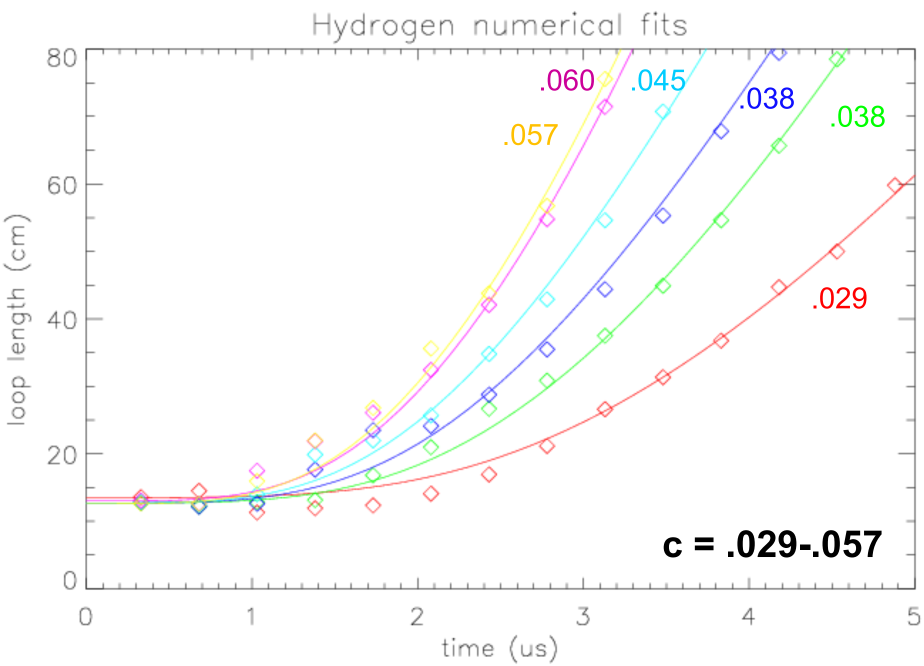
Numerical solutions to the simplified hoop force model are given by:

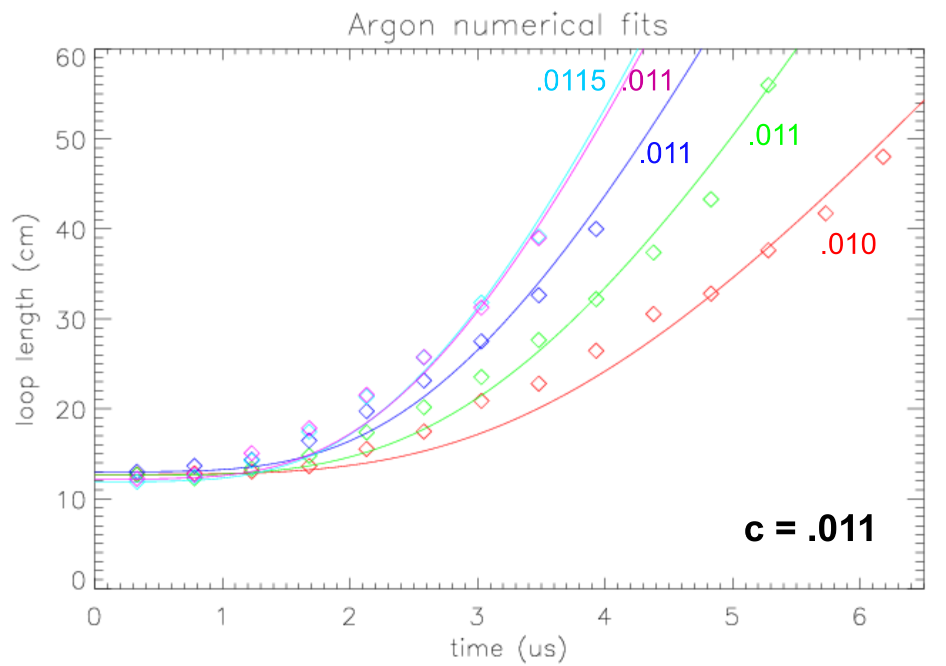
$$\ddot{r}(t) = c\frac{I(t)^2}{r(t)}$$

where each $$c = \frac{\mu_o}{2\pi^2 m_i n a^2}$$

is a free parameter, determined by the best fit for the experimental data.



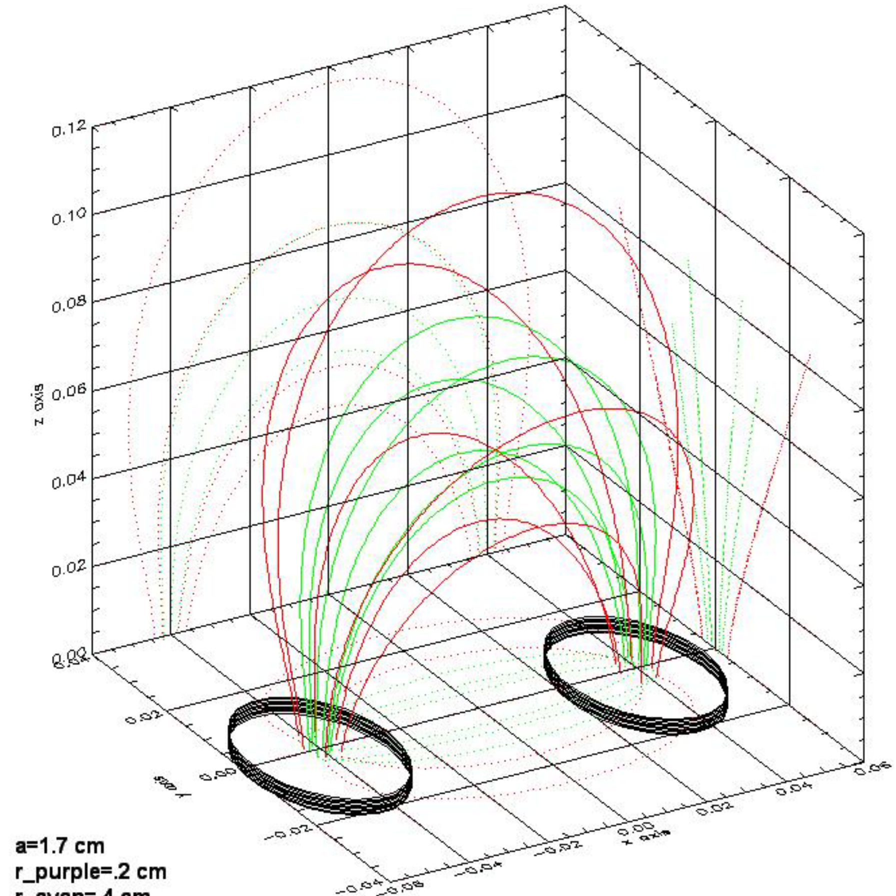




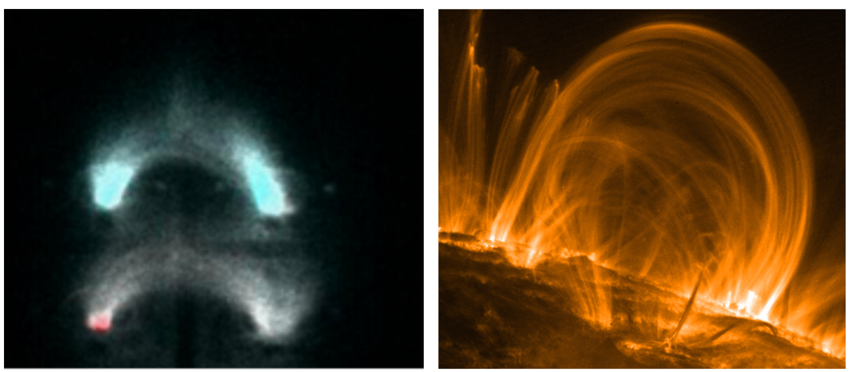
HOWEVER: If the hoop force acted alone, we'd expect the plasma density to drop almost 10-

Model 2: Axial flows via the gobble model

Vacuum field lines spread out significantly



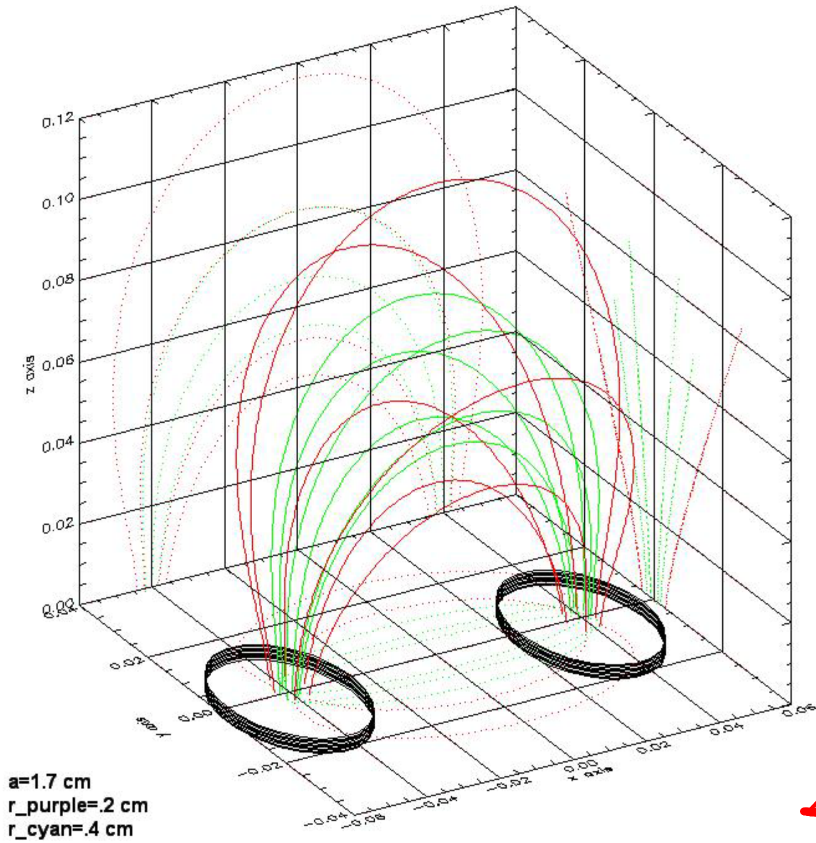
Laboratory and solar loops are collimated



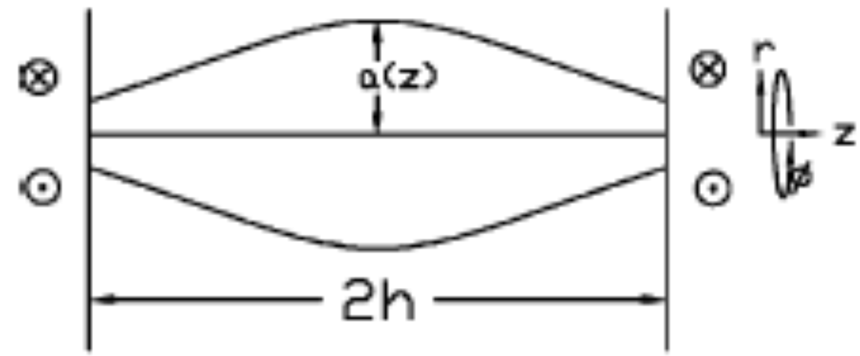
Left: Two adjacent plasma flux tubes (one H, one N).
Right: Solar coronal loops.

P. M. Bellan, *Why current-carrying magnetic flux tubes gobble up plasma and become thin as a result*, Phys. Plasmas **10** Pt 2, 1999 (2003)

Radial (but perhaps not axial) equilibrium:



Switching to a cylindrical geometry to make the analysis tractable.

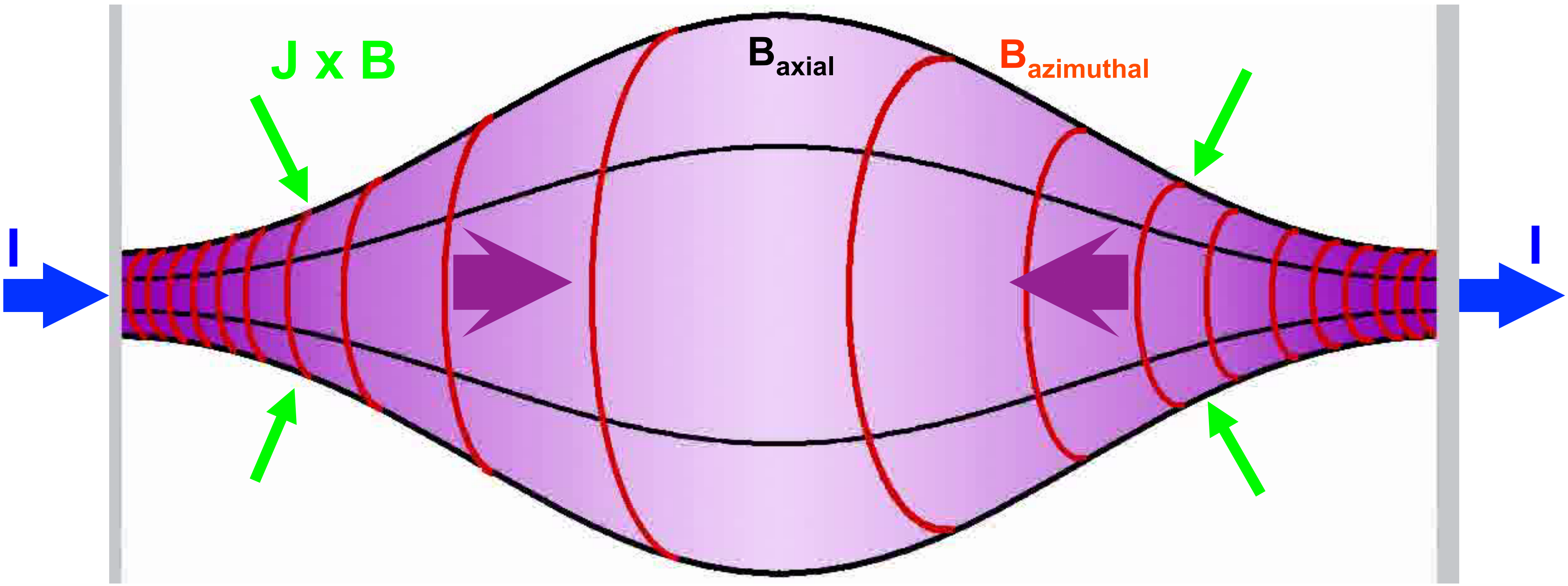


Note that you can't define an equilibrium for just any magnetic field configuration.

$$\mathbf{J} \times \mathbf{B} - \nabla P = \rho \frac{D\mathbf{U}}{Dt} = 0 \quad \longrightarrow \quad \mathbf{J} \times \mathbf{B} = \nabla P \ ?$$

Bellan, P. M. *Phys. Plasmas* **10**, 1999 (2003).

Radial equilibrium implies flows:

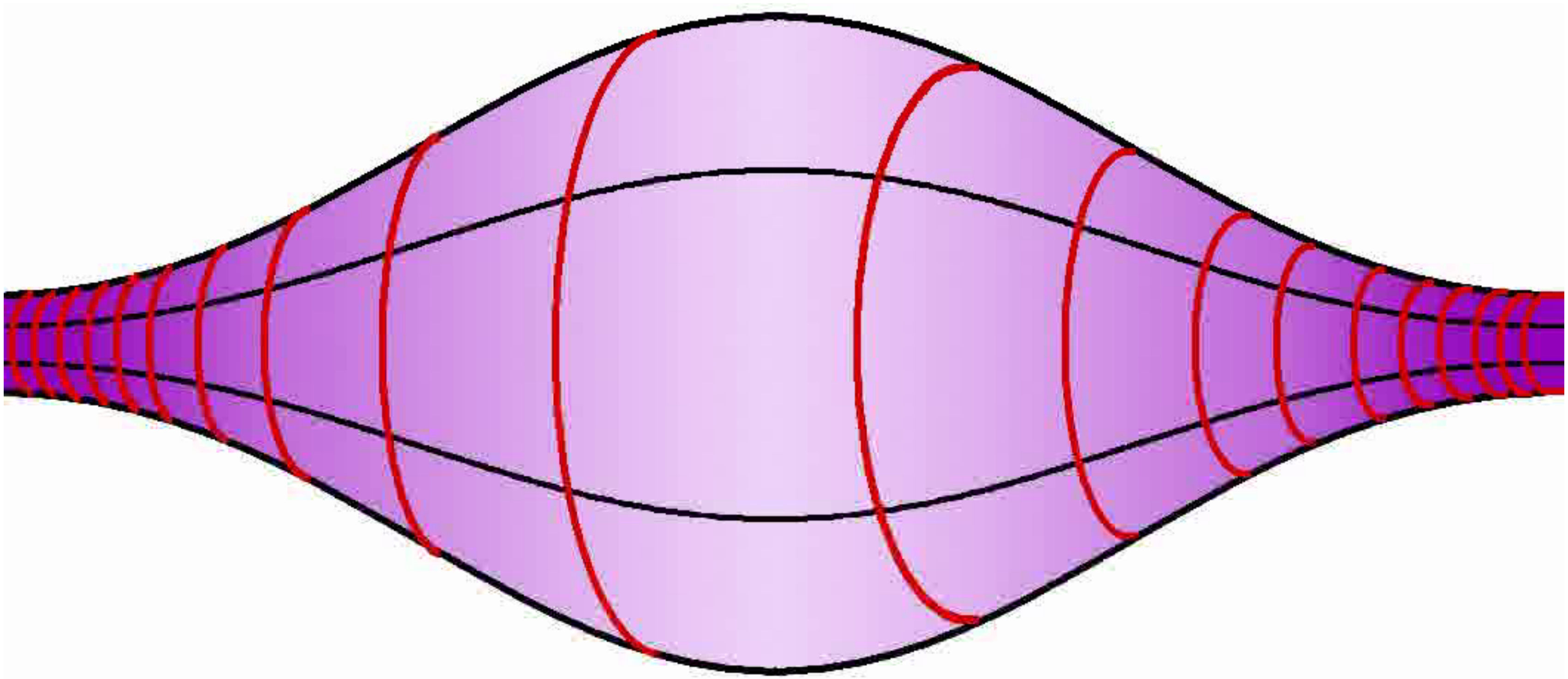


Question: What would happen if you DID have a bulged, current-carrying flux tube?

Answer: It would send plasma into the bulge until it became uniform.

Both the hoop force expansion rate and the gobble model flow rate are proportional to I (and hence $B_{azimuthal}$).

Bellan, P. M. *Phys. Plasmas* **10**, 1999 (2003).
D. Kumar and P. M. Bellan, *Phys. Rev. Lett.* **03**, 105003 (2009).



Both the hoop force expansion rate and the gobble model flow rate are proportional to I (and hence $B_{azimuthal}$).

Thus, flux tube is not diluted when it expands due to hoop force.