US–Japan Workshop on Magnetic Reconnection (MR2012)
PPPL, 23-25 May 2012

Turbulent transport and magnetic reconnection

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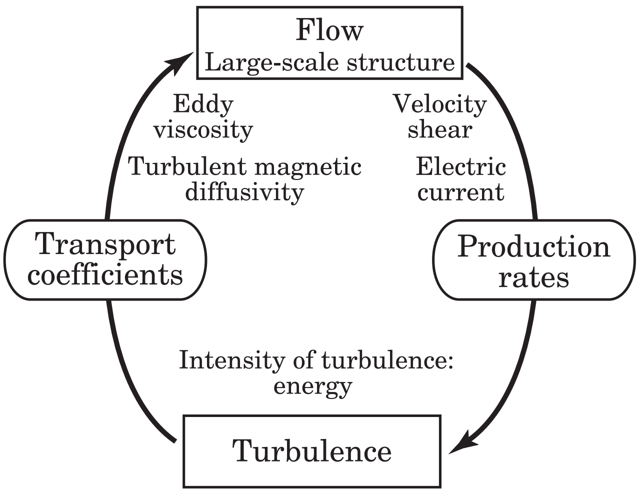
Topics

- Turbulence
- Cross helicity effects
- Flow–turbulence interaction in magnetic reconnection
- Summary

How I feel about turbulence

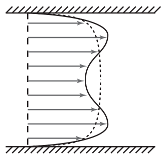
Mean-field structures determine
the properties of turbulence
through production rates

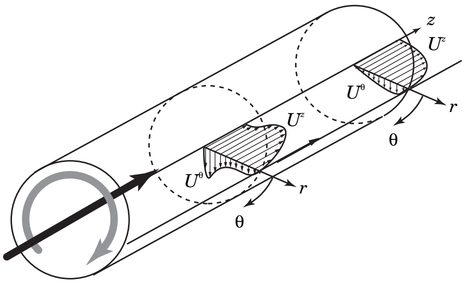


Turbulence properties determine
the mean-field structures
through transport coefficients

Suppression of transport

Turbulent swirling pipe flow





Large-scale structure again! $\longleftarrow$ Additional symmetry breakage

$$\mathcal{R}_{\alpha\beta} \equiv \left\langle u'_\alpha u'_\beta \right\rangle$$
$$= \frac{2}{3} K \delta_{\alpha\beta} - \nu_\mathrm{T} \left(\frac{\partial U_\alpha}{\partial x_\beta} + \frac{\partial U_\beta}{\partial x_\alpha} \right)$$
$$+ \eta \left[\Omega_\alpha \frac{\partial H}{\partial x_\beta} + \Omega_\beta \frac{\partial H}{\partial x_\alpha} - \frac{2}{3} \delta_{\alpha\beta} \left(\mathbf{\Omega} \cdot \nabla \right) H \right]$$

Kinetic helicity

$$H = \langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle$$

Mean vorticity

$$\mathbf{\Omega} = \nabla \times \mathbf{U}$$

Transport suppression due to helicity effect (Yokoi & Yoshizawa, 1993)

Turbulent energy

$$\frac{\partial}{\partial t} \frac{1}{2} \left\langle \mathbf{u}'^2 \right\rangle = - \left\langle u'^a u'^b \right\rangle \frac{\partial U^a}{\partial x^b} + \cdots \qquad \longleftarrow \qquad + \frac{1}{2} \nu_\mathrm{T} \left(\nabla \mathbf{U} \right)^2$$

Turbulent helicity

$$\frac{\partial}{\partial t} \langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle = - \left\langle u'^a u'^b \right\rangle \frac{\partial \Omega^a}{\partial x^b} + \cdots \qquad \longleftarrow \qquad + \frac{1}{2} \nu_\mathrm{T} \left(\nabla \mathbf{U} \right) \left(\nabla \mathbf{\Omega} \right)$$

Turbulent MHD energy

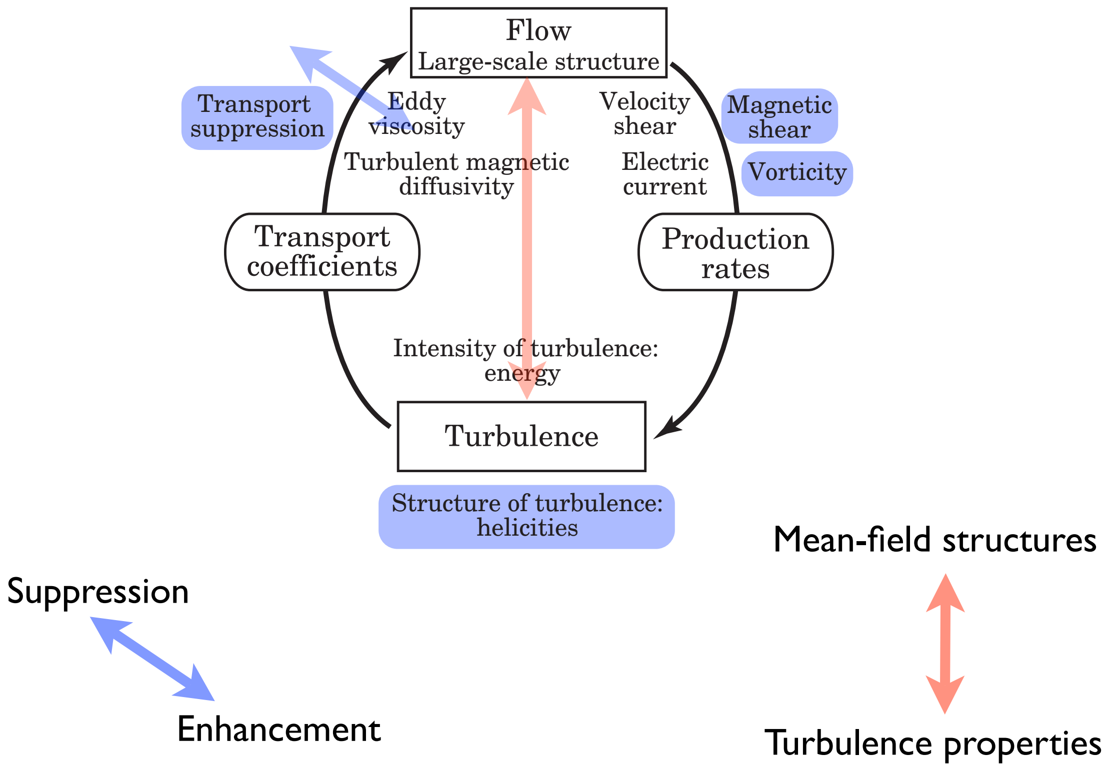
$$\frac{\partial}{\partial t} \frac{1}{2} \left\langle \mathbf{u}'^2 + \mathbf{b}'^2 \right\rangle = - \left\langle u'^a u'^b - b'^a b'^b \right\rangle \frac{\partial U^a}{\partial x^b} - \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \mathbf{J} + \cdots$$
$$\longleftarrow \qquad + \frac{1}{2} \nu_\mathrm{K} \left(\nabla \mathbf{U} \right)^2 + \beta \mathbf{J}^2$$

Turbulent magnetic helicity

$$\frac{\partial}{\partial t} \langle \mathbf{a}' \cdot \mathbf{b}' \rangle = - \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \mathbf{B} + \cdots \qquad \longleftarrow \qquad + \beta \mathbf{J} \cdot \mathbf{B}$$

Turbulent cross helicity

$$\frac{\partial}{\partial t} \langle \mathbf{u}' \cdot \mathbf{b}' \rangle = - \left\langle u'^a u'^b - b'^a b'^b \right\rangle \frac{\partial B^a}{\partial x^b} - \langle \mathbf{u}' \times \mathbf{b}' \rangle \cdot \mathbf{\Omega} + \cdots$$
$$\longleftarrow \qquad + \frac{1}{2} \nu_\mathrm{K} \left(\nabla \mathbf{U} \right) \left(\nabla \mathbf{B} \right) + \beta \mathbf{J} \cdot \mathbf{\Omega}$$



- Mean & Fluctuation
  (Flow & Turbulence)

- Enhancement vs. Suppression
  (Intensity & structural
  information of turbulence)

What is cross helicity?

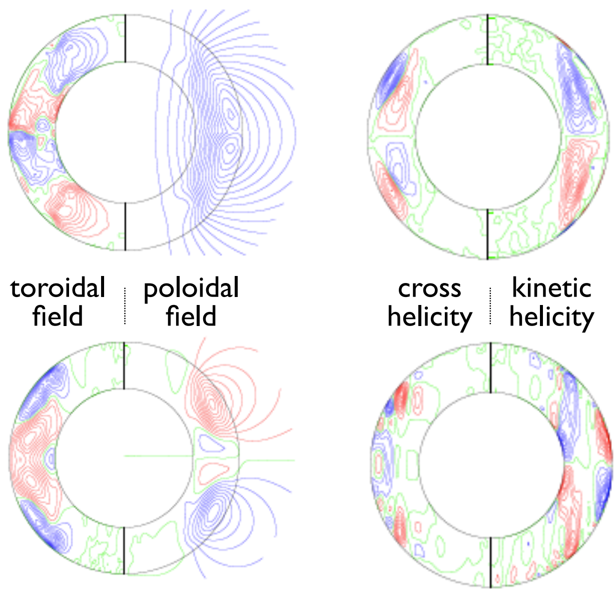
Yokoi (2012) submitted to Geophys. Astrophys. Fluid Dyn.

Pseudoscalar

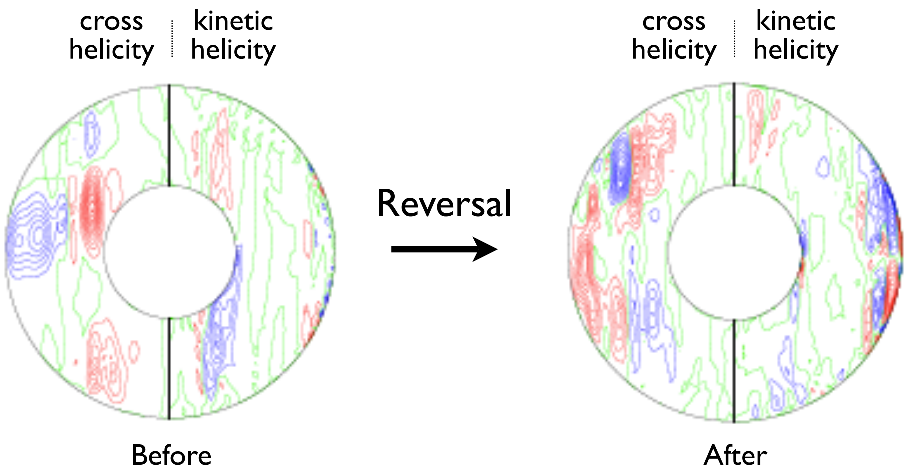
Spatial distribution (with R. Simitev & F. Busse)

Dipole-like case

Quadrupole-like case



Signs of cross helicity and helicity during the polarity reversal



Cross helicity changes its sign

Kinetic helicity does not changes its sign

Turbulence dynamo

Induction equation
$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{b}) + \eta \nabla^2 \mathbf{b}$$

$$\mathbf{u} = \mathbf{U} + \mathbf{u}', \quad \mathbf{b} = \mathbf{B} + \mathbf{b}', \quad \cdots$$

Mean induction equation
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \nabla \times \mathbf{E}_{\mathrm{M}} + \eta \nabla^2 \mathbf{B}$$

turbulent electromotive force

$$\begin{aligned} \mathbf{E}_{\mathrm{M}} &\equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle \\ &= \alpha \mathbf{B} - \beta \nabla \times \mathbf{B} + \gamma \boldsymbol{\Omega} \end{aligned}$$

Mean vorticity

$$\boldsymbol{\Omega} = \nabla \times \mathbf{U}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) - \nabla \times [(\eta + \underline{\beta}) \nabla \times \mathbf{B}] + \nabla \times (\underline{\alpha} \mathbf{B} + \underline{\gamma} \boldsymbol{\Omega})$$

Enhanced resistivity

Generation due to pseudoscalars

Magnetic-flux freezing in turbulence

Interpretation from turbulent transport

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \nabla \times \mathbf{E}_{\rm M} + \eta \nabla^2 \mathbf{B}$$

$$\longrightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) - \nabla \times [(\eta + \beta) \nabla \times \mathbf{B}] + \nabla \times (\alpha \mathbf{B} + \gamma \mathbf{\Omega})$$

Without any pseudoscalar-related effects

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) - \nabla \times [(\eta + \beta) \nabla \times \mathbf{B}]$$

$$\simeq \nabla \times (\mathbf{U} \times \mathbf{B}) - \nabla \times (\beta \nabla \times \mathbf{B}) \qquad \text{large effective resistivity}$$

If the turbulent resistivity is balanced by some pseudo-scalar-related effects

$$\mathbf{E}_{\rm M} \simeq 0 \quad \text{or} \quad \beta \mathbf{J} \simeq \alpha \mathbf{B} + \gamma \mathbf{\Omega}$$

Then $$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \qquad \text{very small effective resistivity}$$

Condition for freezing
dynamically determined by turbulent flow properties

Transport coefficients are determined by the turbulence properties

turbulent magnetic diffusivity

$$\mathbf{E}_{\rm M} \equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle = \alpha \mathbf{B} - \beta \nabla \times \mathbf{B} + \gamma \nabla \times \mathbf{U}$$

helicity effect   cross-helicity effect

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) - \nabla \times [(\eta + \underline{\beta}) \nabla \times \mathbf{B}] + \nabla \times (\underline{\alpha} \mathbf{B} + \underline{\gamma} \mathbf{\Omega})$$

$$\alpha = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[-H_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + H_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right]$$

kinetic helicity   current helicity

$$\beta = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right]$$

kinetic energy   magnetic energy

$$\gamma = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{ub}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bu}(k, \mathbf{x}; \tau, \tau_1, t) \right]$$

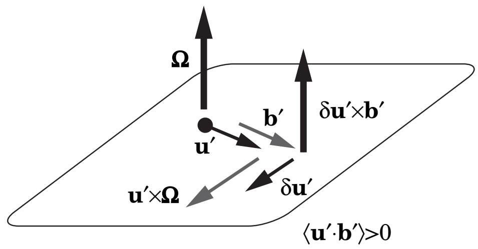
cross helicity

Cross-helicity (γ) effect

(Yoshizawa, 1990; Yoshizawa & Yokoi, 1993)

$$\delta \mathbf{u}' = \tau \mathbf{u}' \times \boldsymbol{\Omega}$$

$$\delta \mathbf{u}' \times \mathbf{b}' = \tau \left(\mathbf{u}' \times \boldsymbol{\Omega} \right) \times \mathbf{b}'$$



(Yokoi, 1999)

$\left\{ \begin{array}{l} \text{Correlation between } \mathbf{u}' \text{ and } \mathbf{b}' \\ \text{Local angular-momentum conservation} \end{array} \right.$

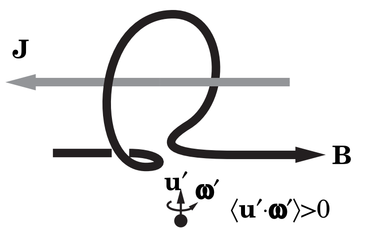
$$[\mathbf{E}_{\mathrm{M}}]_\gamma = \langle \delta \mathbf{u}' \times \mathbf{b}' \rangle = +\tau_\gamma \langle \mathbf{u}' \cdot \mathbf{b}' \rangle \, \boldsymbol{\Omega}$$

$\longrightarrow$ Turbulent electromotive force contribution parallel to the mean vorticity

Helicity and cross-helicity dynamos

$$\mathbf{E}_{\mathrm{M}} = \overbrace{\alpha \mathbf{B}}^{\text{helicity dynamo}} - \beta \nabla \times \mathbf{B} + \underbrace{\gamma \nabla \times \mathbf{U}}_{\text{cross-helicity dynamo}}$$

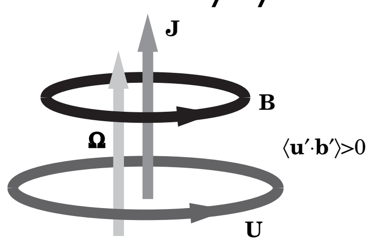
Helicity or α dynamo



(Krause & Rädler, 1980)

$\mathbf{J} \parallel \mathbf{B}$

Cross-helicity dynamo



(Yoshizawa & Yokoi, 1993)

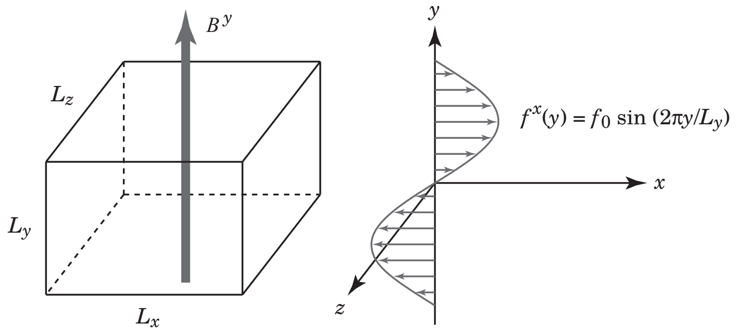
$\mathbf{J} \parallel \boldsymbol{\Omega}$

$\mathbf{B} \parallel \mathbf{U}$

DNS of Kolmogorov flow

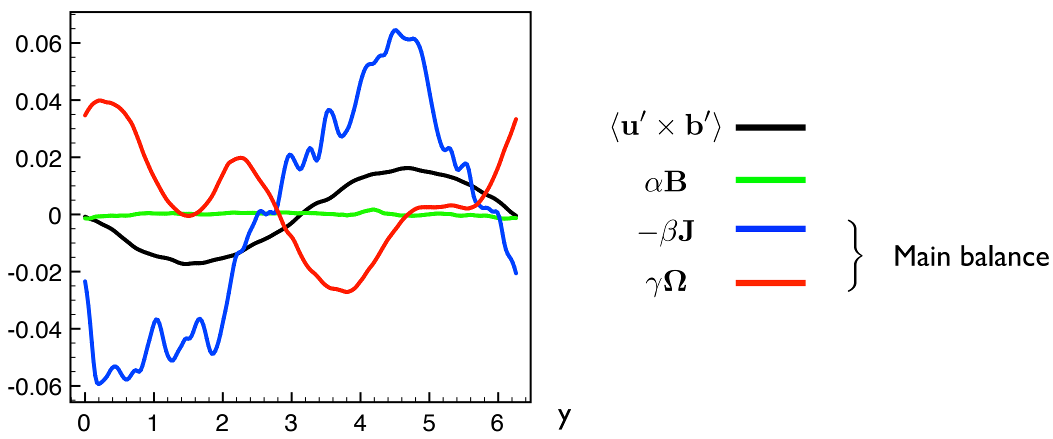
(Yokoi & Balarac, 2011)

- 3D (256^3) periodic flow with external forcing $f^x(y) = f_0 \sin(2\pi y/L_y)$
- Mean shear velocity
- Constant magnetic field imposed [y (inhomogeneous) direction]
- Homogeneous in x and z directions



cf. Archontis flow, a generalization of the Arnold–Beltrami–Childress flow (Sur & Brandenburg, 2009)

Turbulent electromotive force



The cross-helicity effect, rather than the helicity or α effect, plays a dominant role in balancing the turbulent magnetic diffusivity effect

What makes cross helicity?

Cross-helicity generation mechanism

Evolution equation of the turbulent cross helicity $W \equiv \langle \mathbf{u}' \cdot \mathbf{b}' \rangle$

$$\frac{DW}{Dt} \equiv \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) W = P_W - \varepsilon_W + \nabla \cdot \mathbf{T}_W$$

where

$$P_W = -\mathcal{R}^{ab} \frac{\partial B^a}{\partial x^b} - \mathbf{E}_\mathrm{M} \cdot \mathbf{\Omega} \qquad \text{production rate}$$

$$\varepsilon_W = (\nu + \eta) \left\langle \frac{\partial u'^a}{\partial x^b} \frac{\partial b'^a}{\partial x^b} \right\rangle \qquad \text{dissipation rate}$$

$$\mathbf{T}_W = K\mathbf{B} - \left\langle (\mathbf{u}' \cdot \mathbf{b}')\, \mathbf{u}' - \left(\frac{\mathbf{u}'^2 + \mathbf{b}'^2}{2} - p'_\mathrm{M} \right) \mathbf{b}' \right\rangle \qquad \text{transport rate}$$

with

$$\mathcal{R}^{\alpha\beta} = \left\langle u'^\alpha u'^\beta - b'^\alpha b'^\beta \right\rangle \qquad \text{Reynolds stress}$$

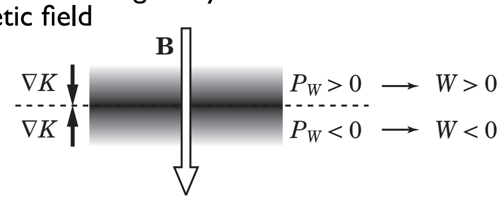
$$\mathbf{E}_\mathrm{M} \equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle \qquad \text{Turbulent electromotive force}$$

- Generation due to vorticity

$$-\mathbf{E}_\mathrm{M} \cdot \mathbf{\Omega} = -\alpha \mathbf{B} \cdot \mathbf{\Omega} + \beta \mathbf{J} \cdot \mathbf{\Omega} - \gamma \mathbf{\Omega}^2$$

- Generation due to inhomogeneity along the magnetic field

$$\nabla \cdot (\mathbf{B} K) = \mathbf{B} \cdot (\nabla K)$$



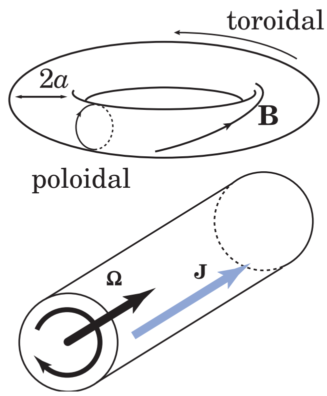
Generation due to vorticity

(Yokoi, 1999)

$$W \equiv \langle \mathbf{u}' \cdot \mathbf{b}' \rangle$$

$$\frac{\partial W}{\partial t} \simeq -\mathbf{E}_{\rm M} \cdot \boldsymbol{\Omega} = -\alpha \mathbf{B} \cdot \boldsymbol{\Omega} + \beta \mathbf{J} \cdot \boldsymbol{\Omega} - \gamma \boldsymbol{\Omega}^2$$

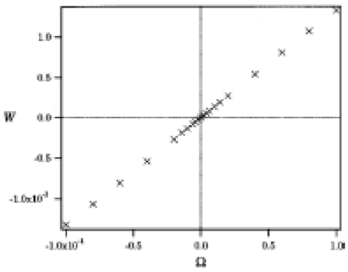
with $\quad \boldsymbol{\Omega} = \nabla \times \mathbf{U} \qquad \mathbf{J} = \nabla \times \mathbf{B}$



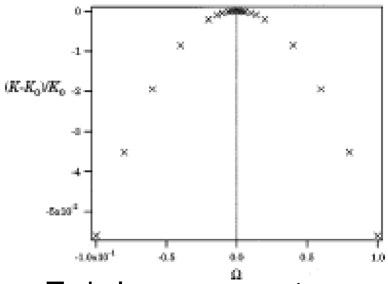
Plasma current + poloidal rotation

$$\mathbf{J} \cdot \boldsymbol{\Omega} > 0 \quad \rightarrow \quad \frac{\partial W}{\partial t} > 0 \quad \rightarrow \quad W > 0$$

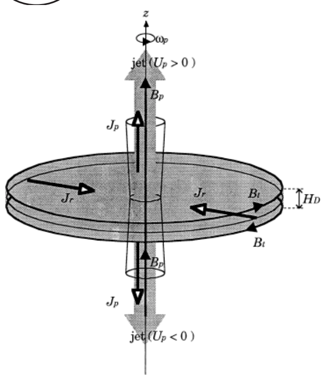
$$\mathbf{J} \cdot \boldsymbol{\Omega} < 0 \quad \rightarrow \quad \frac{\partial W}{\partial t} < 0 \quad \rightarrow \quad W < 0$$



Generated cross helicity against the imposed rotation



Turbulence suppression rate against the imposed rotation



Flow–turbulence interaction in magnetic reconnection

Yokoi & Hoshino (2011) Phys. Plasmas **18**,111208

Too slow

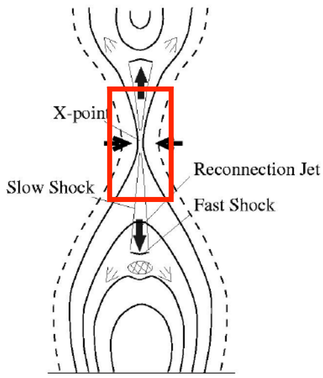
$$M_{\text{in}} = \frac{U_{\text{in}}}{V_{\text{Ain}}} = \frac{\delta}{L} = S^{-1/2}$$

Lundquist number $S = \frac{\mu_0 L V_{\text{A}}}{\eta}$

astrophysical and space plasmas $S \gg 10^6$

$\longrightarrow$ **Fast reconnection**

Gap of scales



Thickness of current sheet

$$\delta = \rho_{\text{i}} \sim 10 \text{ m}$$

Ion Larmor radius ρ_{i}

Flare scale 10^4 km

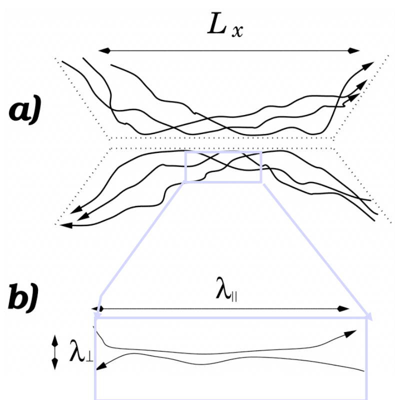
$\longrightarrow$ **Localized resistivity**

$\longrightarrow$ **Matching of scales**

Turbulent reconnection

Matthaeus & Lamkin (1985)

Lazarian & Vishniac (1999)



$$M_{\text{in}} = \frac{U_{\text{in}}}{V_{\text{Ain}}} \leq M_{\text{turb}}^2$$

M_{turb} : large-scale magnetic Mach number of turbulence

Fractal current sheet

Tajima & Shibata (1997)

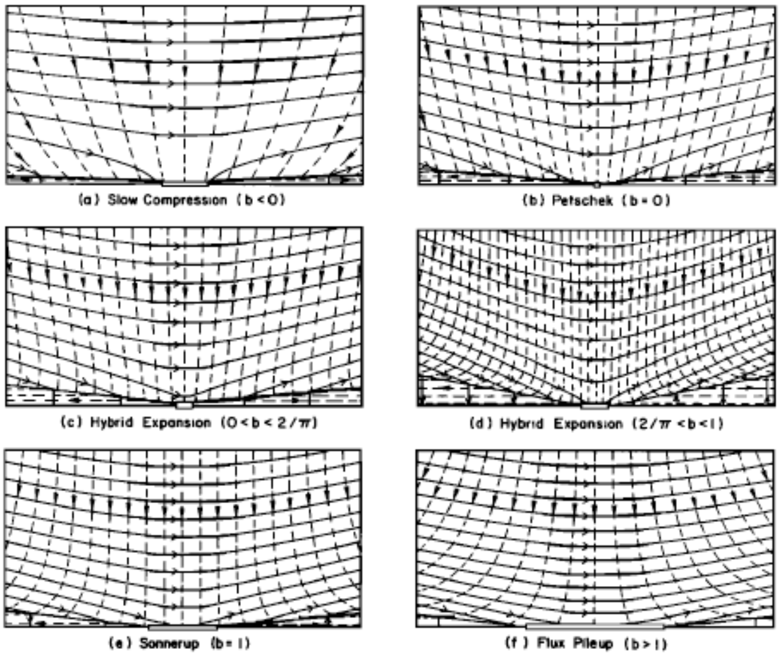


Fig. 5. Magnetic field lines (solid lines) and streamlines (dashed lines) for different classes of solution with external magnetic Reynolds number $R_{me} = 500$. The open rectangular boxes indicate the lengths of the diffusion regions: (*a*) $b = -2.0$, $M_e = 0.043$; (*b*) $b = 0$, $M_e = 0.091$; (*c*) $b = 0.3$, $M_e = 0.100$; (*d*) $b = 0.8$, $M_e = 0.200$; (*e*) $b = 1.0$, $M_e = 0.100$; (*f*) $b = 2.0$, $M_e = 0.100$. Only every third streamline in the outflow jets is shown [from *Priest and Forbes*, 1986].

Forbes & Priest, 1987

Turbulence effects

Reynolds (+ turbulent Maxwell) stress

$$\mathcal{R}^{\alpha\beta} \equiv \langle u'^{\alpha} u'^{\beta} - b'^{\alpha} b'^{\beta} \rangle$$
$$= \frac{2}{3} K_{\mathrm{R}} \delta^{\alpha\beta} - \nu_{\mathrm{K}} \mathcal{S}^{\alpha\beta} + \nu_{\mathrm{M}} \mathcal{M}^{\alpha\beta} + \left[\Gamma^{\alpha} \Omega^{\beta} + \Gamma^{\beta} \Omega^{\alpha} \right]_{\mathrm{D}}$$

Enhancement Suppression

Turbulent electromotive force

$$\mathbf{E}_{\mathrm{M}} \equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle$$
$$= -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \boldsymbol{\Omega}$$

Enhancement Suppression

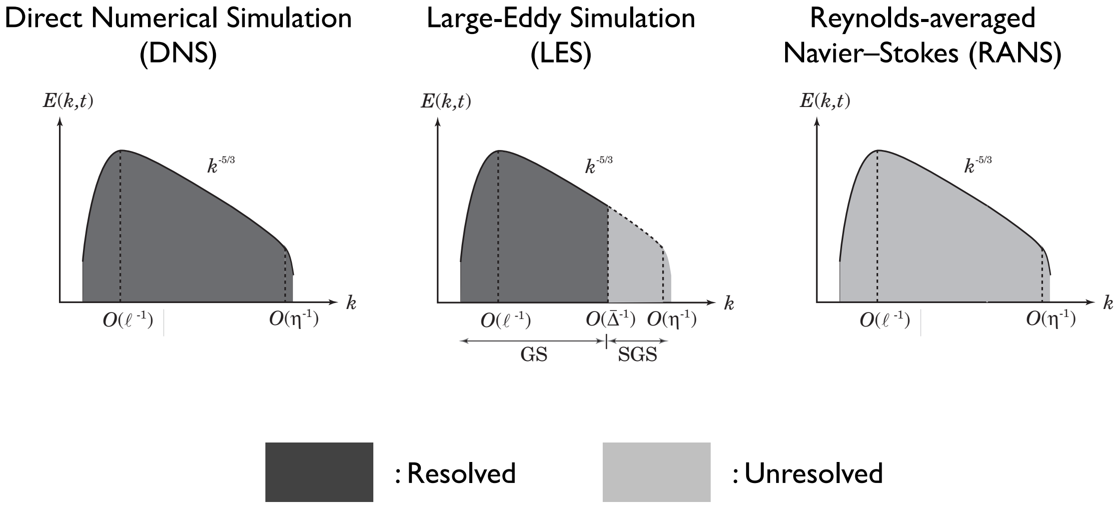
$$\alpha = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[-H_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + H_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right]$$

$$\beta = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{uu}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bb}(k, \mathbf{x}; \tau, \tau_1, t) \right] = \frac{5}{7} \nu_{\mathrm{K}}$$

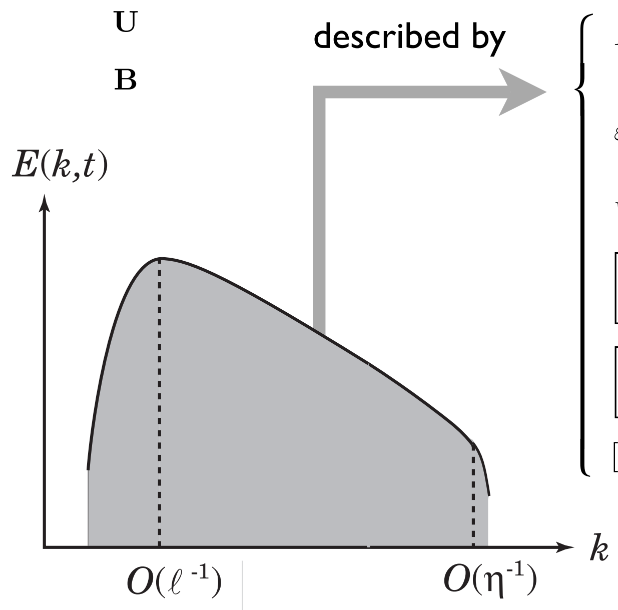
$$\gamma = \frac{1}{3} \int d\mathbf{k} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \left[Q_{ub}(k, \mathbf{x}; \tau, \tau_1, t) + Q_{bu}(k, \mathbf{x}; \tau, \tau_1, t) \right] = \frac{5}{7} \nu_{\mathrm{M}}$$

$$\boldsymbol{\Gamma} = \frac{1}{15} \int d\mathbf{k}\, k^{-2} \int_{-\infty}^{\tau} d\tau_1 G(k, \mathbf{x}; \tau, \tau_1, t) \nabla H_{uu}(k, \mathbf{x}; \tau, \tau_1, t)$$

Simulations and Turbulence Models



Turbulent statistical quantities



$$
\begin{cases}
K \equiv \frac{1}{2}\left\langle \mathbf{u}'^2 + \mathbf{b}'^2 \right\rangle \\
\varepsilon \equiv \nu \left\langle \left(\frac{\partial u'^a}{\partial x^b} \right)^2 \right\rangle + \lambda \left\langle \left(\frac{\partial b'^a}{\partial x^b} \right)^2 \right\rangle \\
W \equiv \langle \mathbf{u}' \cdot \mathbf{b}' \rangle \\
\left[K_{\mathrm{R}} \equiv \frac{1}{2}\left\langle \mathbf{u}'^2 - \mathbf{b}'^2 \right\rangle \right] \\
\left[\varepsilon_W \equiv (\nu + \lambda) \left\langle \frac{\partial u'^a}{\partial x^b} \frac{\partial b'^a}{\partial x^b} \right\rangle \right] \\
\left[H \equiv \langle -\mathbf{u}' \cdot \boldsymbol{\omega}' + \mathbf{b}' \cdot \mathbf{j}' \rangle \right]
\end{cases}
$$

What should be solved ...

$$\frac{\partial \mathbf{U}}{\partial t} = \cdots + \mathbf{J} \times \mathbf{B} - \nabla \cdot \boldsymbol{\mathcal{R}} + \cdots$$

$$\frac{\partial \mathbf{B}}{\partial t} = \cdots + \nabla \times \mathbf{E}_{\mathrm{M}} + \cdots$$

$$\begin{aligned}\mathcal{R}^{\alpha\beta} &\equiv \left\langle u'^{\alpha} u'^{\beta} - b'^{\alpha} b'^{\beta} \right\rangle \\ &= \frac{2}{3} K_{\mathrm{R}} \delta^{\alpha\beta} - \nu_{\mathrm{K}} \mathcal{S}^{\alpha\beta} + \nu_{\mathrm{M}} \mathcal{M}^{\alpha\beta} + \left[\Gamma^{\alpha} \Omega^{\beta} + \Gamma^{\beta} \Omega^{\alpha}\right]_{\mathrm{D}}\end{aligned}$$

$$\begin{aligned}\mathbf{E}_{\mathrm{M}} &\equiv \langle \mathbf{u}' \times \mathbf{b}' \rangle \\ &= -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \boldsymbol{\Omega}\end{aligned}$$

$\frac{\partial \beta}{\partial t} = \cdots$ (turbulent correlation) x (mean-field inhomogeneity)

$\frac{\partial \gamma}{\partial t} = \cdots$ (turbulent correlation) x (mean-field inhomogeneity)

$\frac{\partial \alpha}{\partial t} = \cdots$ (turbulent correlation) x (mean-field inhomogeneity)

Analytical solution

Mean induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} + \mathbf{E}_\mathrm{M}) + \eta \nabla^2 \mathbf{B} \qquad \mathbf{E}_\mathrm{M} = -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \mathbf{\Omega}$$

$$\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}, \quad \mathbf{J} = \mathbf{J}_0 + \delta \mathbf{J}$$

Reference $$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}_0) + \nabla \times (\alpha \mathbf{B}_0 - \beta \nabla \times \mathbf{B}_0)$$

Modulation $$\frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \delta \mathbf{B}) - \nabla \times \left[\beta \left(\nabla \times \delta \mathbf{B} - \frac{\gamma}{\beta} \nabla \times \mathbf{U} \right) \right]$$

$$\rightarrow \quad \delta \mathbf{B} = \frac{\gamma}{\beta} \mathbf{U} = C_\mathrm{W} \frac{W}{K} \mathbf{U} \qquad \frac{|W|}{K} = \frac{|\langle \mathbf{u}' \cdot \mathbf{b}' \rangle|}{\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2} \le 1$$

c.f. $$\nabla \times \left(\frac{\gamma}{\beta} \mathbf{U} \right) = \frac{\gamma}{\beta} \nabla \times \mathbf{U} + \nabla \left(\frac{\gamma}{\beta} \right) \times \mathbf{U}$$

Mean momentum equation

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{U} \times \mathbf{\Omega} + \mathbf{J} \times \mathbf{B} - \nabla \cdot \boldsymbol{\mathcal{R}} + \mathbf{F} - \nabla \left(P + \frac{1}{2} \mathbf{U}^2 + \left\langle \frac{1}{2} \mathbf{b}'^2 \right\rangle \right)$$

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{U} \times \mathbf{B} + \mathbf{E}_\mathrm{M}) \qquad \left\{ \begin{array}{l} \mathcal{R}^{\alpha\beta} = \frac{2}{3} K_\mathrm{R} \delta^{\alpha\beta} - \nu_\mathrm{K} \mathcal{S}^{\alpha\beta} + \nu_\mathrm{M} \mathcal{M}^{\alpha\beta} \\ \mathbf{E}_\mathrm{M} = -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \mathbf{\Omega} \end{array} \right.$$

Mean Lorentz force $$\mathbf{J} \times \mathbf{B} = \frac{1}{\beta} (\mathbf{U} \times \mathbf{B}) \times \mathbf{B} + \frac{\gamma}{\beta} \mathbf{\Omega} \times \mathbf{B} - \frac{1}{\beta} \frac{\partial \mathbf{A}}{\partial t} \times \mathbf{B}$$

$$\mathbf{U} = \mathbf{U}_0 + \delta \mathbf{U}, \quad \mathbf{\Omega} = \mathbf{\Omega}_0 + \delta \mathbf{\Omega}$$

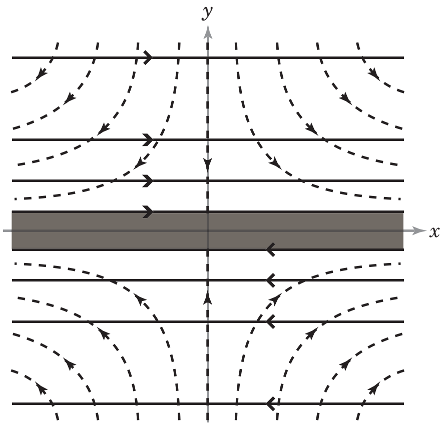
Reference $$\frac{\partial \mathbf{\Omega}_0}{\partial t} = \nabla \times \left(\mathbf{U}_0 \times \mathbf{\Omega}_0 + \nu_\mathrm{K} \nabla^2 \mathbf{U}_0 + \mathbf{F} \right)$$

Modulation $$\frac{\partial \delta \mathbf{\Omega}}{\partial t} = \nabla \times \left[\left(\delta \mathbf{U} - \frac{\gamma}{\beta} \mathbf{B} \right) \times \mathbf{\Omega}_0 + \nu_\mathrm{K} \nabla^2 \left(\delta \mathbf{U} - \frac{\gamma}{\beta} \mathbf{B} \right) \right]$$

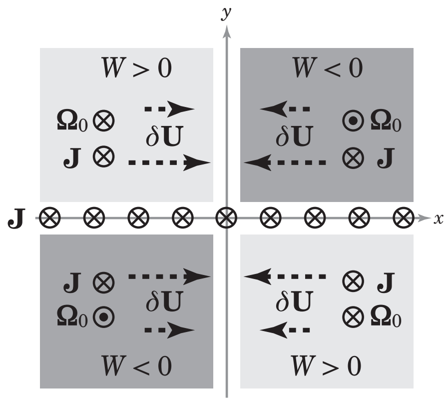
$$\rightarrow \quad \delta \mathbf{U} = \frac{\gamma}{\beta} \mathbf{B} = C_\mathrm{W} \frac{W}{K} \mathbf{B} \qquad \frac{|W|}{K} = \frac{|\langle \mathbf{u}' \cdot \mathbf{b}' \rangle|}{\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2} \le 1$$

Mean-field configuration and turbulence

Typical inflow and outflow

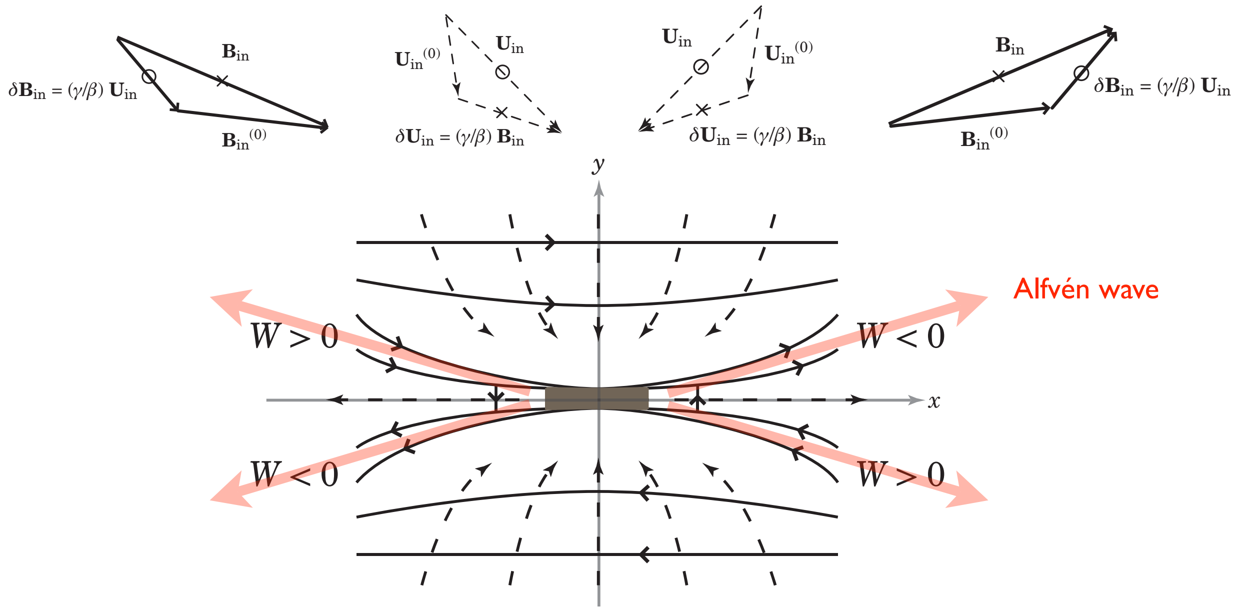


Spatial distribution of turbulent cross helicity



$$\mathbf{J}\cdot\boldsymbol{\Omega} > 0 \;\rightarrow\; P_{W2} > 0 \;\rightarrow\; W > 0$$
$$\mathbf{J}\cdot\boldsymbol{\Omega} < 0 \;\rightarrow\; P_{W2} < 0 \;\rightarrow\; W < 0$$

$$\delta\mathbf{B} = \frac{\gamma}{\beta}\mathbf{U} \qquad \delta\mathbf{U} = \frac{\gamma}{\beta}\mathbf{B} \qquad \delta\mathbf{B} = \frac{\gamma}{\beta}\mathbf{U}$$

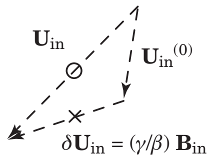


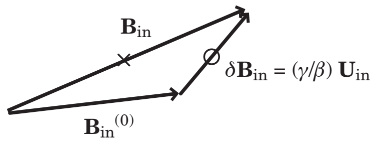
Reconnection rate

$$M_{\rm in} \equiv \frac{U_{\rm in}}{V_{\rm Ain}} = \frac{U_{\rm in}}{B_{\rm in}}$$

$$\mathbf{U}_{\rm in} = \mathbf{U}_{\rm in}^{(0)} + \delta\mathbf{U}_{\rm in} = \mathbf{U}_{\rm in}^{(0)} + \frac{\gamma}{\beta}\mathbf{B}_{\rm in}$$

$$\mathbf{B}_{\rm in} = \mathbf{B}_{\rm in}^{(0)} + \delta\mathbf{B}_{\rm in} = \mathbf{B}_{\rm in}^{(0)} + \frac{\gamma}{\beta}\mathbf{U}_{\rm in}$$





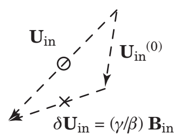
$$\begin{pmatrix} 1 & -\frac{\gamma}{\beta} \\ -\frac{\gamma}{\beta} & 1 \end{pmatrix} \begin{pmatrix} {}^t\mathbf{U}_{\rm in} \\ {}^t\mathbf{B}_{\rm in} \end{pmatrix} = \begin{pmatrix} {}^t\mathbf{U}_{\rm in}^{(0)} \\ {}^t\mathbf{B}_{\rm in}^{(0)} \end{pmatrix}$$

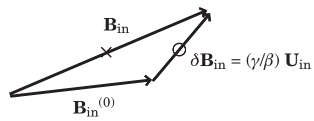
$$\begin{pmatrix} {}^t\mathbf{U}_{\rm in} \\ {}^t\mathbf{B}_{\rm in} \end{pmatrix} = \left[1 - \left(\frac{\gamma}{\beta}\right)^2\right]^{-1} \begin{pmatrix} 1 & \frac{\gamma}{\beta} \\ \frac{\gamma}{\beta} & 1 \end{pmatrix} \begin{pmatrix} {}^t\mathbf{U}_{\rm in}^{(0)} \\ {}^t\mathbf{B}_{\rm in}^{(0)} \end{pmatrix}$$

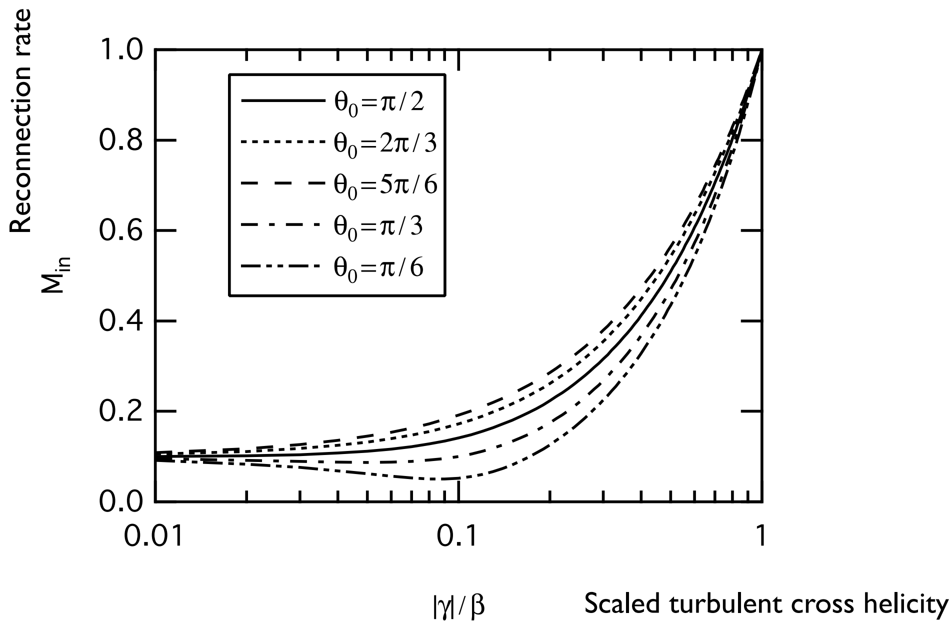
$$M_{\rm in} = \left[\frac{{M_{\rm in}^{(0)}}^2 + (\gamma/\beta)^2 + (2\gamma/\beta)M_{\rm in}^{(0)}\cos\theta_0}{(\gamma/\beta)^2{M_{\rm in}^{(0)}}^2 + 1 + (2\gamma/\beta)M_{\rm in}^{(0)}\cos\theta_0}\right]^{1/2}$$

$$\frac{|W|}{K} = \frac{|\langle\mathbf{u}'\cdot\mathbf{b}'\rangle|}{\langle\mathbf{u}'^2 + \mathbf{b}'^2\rangle/2} \leq 1$$

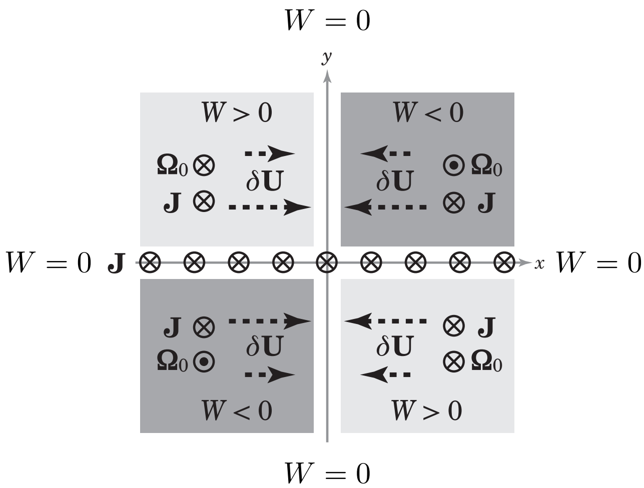
$$M_{\rm in} = \left[\frac{{M_{\rm in}^{(0)}}^2 + (\gamma/\beta)^2 + (2\gamma/\beta)M_{\rm in}^{(0)}\cos\theta_0}{(\gamma/\beta)^2{M_{\rm in}^{(0)}}^2 + 1 + (2\gamma/\beta)M_{\rm in}^{(0)}\cos\theta_0}\right]^{1/2}$$

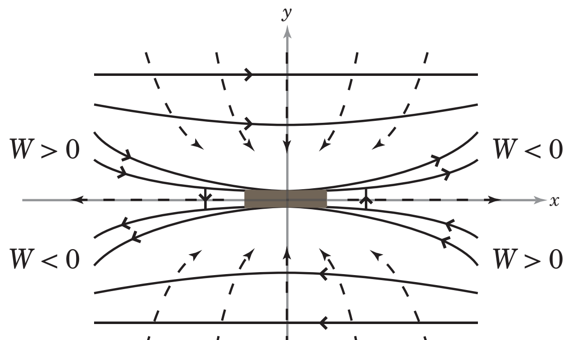






Localization





Numerical test

Basic Equations to Solve

4th-Order Runge-Kutta Scheme in Time

4th-Order Centered Difference in Space

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{V})$$

$$\frac{\partial}{\partial t}(\rho \boldsymbol{V}) = -\boldsymbol{\nabla} \cdot \left[\rho \boldsymbol{V}\boldsymbol{V} + \left(p + \frac{B^2}{2}\right)\overleftrightarrow{\boldsymbol{I}} - \boldsymbol{B}\boldsymbol{B}\right]$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\gamma - 1} + \frac{\rho}{2}V^2 + \frac{B^2}{2}\right) = -\boldsymbol{\nabla} \cdot \left[\left(\frac{\gamma}{\gamma - 1}p + \frac{\rho}{2}V^2\right)\boldsymbol{V} + \boldsymbol{E} \times \boldsymbol{B}\right]$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\boldsymbol{\nabla} \times \boldsymbol{E}$$

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta \boldsymbol{J} - \tau \left(W\boldsymbol{\Omega} - K\boldsymbol{J}\right)$$

$$\frac{\partial K}{\partial t} = \tau K \boldsymbol{J}^2 - \tau W \boldsymbol{\Omega} \cdot \boldsymbol{J} + \boldsymbol{B} \cdot (\boldsymbol{\nabla} W) - \boldsymbol{V} \cdot (\boldsymbol{\nabla} K)$$

$$\frac{\partial W}{\partial t} = \tau K \boldsymbol{\Omega} \cdot \boldsymbol{J} - \tau W \boldsymbol{\Omega}^2 + \boldsymbol{B} \cdot (\boldsymbol{\nabla} K) - \boldsymbol{V} \cdot (\boldsymbol{\nabla} W)$$

K: Turbulent energy, $\langle v'^2 + b'^2 \rangle / 2$

W: Cross helicity, $\langle \boldsymbol{v}' \cdot \boldsymbol{b}' \rangle$

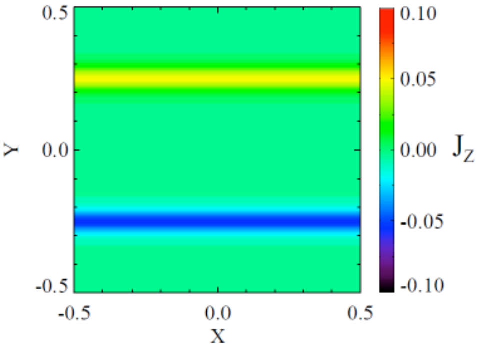
Initial Conditions

Adiabatic index: $\gamma = 1.01$

Initial plasma beta (inflow region): $\beta_0 = 0.3$

Boundary conditions: Periodic in both X and Y

Number of grids: 1024x512 (Test run)



Only the portion of the simulation box, Y<0, are shown in results.

$$\boldsymbol{V} = \boldsymbol{0}$$

$$\boldsymbol{B} = \hat{\boldsymbol{x}} B_0 \left[\tanh\left(\frac{y+0.25}{\delta}\right) - \tanh\left(\frac{y-0.25}{\delta}\right) - 1 \right]$$

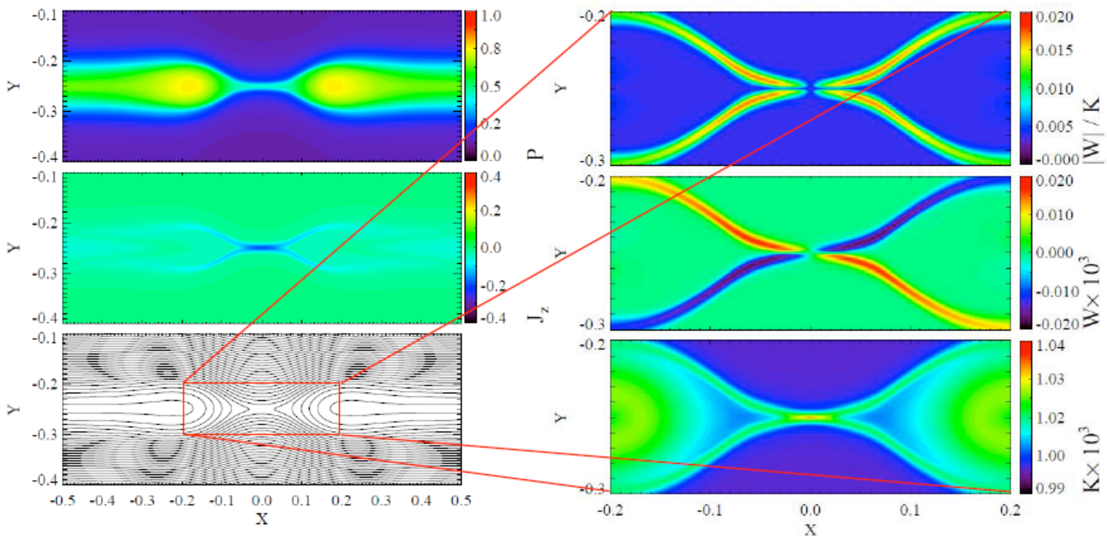
$$\eta(x,y,t) = \eta_0 + \eta_{\rm anr} \cosh^{-2}\left[-\frac{x^2}{l_x^2} - \frac{(y+0.25)^2}{l_y^2} \right] f(t)$$

η_0 and $\eta_{\rm anr}$ are chosen so that the corresponding magnetic Reynolds numbers, $R_{m,0}$ and $R_{m,{\rm anr}}$, are equivalent to 10^3 and 10, respectively.

Numerical test

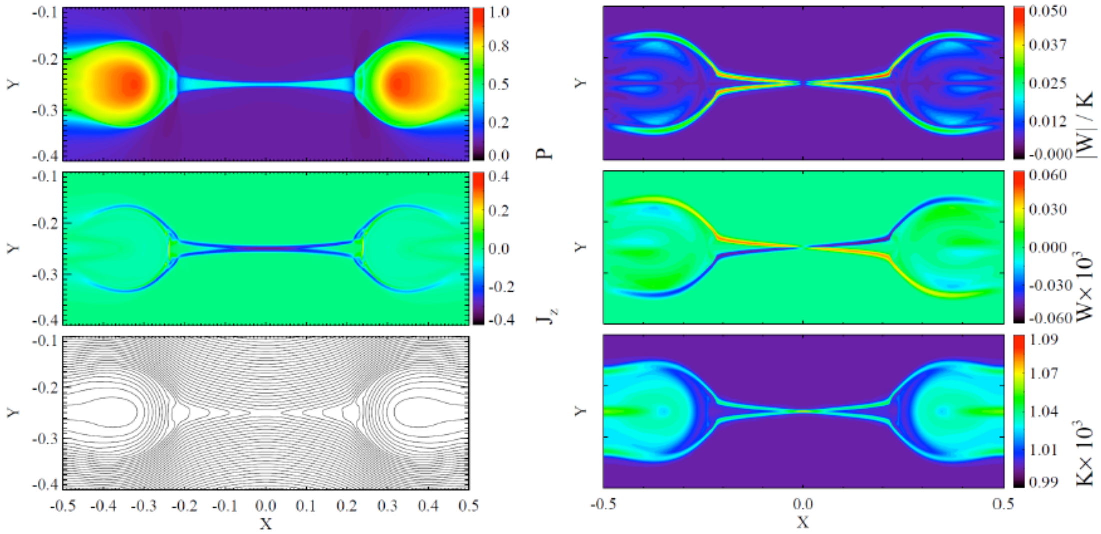
Results

Started with initial parameters: W=0, K=10^{-3}, and τ=10^{-2}



Results

Started with initial parameters: W=0, K=10^{-3}, and τ=10^{-2}



Summary

- Turbulence
  - Large and small scales
  - Enhancement and suppression   Breakage of symmetry
- Turbulent cross helicity
  - Suppress the turbulent transport
  - Sustain and generate the mean-field structures
- Production of turbulent cross helicity
  - Quadruple distribution around the reconnection point
- Analytical solution of turbulent reconnection equations
  - Configuration favorable for fast reconnection
- Balance between transport enhancement and suppression
  - Localized reconnection region
- Numerical test
- Suggestions for observation and experiment

Vortical motion in the tearing mode