Theory of ideal MHD wave propagation in Harris type current sheet

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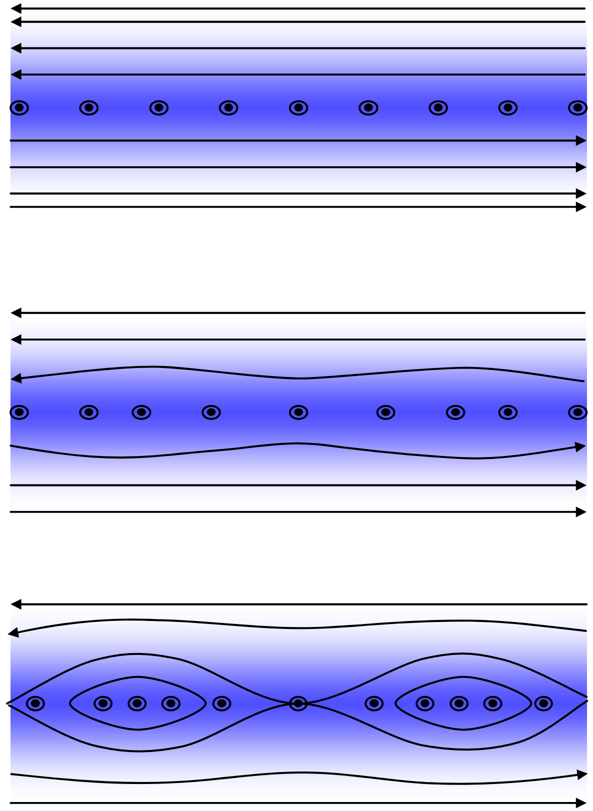
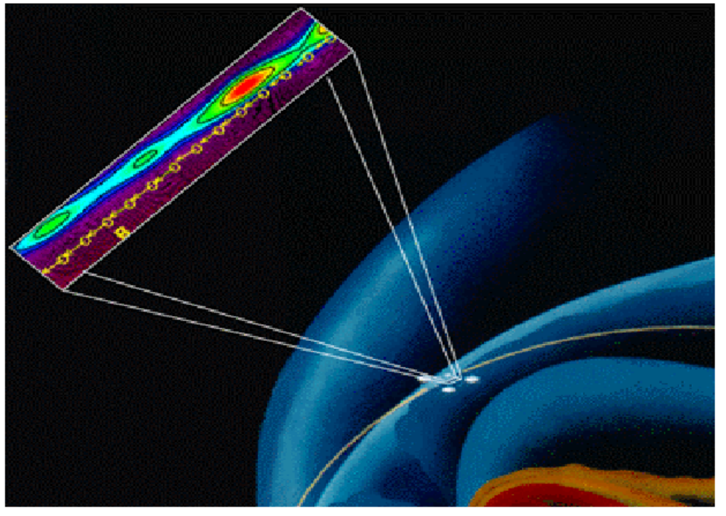
Plasma Model

- Ideal MHD
- Resistive MHD、Hall MHD
- Resistive two-fluid 、multi-fluid
- Vlasov-Maxwell
- Hybrid (kinetic ion 、fluid electron)
- Full particle (PIC)

Harris Type Current Sheet and Magnetic Reconnection



First Evidence for 2D Magnetic Reconnection at Magnetopause Current Sheet



Hau and Sonnerup (1999)

Generalized Ohm's Law

ion inertia

$$\vec{E} + \vec{u} \times \vec{B} = \eta \vec{j} + \frac{m_i}{\rho e}\left(\vec{j} \times \vec{B} - \nabla \cdot \vec{p}_e\right) + \frac{m_e}{ne^2}\frac{\partial \vec{j}}{\partial t}$$

resistivity

electron inertia

Issues

- Physics of Harris current sheet in ideal MHD limit

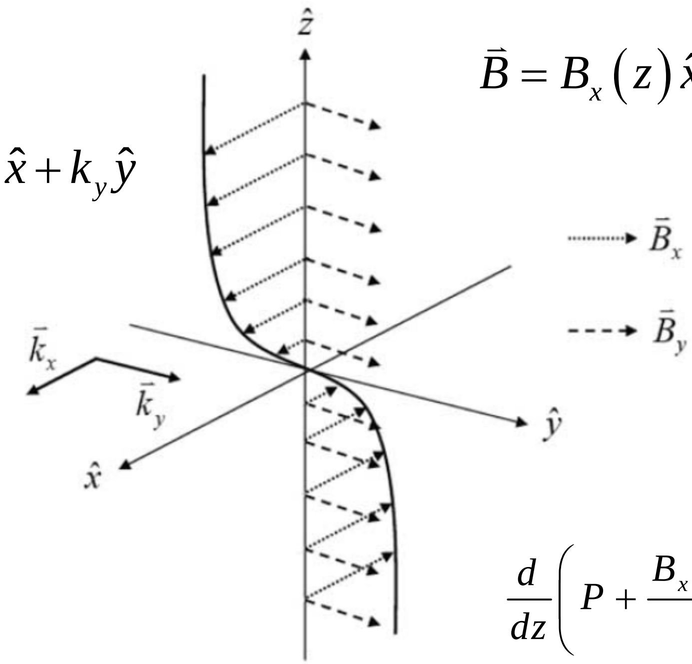
  -- MHD wave propagation in inhomogeneous plasmas

  -- the characteristic modes and resonant effects of Harris current sheet

- Implications for collisionless heating and magnetic reconnection

Magnetic Field Geometry

$$\vec{k} = k_x \hat{x} + k_y \hat{y}$$



$$\vec{B} = B_x\left(z\right)\hat{x} + B_y\left(z\right)\hat{y}$$

$$C_A(z) = \frac{B}{\sqrt{\mu_o \rho}}$$

$$C_S(z) = (\frac{\gamma P}{\rho})^{1/2}$$

$$\theta(z)$$

$$\frac{d}{dz}\left(P + \frac{B_x{}^2 + B_y{}^2}{2\mu_0}\right) = 0$$

$$\delta A(z)\; e^{i(\vec{k}\bullet\vec{x} - \omega t)}$$

Characteristic Speeds

$$V_{f,s}^2 = \frac{1}{2}\left(1+\frac{k_\perp^2}{k_\parallel^2}\right)\left(C_S^2 + C_A^2\right)\left(1 \pm \sqrt{1-\frac{4C_S^2 C_A^2}{\left(1+\frac{k_\perp^2}{k_\parallel^2}\right)\left(C_S^2+C_A^2\right)^2}}\right)\cos^2\theta$$

$$V_i^2 = C_A^2 \cos^2\theta$$

$$k_\parallel(z) \quad , \quad k_\perp(z)$$

$$V_{cp}^2 = \frac{C_A^2 C_S^2}{C_A^2 + C_S^2}\cos^2\theta$$

Wave Equation

$$\frac{d^2\xi_z}{dz^2} + \frac{f'(z)}{f(z)}\frac{d\xi_z}{dz} - \frac{\varepsilon(z)}{f(z)}\xi_z = 0$$

$$f(z) = -\rho C_{ms}^2 \frac{\left(\omega^2/k^2 - V_i^2\right)\left(\omega^2/k^2 - V_{cp}^2\right)}{\left(\omega^2/k^2 - V_f^2\right)\left(\omega^2/k^2 - V_s^2\right)}.$$

$$\frac{\varepsilon}{f} = -\frac{k^2\left(\omega^2/k^2 - V_f^2\right)\left(\omega^2/k^2 - V_s^2\right)}{C_{ms}^2\left(\omega^2/k^2 - V_{cp}^2\right)}$$

$$m_B^2(z) = \frac{\left(\omega^2 - k_{\|}^2 C_A^2\right)\left(C_S^2 - \frac{\omega^2}{k_{\|}^2}\right)}{\left(C_S^2 + C_A^2\right)\left(\frac{\omega^2}{k_{\|}^2} - C_{cp}^2\right)}$$

$$\frac{f'}{f} = \frac{\varepsilon' - 2f\left(k_\perp k'_\perp + m_B m'_B\right)}{\varepsilon}.$$

Mathematical Singularities

$$\frac{d^2\xi_z}{dz^2} + \frac{f'(z)}{f(z)}\frac{d\xi_z}{dz} - \frac{\varepsilon(z)}{f(z)}\xi_z = 0$$

- Singularity arises for

$$\frac{\omega}{k} = V_f \;,\; V_s \;,\; V_i \;,\; V_{cp}$$

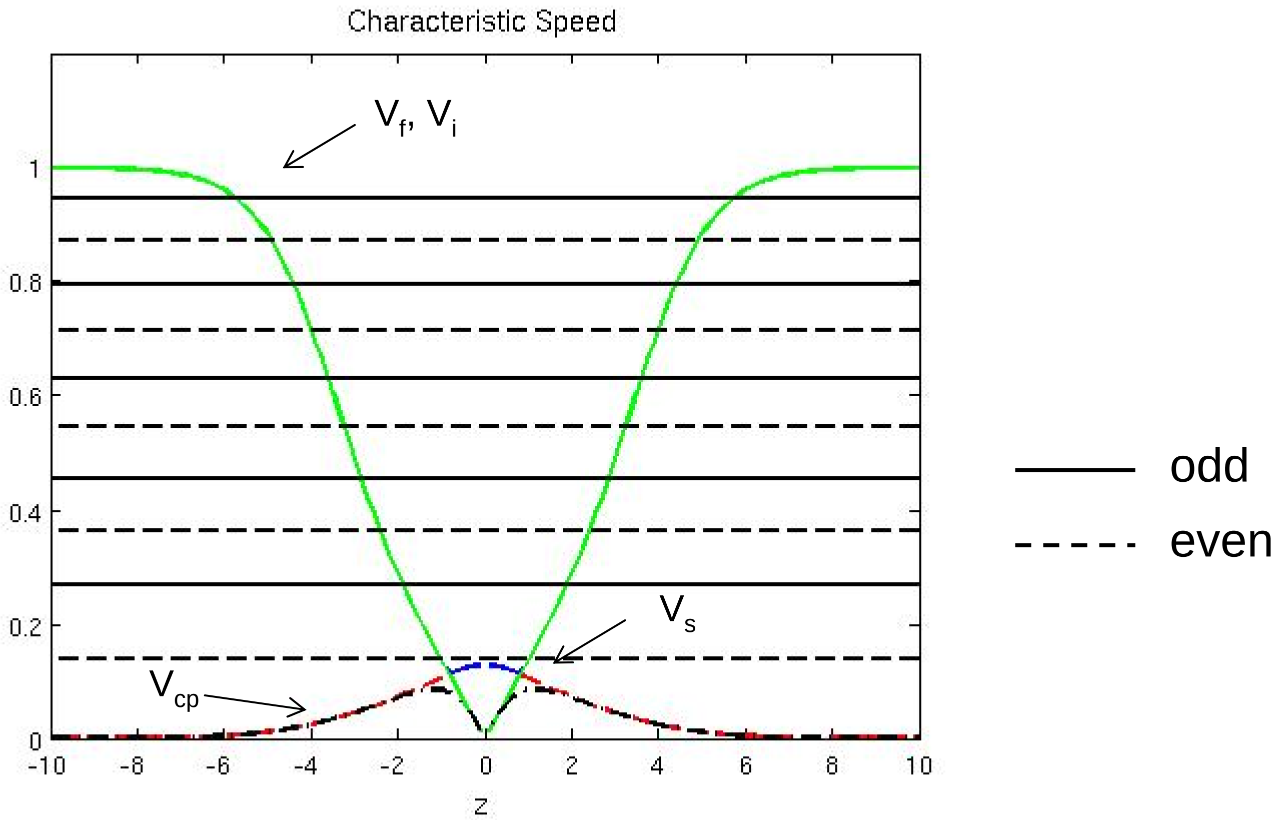
- Cusp resonance occurs for

$$\frac{\omega}{k} = V_{cp}$$

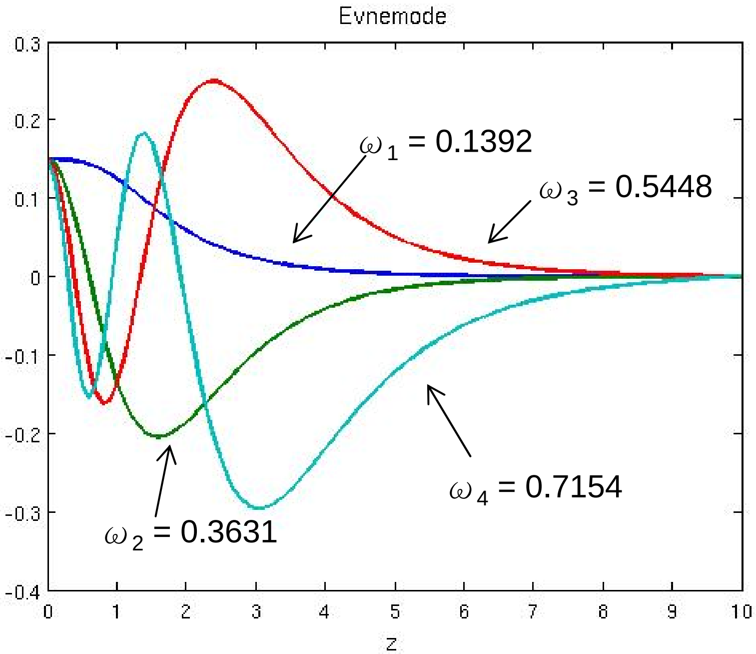
- Alfven or field-line resonance occurs for

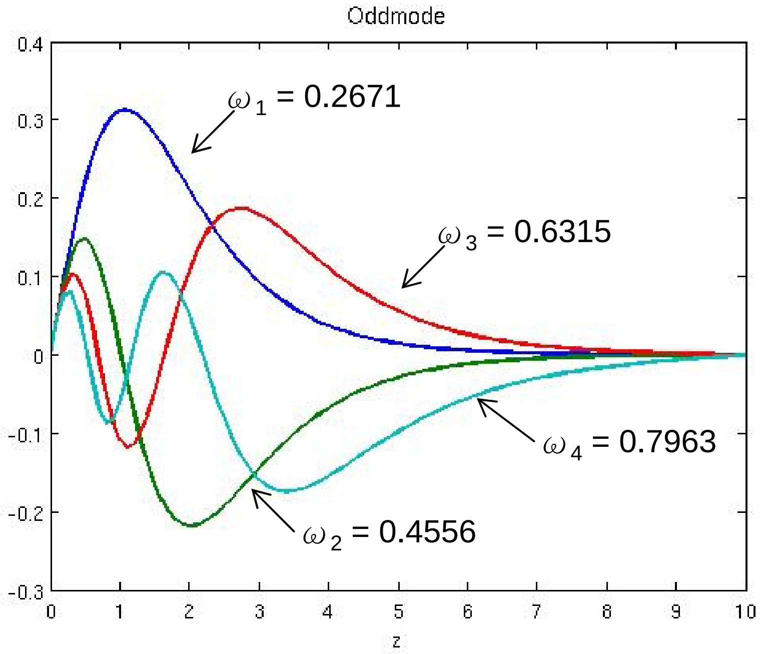
$$k_\perp \neq 0 \;, \qquad \frac{\omega}{k} = C_A \cos\theta$$

$B_y = 0 \;\;, \;\; k_\perp = 0 \;\;, \;\; \theta = 0$ for the entire current layer

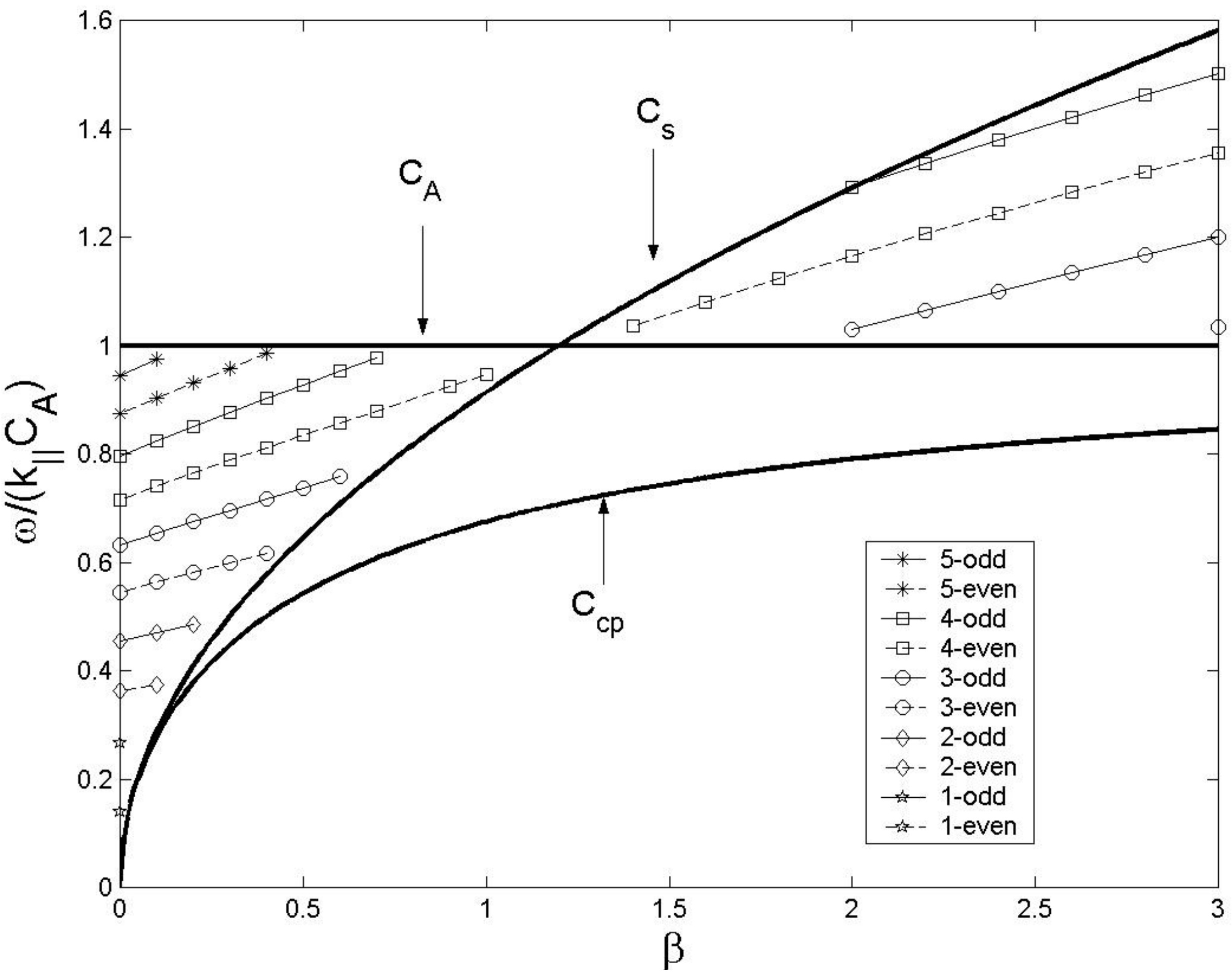


$k_{\perp} = 0$ for the entire current layer

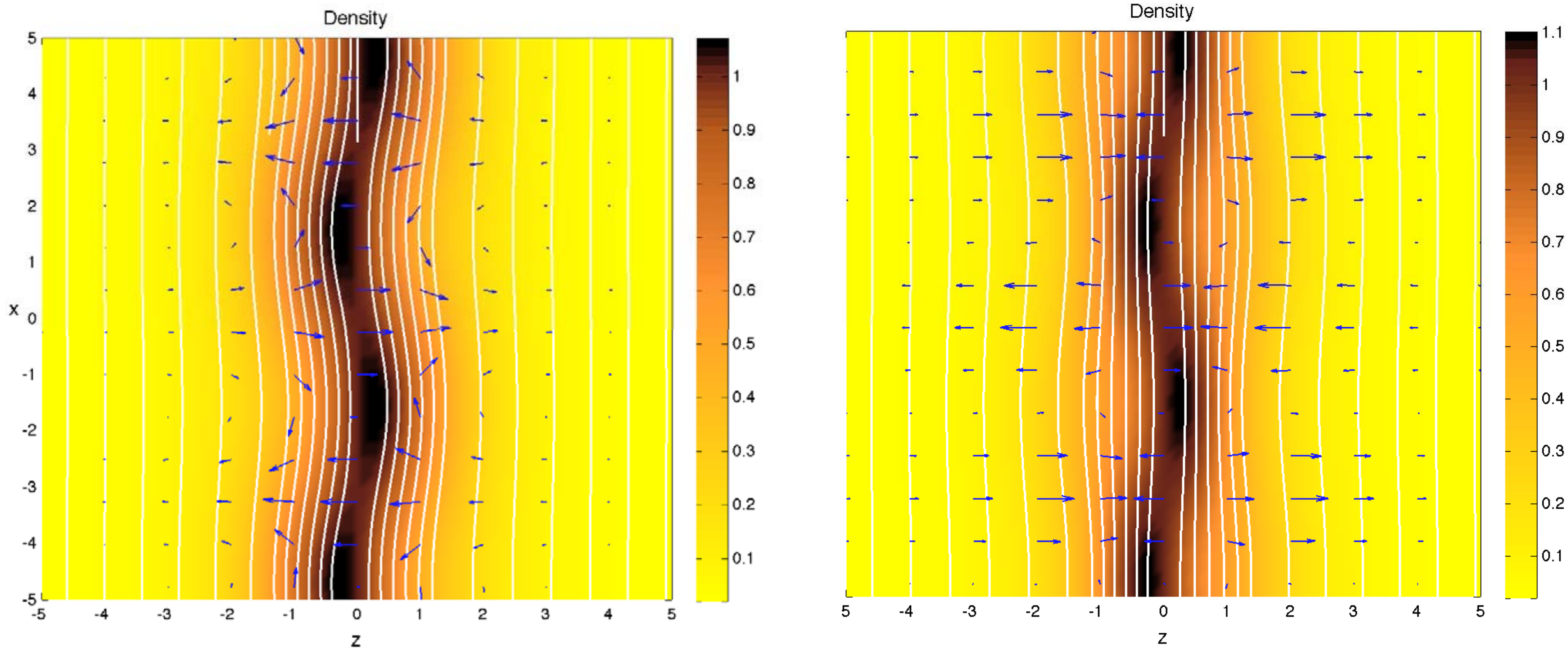




Beta Dependence



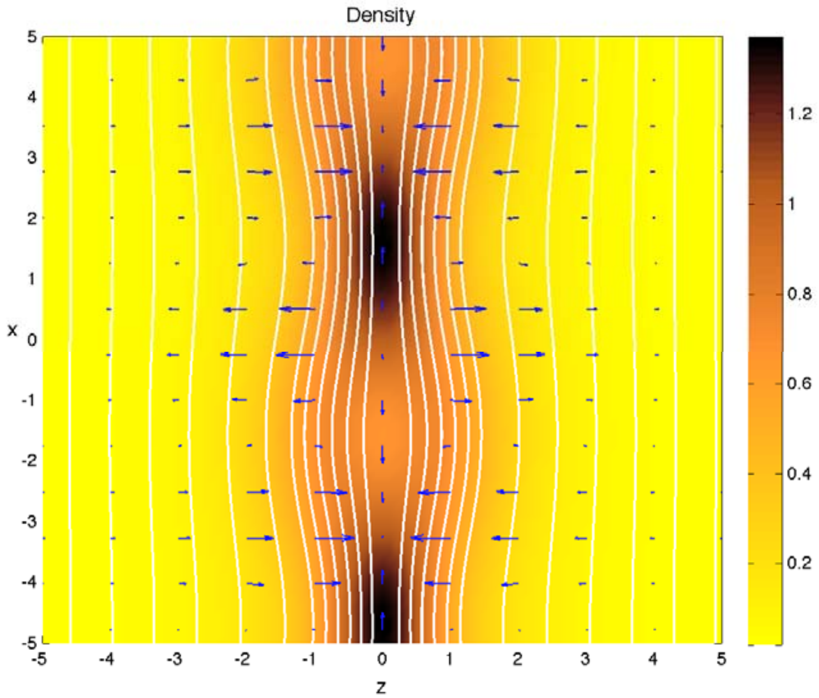
$k_\perp = 0$ for the entire current layer



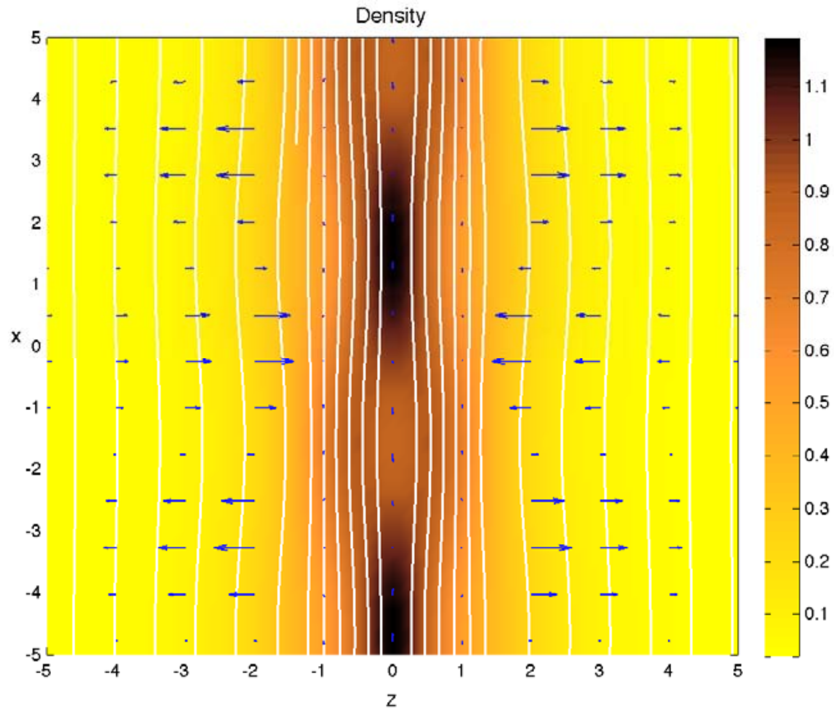
Mode 1 Mode 2

even mode

$k_{\perp} = 0$ for the entire current layer



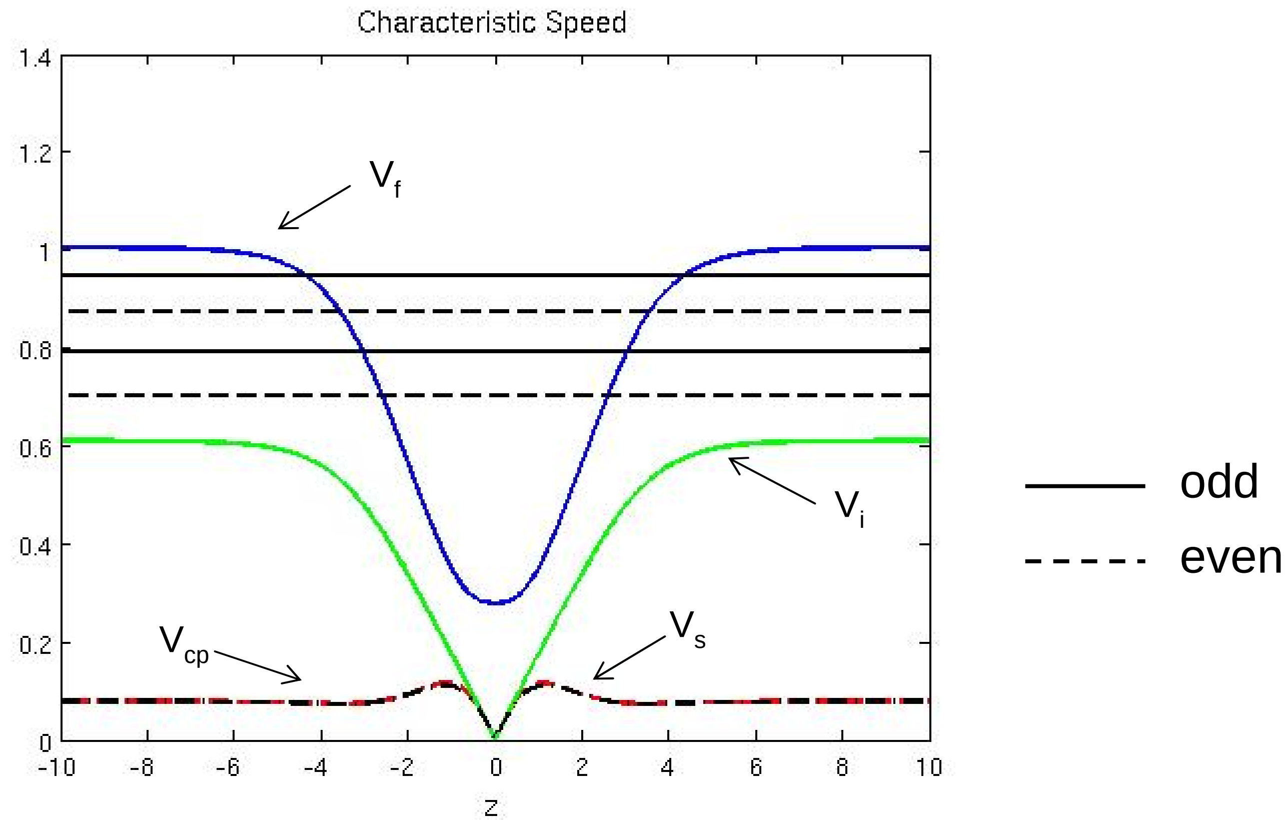
Mode 1



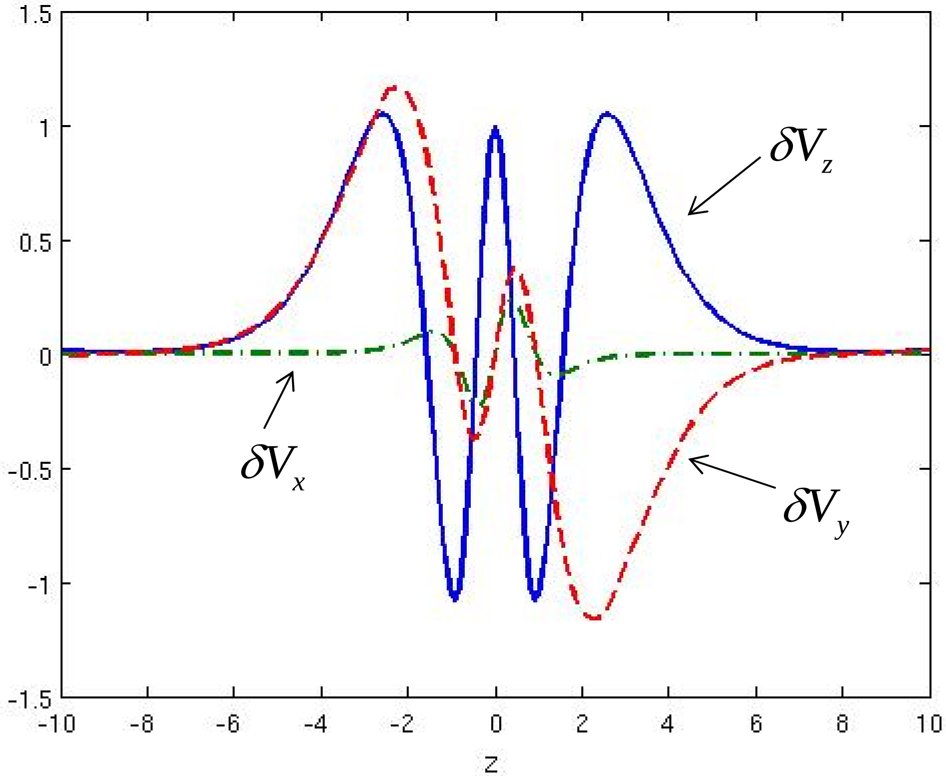
Mode 2

odd mode

$$B_y = 0 \ , \ k_\perp \neq 0 \ , \ \theta \neq 0$$

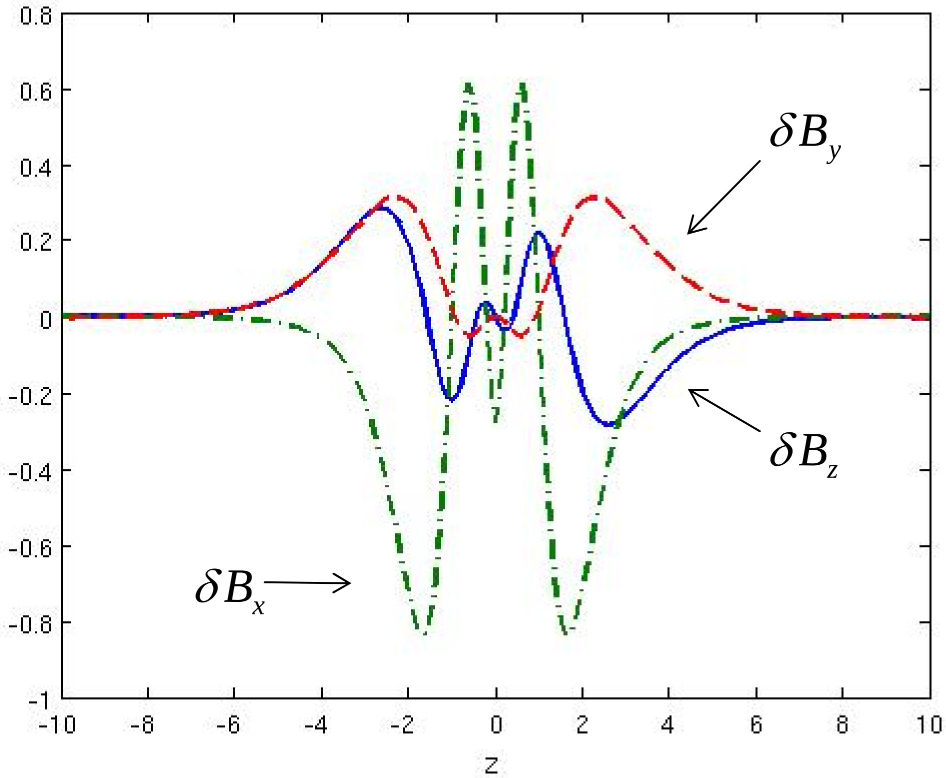


$k_\perp \neq 0$ for the entire current layer



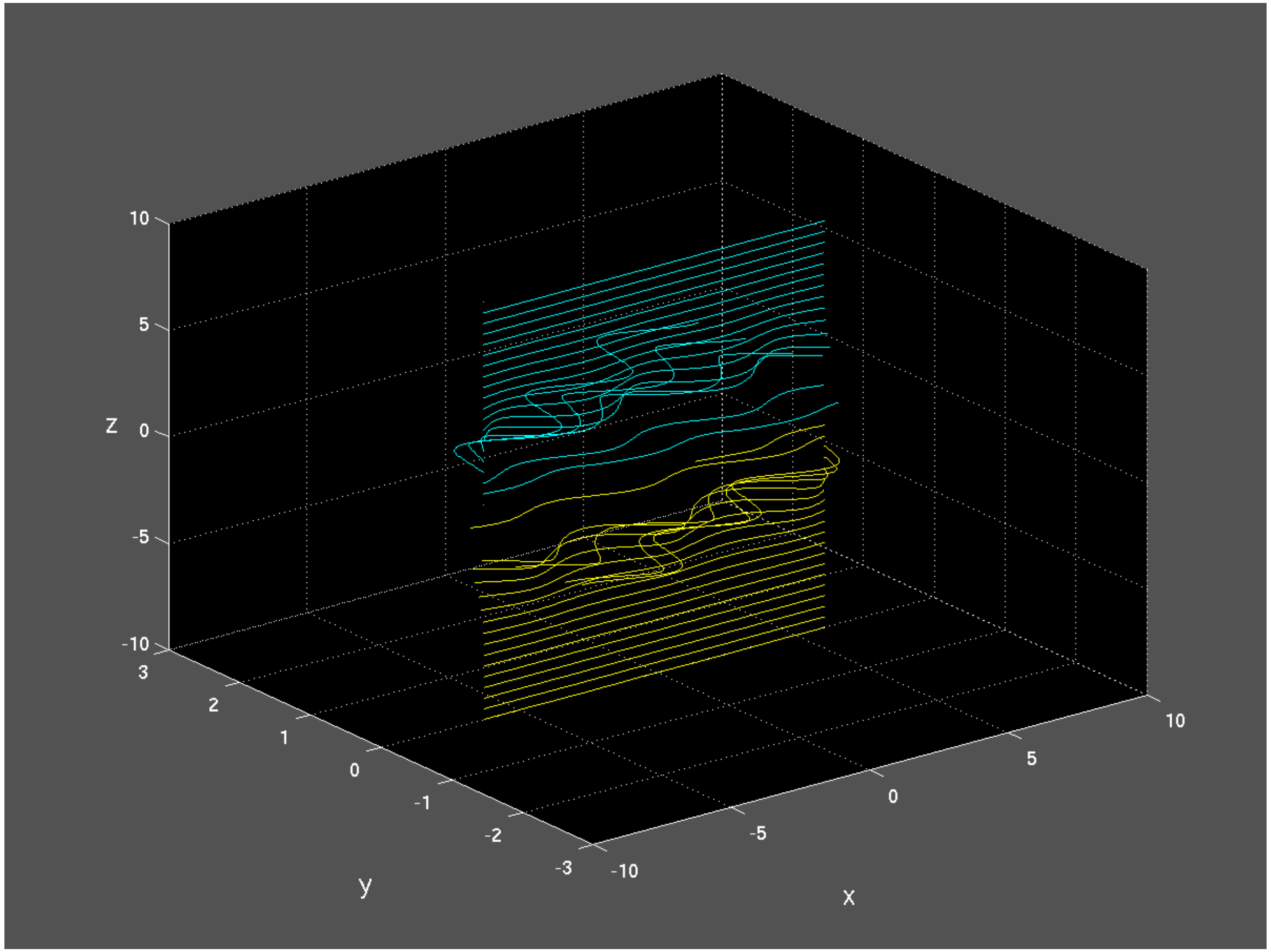
$\delta\vec{V}$ for the first even mode

$k_\perp \neq 0$ for the entire current layer



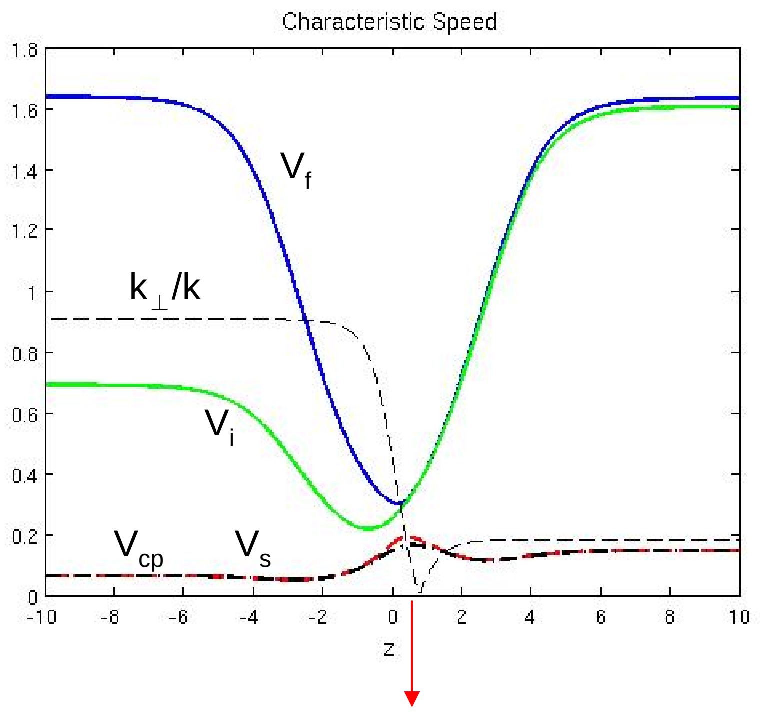
$\delta\vec{B}$ for the first even mode

$B_x = B_x(z)\hat{x},\ k_x / k_y = 0.77,\ \beta_\infty$=0.02, first even-mode



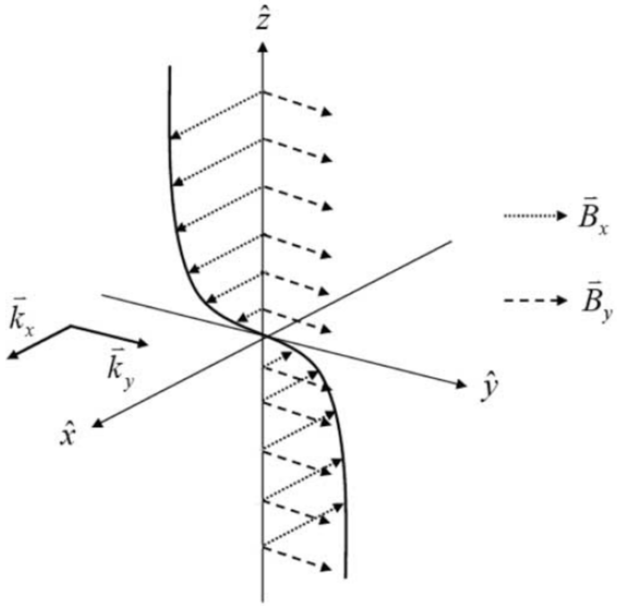
$$B_y \neq 0 \ , \ k_\perp \neq 0 \ , \ \theta(z)$$

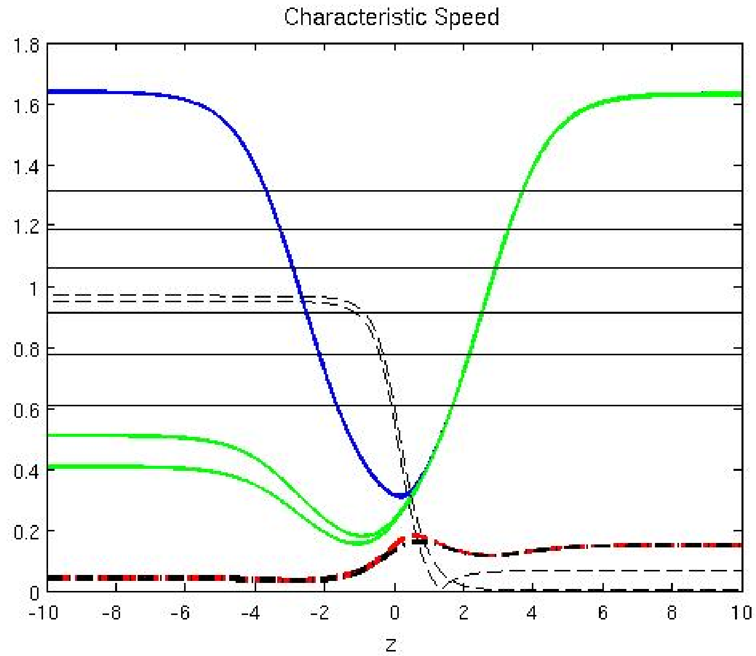
$$k_\perp = 0 \quad \text{at} \quad z = z_t \qquad\qquad k_\perp \neq 0 \quad \text{at} \quad z \neq z_t$$

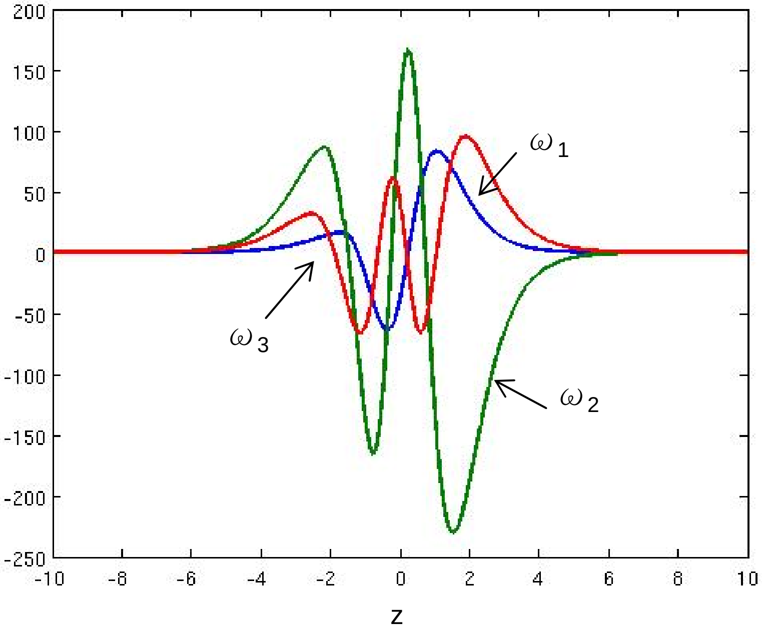


Tunnel Point

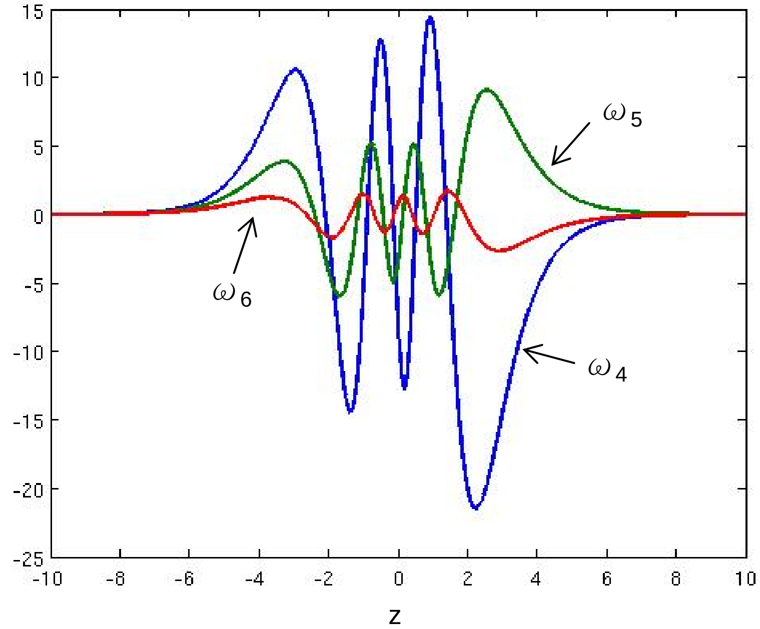
Varying $\vec{k}$ $\Longrightarrow$ $k_{\perp} = 0$ at $z = z_{t1}, z_{t2}, z_{t3}.....$







200
150
100
50
0
-50
-100
-150
-200
-250
-10
-8
-6
-4
-2
0
2
4
6
8
10
z
ω_1
ω_2
ω_3



15
10
5
0
-5
-10
-15
-20
-25
-10
-8
-6
-4
-2
0
2
4
6
8
10
z
ω_4
ω_5
ω_6

Resonance Heating

- Cusp resonance

$$\frac{\omega^2}{k^2} = C_{cp}^2 \cos^2\theta$$

- Alfven or field-line resonance

$$k_\perp \neq 0 \ , \qquad \frac{\omega^2}{k^2} = C_A^2 \cos^2\theta$$

$$d\xi_z / dz \to \infty$$

$\Downarrow$

Plasma heating

- Effects of B_y

$B_y = 0$ and $k_\perp = 0$ -- No field-line resonance

cusp resonance at $z \neq 0$

$B_y = 0$ and $k_\perp \neq 0$ -- field-line resonance at $z \neq 0$

cusp resonance at

$B_y \neq 0$ and $k_\perp \neq 0$ -- field-line resonance at $z \neq 0$

cusp resonance at $z = 0$

Effects of Temperature Anisotropy

- Slow, intermediate and cusp modes may become unstable.
- Firehose and mirror instabilities may develop.
- Both discrete and resonant modes may become unstable at firehose and mirror instability thresholds.
- Profiles of characteristic speeds depend on the energy laws.
- Analyses involve more free parameters.

Conclusions

- Thin current sheet may support the propagation of discrete wave modes.
- Singularities arise in ideal MHD theory implying for plasma heating via resonance.
- Introducing a guide magnetic field in the current sheet may lead to cusp and Alfven resonances there.
- There is a way to propagate shear Alfven wave in the current sheet without encountering resonance.
- Both discrete and resonant modes may develop firehose and mirror instabilities.
- **Perturbations can be excited and amplified at the central sheet via external source or internal instability.**

References

- Hau and Lai, manuscript, 2012.
- Lai and Hau, manuscript, 2012.
- Lee and Hau, JGR, 2008.
- Hau and Lin, PoP, 1995.