DE LA RECHERCHE À L'INDUSTRIE

A EUROPEAN EFFORT FOR KINETIC MODELLING OF RUNAWAY ELECTRON DYNAMICS

(EUROFUSION, ER15-CEA-09)

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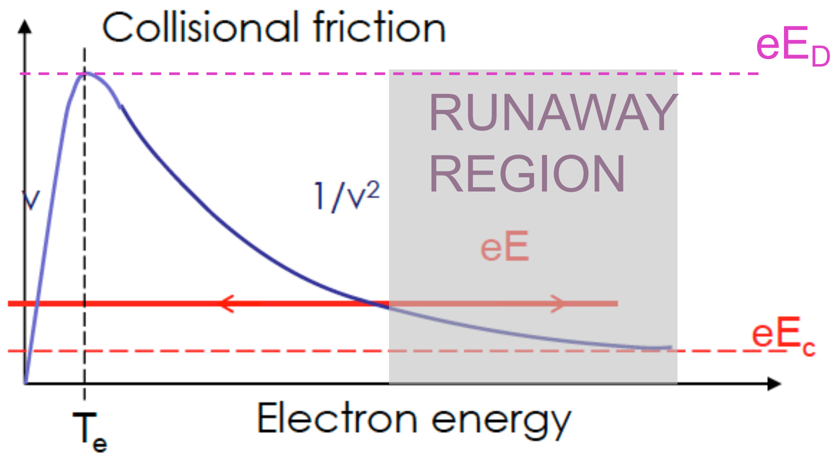
https://www2.euro-fusion.org/erwiki

The Runaway Electron (RE) dynamic

- Collisional friction decreases with electron velocity
- If electric field $E_{||} > E_{critical} \sim n_e$, some electrons can be continuously accelerated → energies of 10-100 MeV range → **runaway electrons (RE).** *Old problem from the early days of tokamaks.*
- Plasmas conditions favorable for RE generation may be *controled* (low-density at ramp-up,…) or *uncontroled* (fast disruption).



RE generated if $E > E_c$

$$E_c = n_e\, e^3 ln\Lambda/(4\varepsilon_0{}^2 mc^2)$$

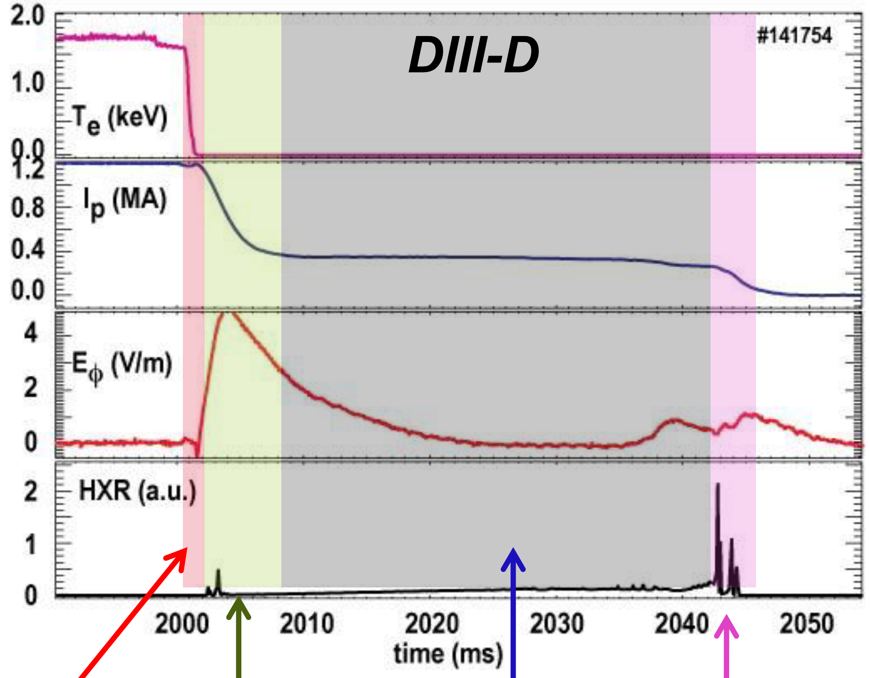
At the Dreicer field even thermal electrons run away

$$E_D = E_c mc^2/T_e$$

[Dreicer, Phys. Rev. ***115*** *(1959)]*

Disruptive or non-disruptive RE

Disruptive RE



Thermal quench

Current quench

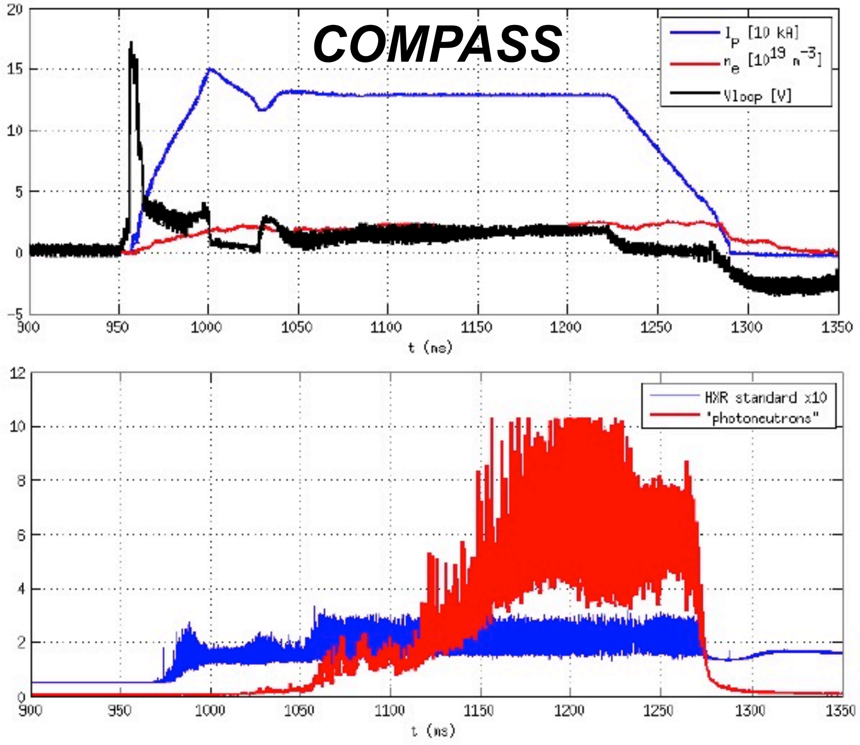
RE plateau

RE final loss

$$\boxed{\rho \sim T_e^{-\frac{3}{2}} \qquad \mathbf{E} = \rho \mathbf{j}}$$

Courtesy of E. Hollmann

Non-disruptive RE

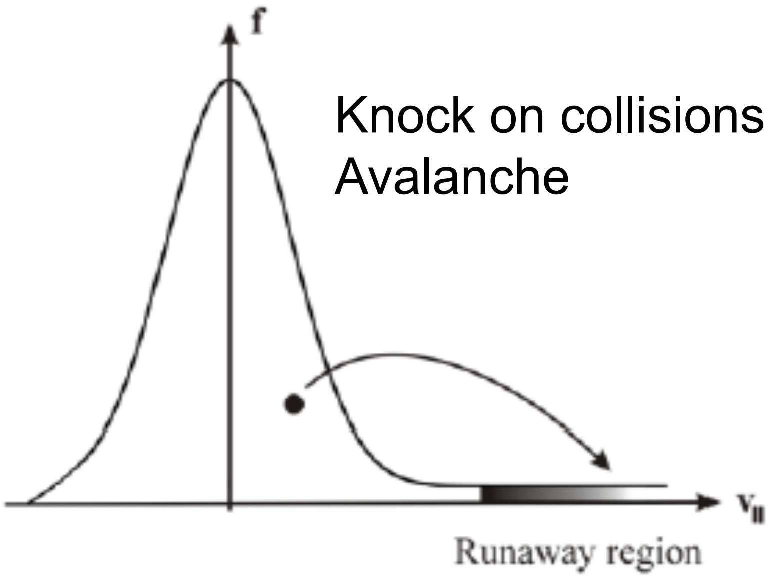


shot #8555

E. Nilsson, PhD thesis, Ecole Polytechnique, (2015)

Amplification of RE growth rate by the avalanche mechanism

- Fast electrons kick others into the runaway region
- Population of runaway electrons grows exponentially in strong electric field
- Avalanche multiplication scales with pre-disruptive plasma current (problem for large tokamaks)



Knock on collisions → Avalanche

Rosenbluth & Putvinski, Nucl. Fusion ***37*** *(1997)*

Needs for modeling the RE dynamics

- If RE carry a large fraction of the plasma current (post-disruption phase), they represent a major threat for a tokamak → beam of RE which can become unstable, hit the wall can cause large damages: **serious issue for the ITER tokamak**.

- In present-day tokamaks also, the danger of runaway-induced damage often limits the range of operation parameters

- ***Aim of the kinetic modeling of RE dynamics***: describe the formation of the suprathermal beam (from the early stage), taking into account selfconsistently of transport and non-linear effects especially as RE carry a large fraction of the plasma current → *strong interplay of momentum and configuration spaces.*

- *Main keywords for characterizing RE dynamics*: **critical electric field, population growth rate, spatial location (beam), upper energy limit**,…

What do we want understand about RE ?

Principally disruptive RE → challenge for ITER

- Under which conditions do disruptions give rise to a runaway beam?
- Can this process be prevented or mitigated?
- Is it possible to transport runaway electrons as soon as they are generated?
- If a runaway beam nonetheless forms, what are its characteristics, i.e. what is the electron energy distribution?
- Is it possible to slow it down progressively?
- What are the effects of mitigation techniques such as massive material injection?

Towards a disruption simulator

- Answering all these questions ultimately requires a complete disruption simulator solving both a kinetic equation for the runaway dynamics, and a fluid-MHD evolution including massive gas or pellet injection, ionization physics, impurity transport, etc. → ***This is a long term objective for the community***.

- ***But short term objectives must be reached especially for ITER needs****,* *which may be summarized by the two goals*:

  1) how to prevent the formation of an energetic beam of RE

  2) how to mitigate an already existing energetic beam of RE.

Modeling the RE dynamics

The short time scale to get answers on RE physics in a disruption, in particular for ITER has led to split the problem in two projects

- The kinetic description of runaways is studied in an independent project, → **the kinetic description of runaways is the subject of the *ER15-CEA-09 project.***
- The fluid-MHD disruption modelling is the subject of another project (C. Sommariva talk)

Tools for modeling the RE dynamics

- *Which modeling tools do we want to build or for which goals ?*
  - **short term** (~ 3 years) → improve existing codes (**LUKE, CODE, GO**, **METIS** …) , incorporate some new physical processes, perform collaborative work for integrated modeling (with ITM ?), benchmark tools against experimental results to validate extrapolation capabilities.
  - **medium term** (~ 5 years) → prepare the development of new tools (**LUKE 2**,…) able to describe more accurately the particle dynamics (orbits, collisionality,…).
  - **long term** → perform self-consistent MHD + kinetic calculations

Modeling the RE dynamics: what can be done ?

- The validity of existing tools for kinetic calculations of disruptive RE is questionable as it is principally a MHD problem at least during the thermal quench: *magnetic topology with nested magnetic flux surfaces is assumed, while strong ergodization is known to take place, with strong losses of RE.*

- Even if a confining magnetic configuration is rebuild in the current plateau phase, the toroidal MHD quasi-equilibrium to consider for the kinetic calculations is still an open question. Which tool is appropriate (JOREK, …) ?

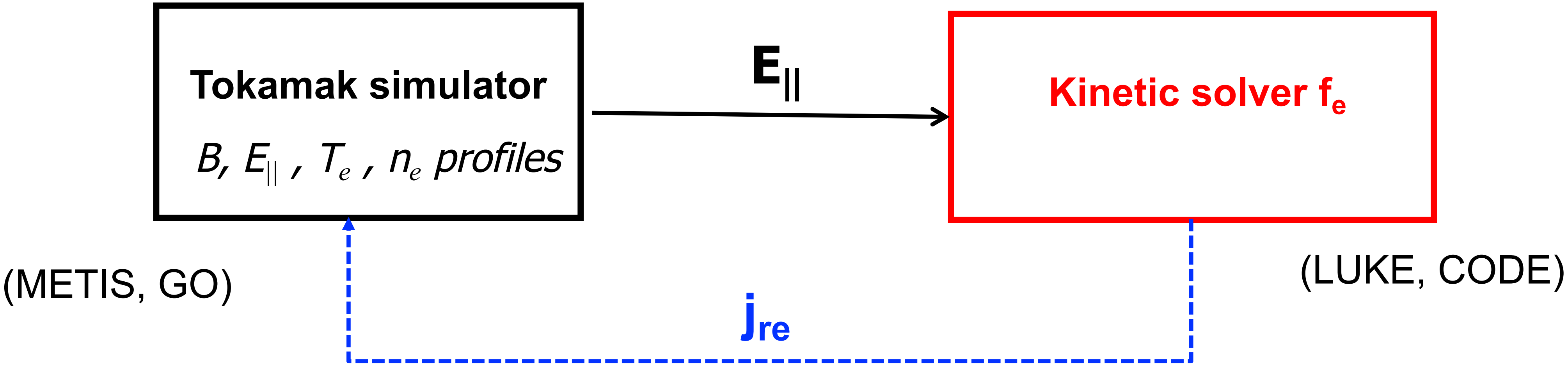
- Existing kinetic codes can be principally used for building *a* ***heuristic fast simulator for describing time evolution of the RE population in disruptions*** which incorporate main transport effects in momentum and configuration spaces such that critical field, population growth rate, spatial location and upper energy of RE may be well reproduced despite the oversimplified description. → **validation against experiments and more advanced studies using exact MHD codes (JOREK, NIMROD,…) coupled to appropriate kinetic solvers that can deal with complex topology (Monte-Carlo,…).**

Global objectives of ER15-CEA-09

The European effort 2015-2017 aims at improving our understanding and modelling of runaway electron dynamics :

1) first focusing on the generation and transport mechanisms (avalanches, additional forces that could limit the formation of a runaway beam,…)

2) later integrating the various processes self-consistently in simulations for comparison with experiments.



Tokamak simulator

B, $E_{||}$, T_e , n_e *profiles*

(METIS, GO)

$\mathbf{E_{||}}$

Kinetic solver $\mathbf{f_e}$

(LUKE, CODE)

$\mathbf{j_{re}}$

Objectives: step 1

Generation and transport mechanisms

- Account for the effect of pre-existing fast electrons and describe the role of hot-tail dynamics in the primary runaway generation.
- Improve the knock-on collisions model for secondary generation process by including the effect of finite incident momentum
- Account for finite orbit width effects in the runaway dynamics.
- Implement the quasilinear formalism of kinetic instabilities in LUKE, and include the resulting radial transport in the runaway dynamics.
- Account for the effect of magnetic turbulence, magnetic ripple and RMP in the form of an equivalent radial transport model
- Account for the interaction between runaway electrons with Alfvenic fluctuations.

Objectives: step 2

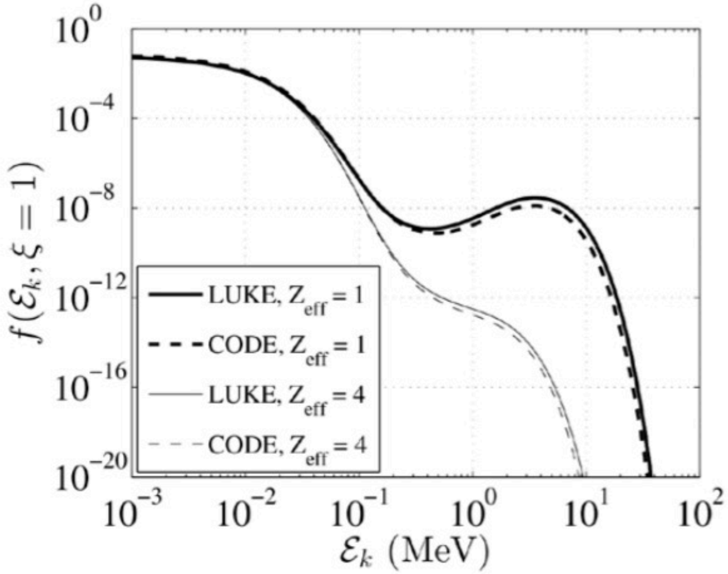
Integration of the various processes for self-consistent simulations of RE and comparison with experiments

- Build a self-consistent solver for the evolution of the runaway electrons, parallel electric field, and plasma equilibrium (within or outside ITM)
- Investigate the possibility of new synthetic diagnostics specific to characterize the runaway dynamics, especially the crucial build-up phase
- Validate the simulations by comparing with approriate experimental observations.

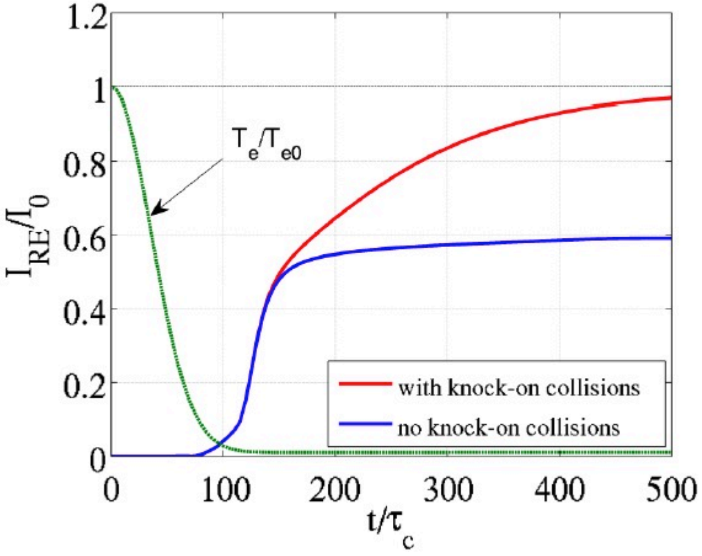
Mid-term main achievements

- **The physics of synchrotron radiation reaction force (ALD)**, which is found to limit the runaway electron energy, and leads to significant modifications in the runaway electron distribution. Includes theoretical work and numerical implementation in kinetic solvers (CODE, LUKE)

- Theoretical calculation of **the bounce-averaged knock-on collision operator** for describing runaway avalanches in realistic magnetic configuration of tokamaks, and implementation in kinetic solver (LUKE).



Bump in tail



The knock-on electrons emerge highly magnetized

Work in progress

- **Observations and scenarios**

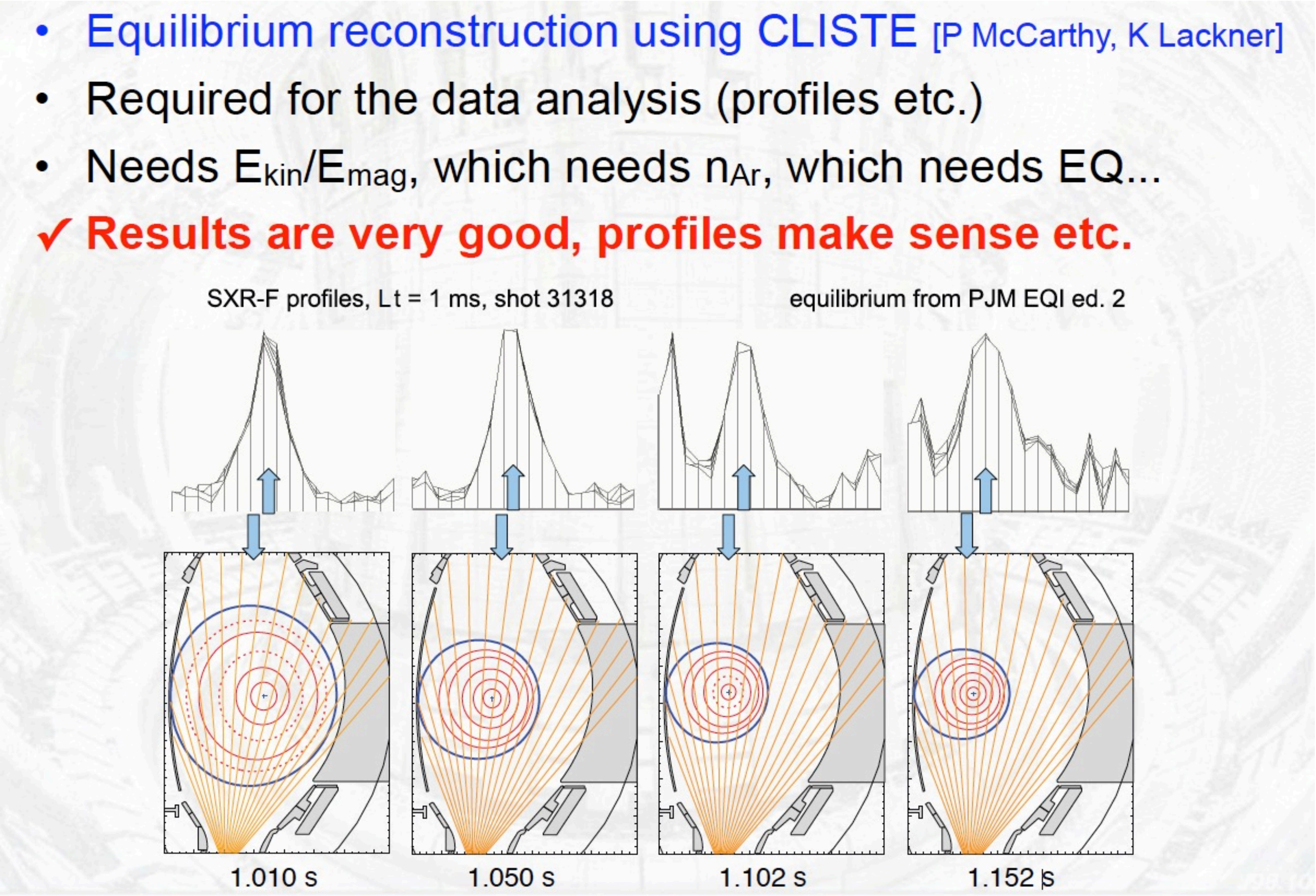
  - *Equilibrium reconstruction after thermal quench (ASDEX Upgrade)*
  - *Core confinement after thermal quench (ASDEX Upgrade)*
  - *RE generation and plasma elongation (COMPASS)*
  - *RE generation and initial fuelling (COMPASS)*
  - *Full conversion from OH to runaway current (TCV)*
  - *Beam mitigation (JET)*
  - *…*

- **Theory, modeling and data analysis**

  - *Synchrotron reaction force (ALD) + implementation in CODE (Alcator C-Mod)*
  - *Bounce-averaged ALD force + implementation in LUKE*
  - *Bremsstrahlung reaction force + implementation in CODE*
  - *Bounce-averaged knock-on collisions + implementation in LUKE*
  - *Toroidal effect on primary and secondary RE generation (LUKE)*
  - *Relative importance between primary and secondary RE generation (LUKE)*
  - *Near critical field, time and plasma tempearture (LUKE)*
  - *Ware pinch effect on runaway electrons dynamics*
  - *Drift diffusion model for RE*
  - *Modeling of non-disruptive RE discharges (Tore Supra with LUKE)*
  - *Modeling RE in realistic fields (with JOREK)*
  - *…*

Equilibrium reconstruction post thermal quench (ASDEX-U)

G. Papp
REM-2015



- Equilibrium reconstruction using CLISTE [P McCarthy, K Lackner]
- Required for the data analysis (profiles etc.)
- Needs E_{kin}/E_{mag}, which needs n_{Ar}, which needs EQ...

✓ **Results are very good, profiles make sense etc.**

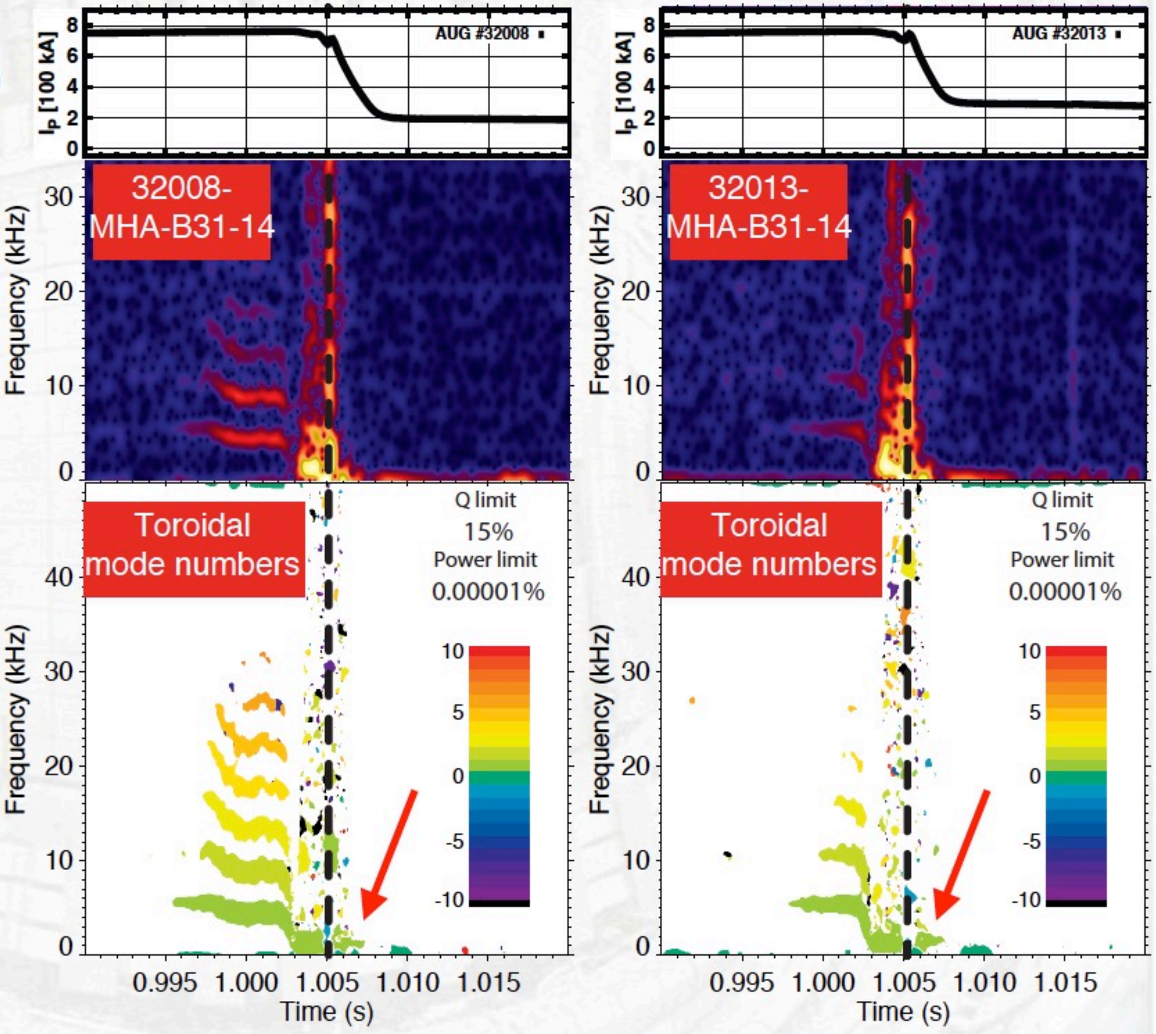
SXR-F profiles, Lt = 1 ms, shot 31318

equilibrium from PJM EQI ed. 2

1/1 mode survives the thermal quench (ASDEX-U)

G. Papp
REM-2016

- **1/1 mode develops (due to low density?) <u>before</u> injection**
- Becomes anharmonic and slows down
- **In most cases 1/1 survives the TQ**
  - ➡ Core confined?
- So far no clear connection between mode parameters (*A*, *f*, etc) and REs
- Further analysis is ongoing



[with P. Zs. Pölöskei & G. I. Pokol @ BME]

Elongation effect on RE generation (COMPASS)

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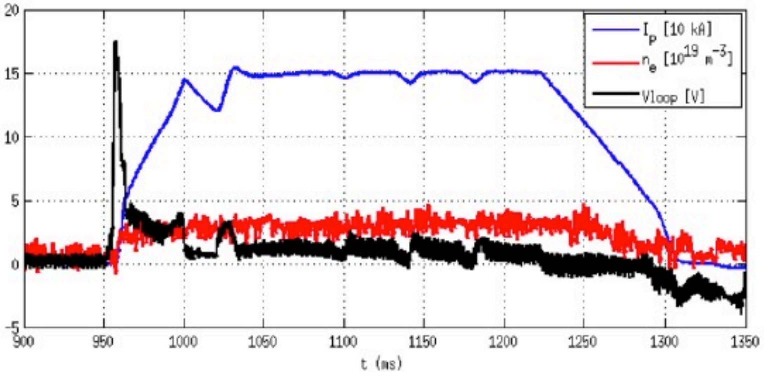
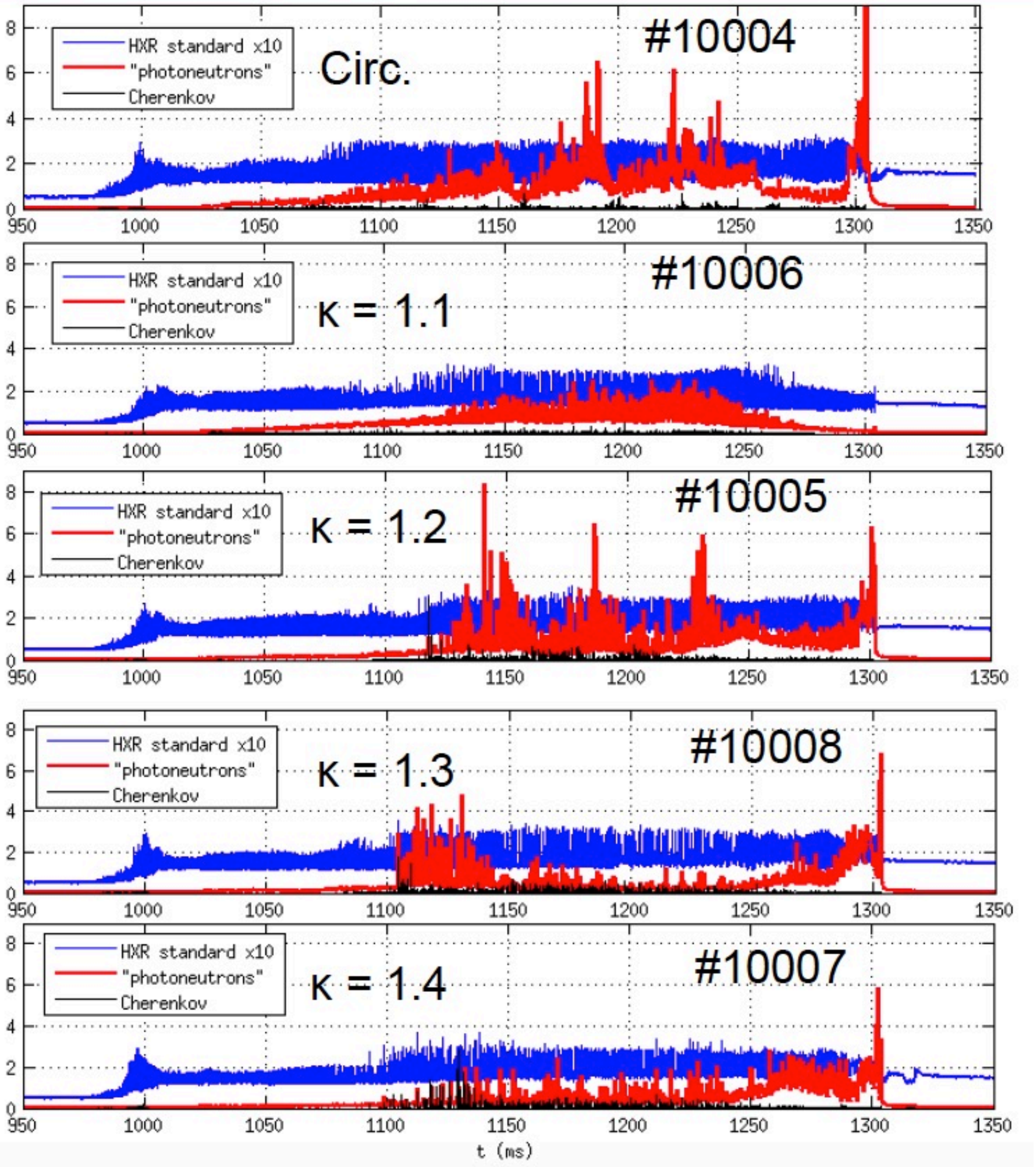
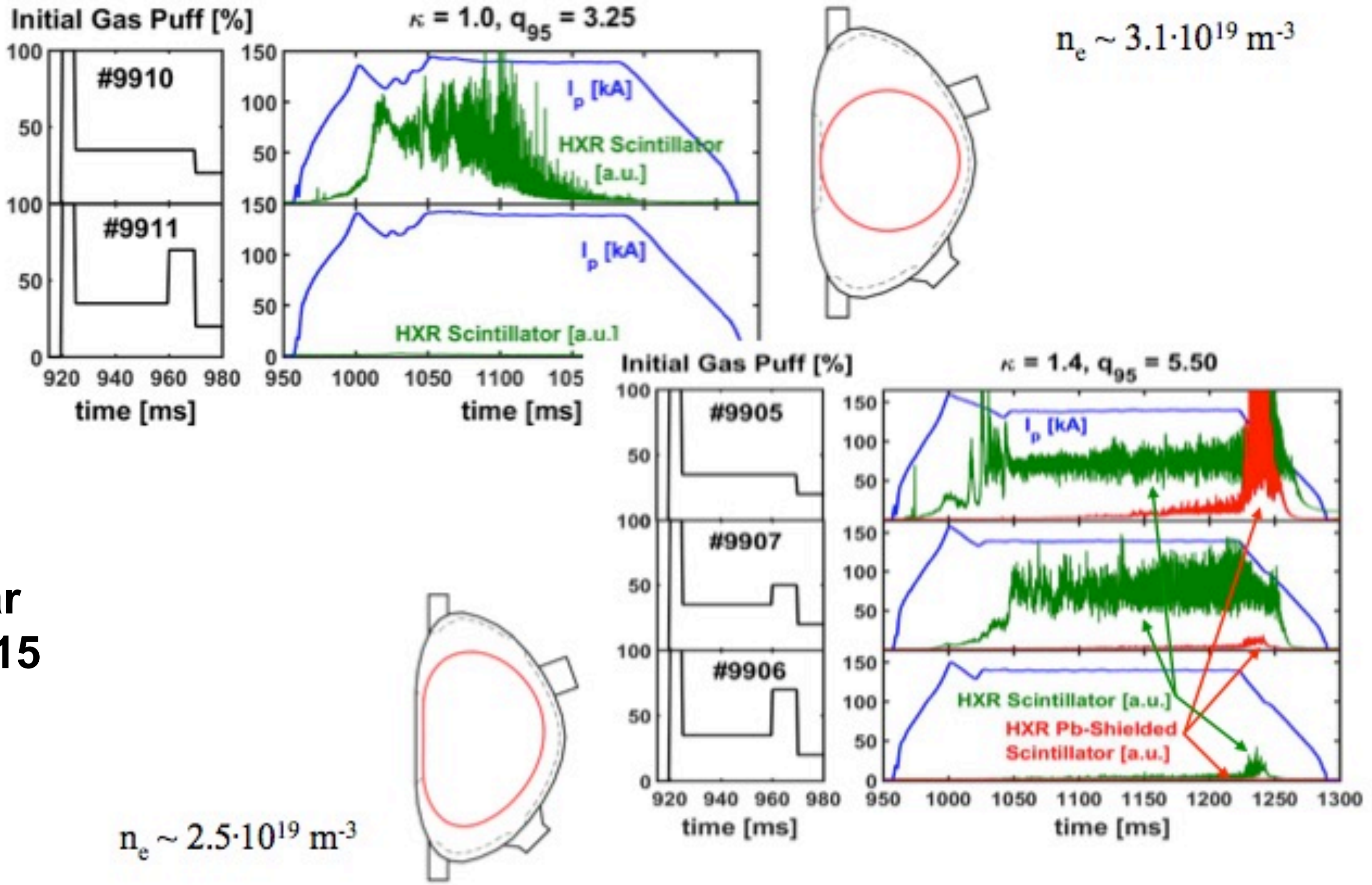




Effect of elongation on formation and confinement of REs

Gas fuelling and RE generation (COMPASS)

Effects of the initial fuelling

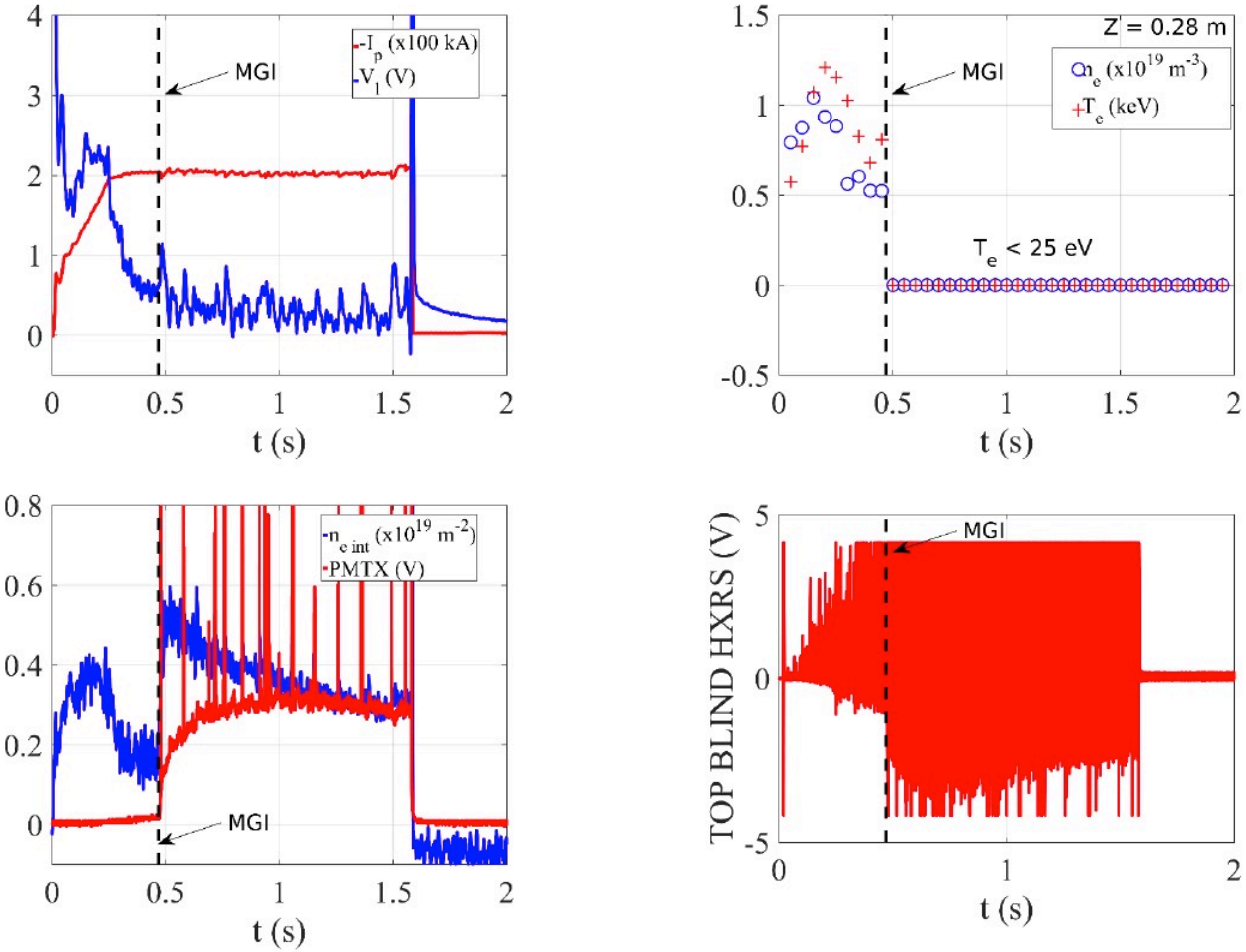


J. Mlynar
REM-2015

Full conversion to RE electron beam (TCV)

SWISS PLASMA CENTER

52716 : full OH->RE conversion

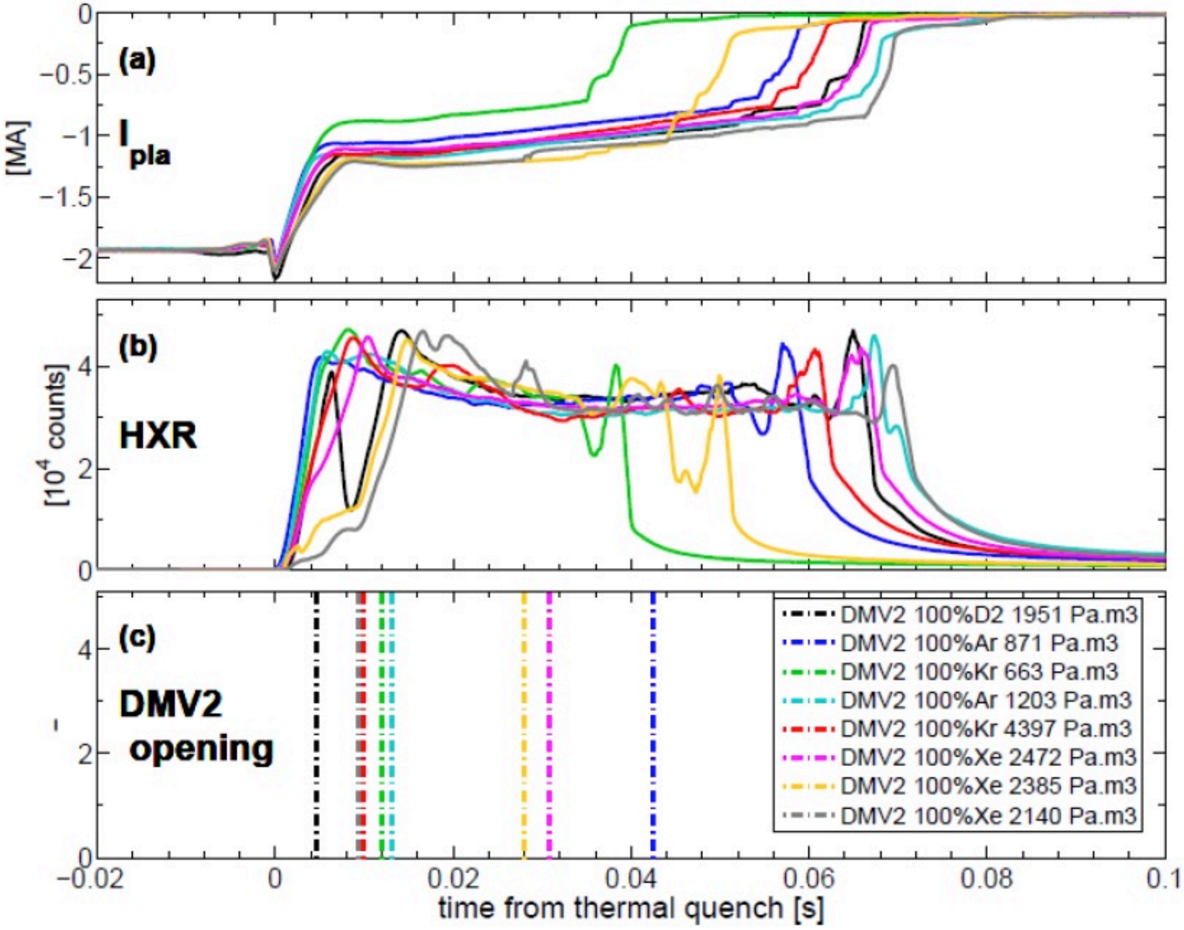


J. Decker
REM-2016

- Full conversion from OH to runaway current
- Robust recipe for RE generation
- Well suited discharges for self-consistent-modeling

Mitigation of RE with the ITER-Like wall (JET)

- Idea: **mitigate a fully accelerated RE beam**
- Use the same RE beam scenario
- Fire DMV2 filled with Ar/Kr/Xe at high pressures during the runaway beam
- Overall result: **no mitigation**



C. Reux REM-2015

Synchrotron reaction force (ALD) + implementation in CODE

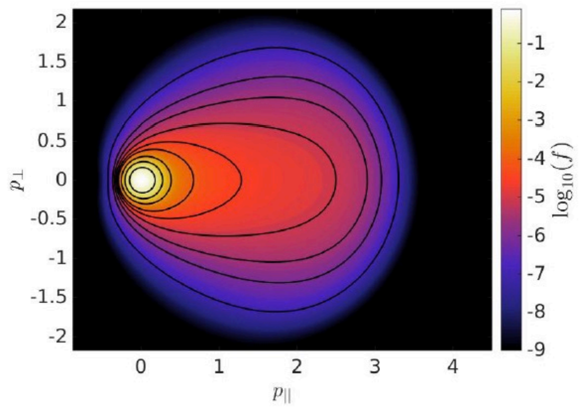
- Electron emits synchrotron radiation – experiences reaction force. Acts as effective friction at high energies
- Derived from the Lorentz-Abraham-Dirac force under the assumption that magnetic force dominates dynamics ($F_m \gg F_E, F_{RR}$)
- Enters the kinetic equation as

$$\frac{\partial}{\partial \mathbf{p}} \cdot (\mathbf{F}_{\rm rad} f) = -\frac{1}{p^2}\frac{\partial}{\partial p}\left(\frac{\gamma p^3(1-\xi^2)}{\tau_r} f\right) + \frac{\partial}{\partial \xi}\left(\frac{\xi(1-\xi^2)}{\gamma \tau_r} f\right)$$

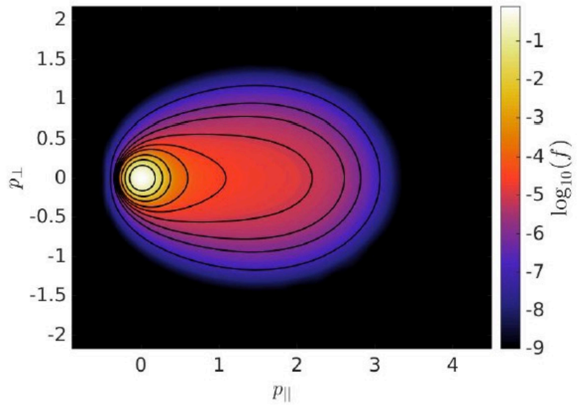
with

$$\tau_r = \frac{6\pi\varepsilon_0 (m_e c)^3}{e^4 B^2}, \quad p = \gamma v/c, \quad \xi = p_\parallel / p = \cos\theta$$

[Stahl, Hirvijoki, Decker, Embréus and Fülöp, PRL **114**, 115002 (2015)]



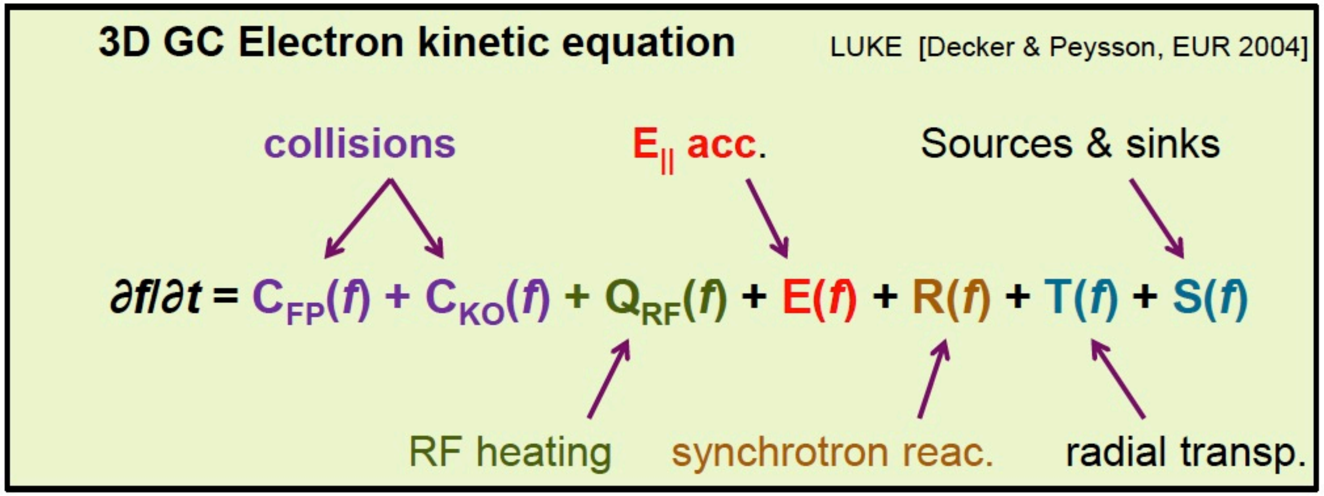
Without radiation reaction



With radiation reaction

A. Stahl, REM-2015

Bounce-averaged ALD force + implementation in LUKE



3D GC Electron kinetic equation

LUKE [Decker & Peysson, EUR 2004]

collisions

$E_{||}$ acc.

Sources & sinks

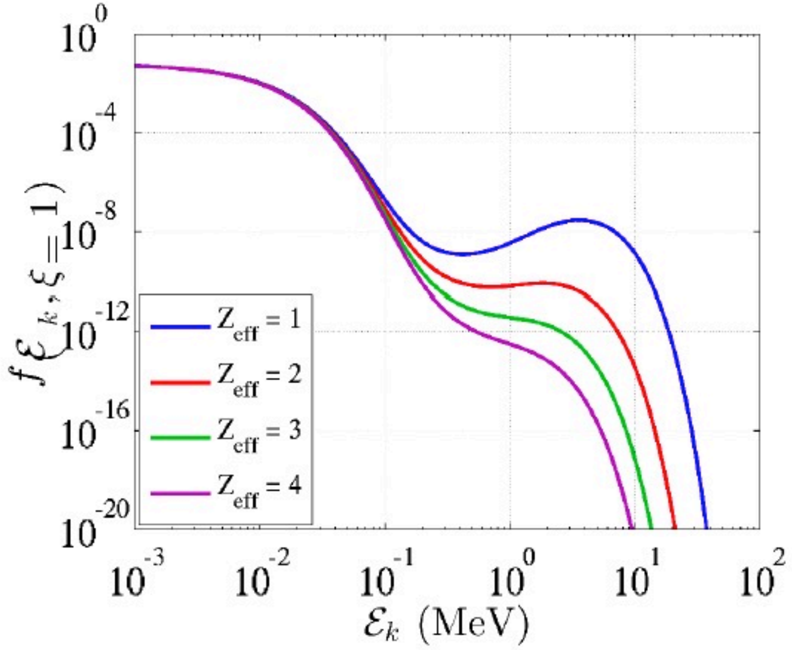
$$\partial f/\partial t = C_{FP}(f) + C_{KO}(f) + Q_{RF}(f) + E(f) + R(f) + T(f) + S(f)$$

RF heating

synchrotron reac.

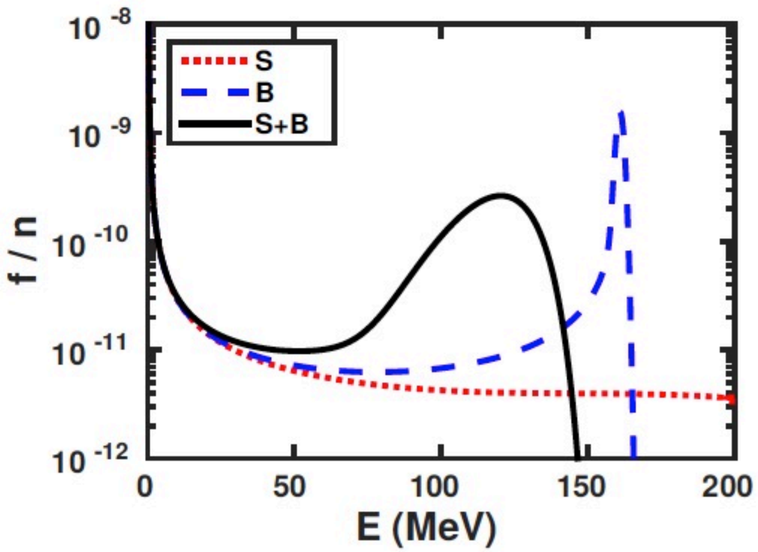
radial transp.

J. Decker
REM-2015



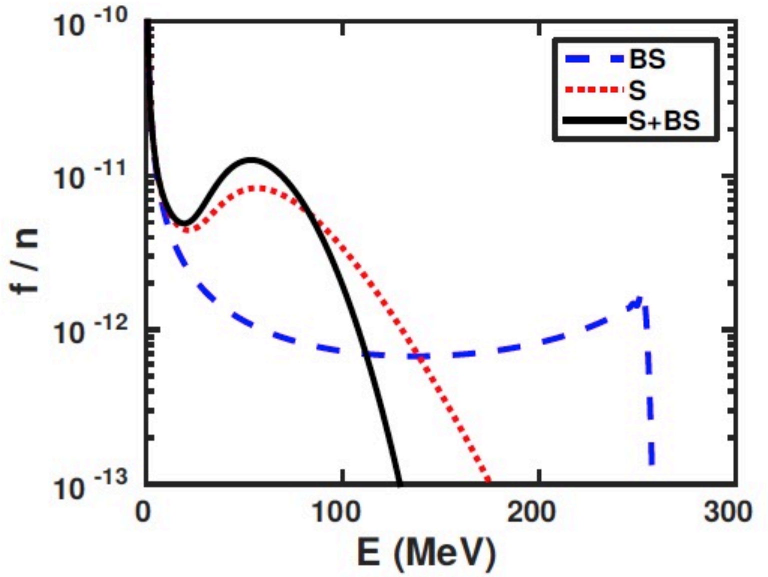
Bremsstrahlung reaction force + implementation in CODE

A. Stahl, REM-2015



Parameters:

$n_e = 1 \cdot 10^{20}$ m^{-3}, $T_e = 10$ keV,
$B = 0.5$ T, $E/E_c = 2$, $Z_{\text{eff}} = 3$



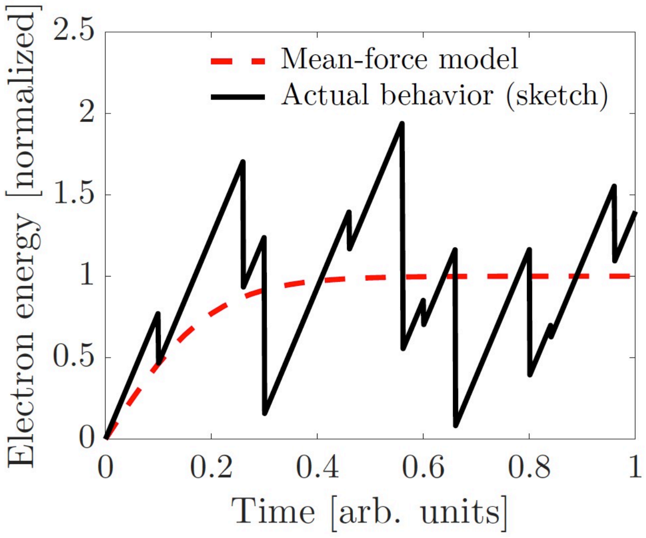
Parameters:

$n_e = 1 \cdot 10^{20}$ m^{-3}, $T_e = 5$ keV,
$B = 2$ T, $E/E_c = 3$, $Z_{\text{eff}} = 3$

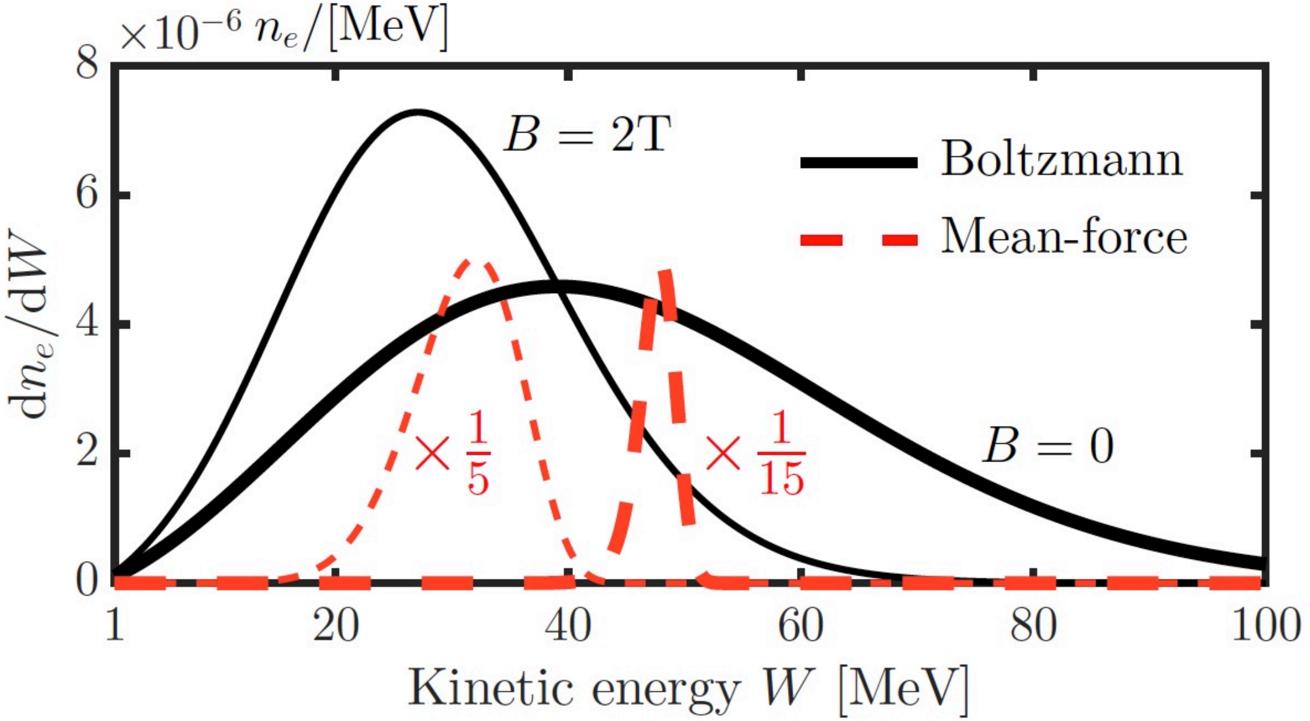
Conclusion: Bremsstrahlung significant when $E \sim E_c$ and B small.

Work in progress: Study characteristics of emitted radiation

Effect of bremsstrahlung emission on runaway electrons



Post-disruption scenario with successful MGI:
Simulations using CODE with $n_e = 3 \cdot 10^{21}$ m^{-3}, $Z_{\text{eff}} = 10$, $E = 2E_c$.



1) New description of RE energy losses by BE
2) Full Boltzman description necessary

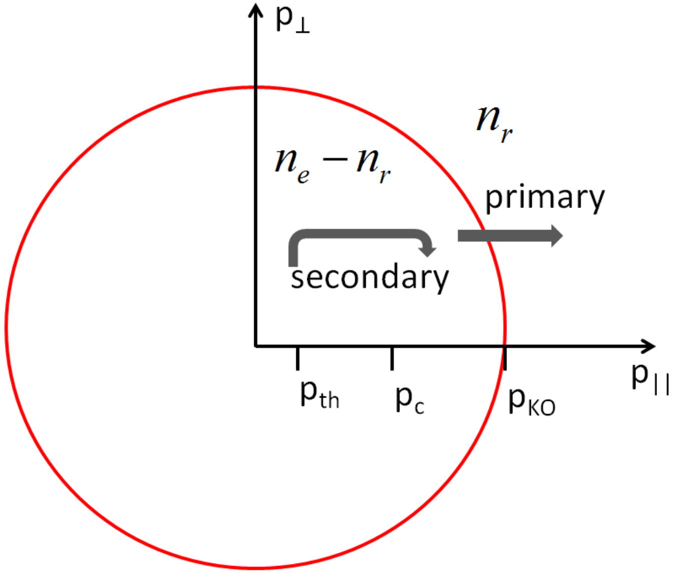
O. Embreus
REM-2016

Knock-on collisions in LUKE code

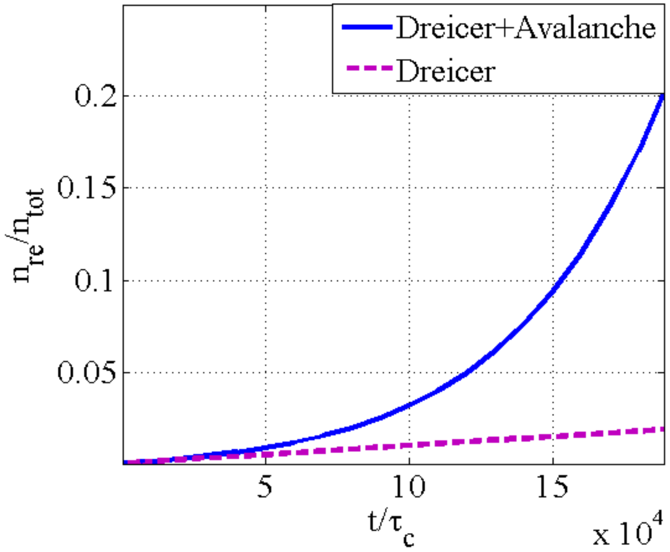
- p_{KO} : E_k =1 MeV, v/c=0.94

Particle conserving form of avalanche process:

$$S = S_+ \quad \langle S_+ \rangle \bar{f}_M \qquad \left\{ \begin{array}{l} S_+ = n_e n_r c \dfrac{d}{d} \\ n_r + n_e = const \end{array} \right.$$



- Exponential RE growth when avalanches dominate



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Influence of toroidicity on primary and secondary RE

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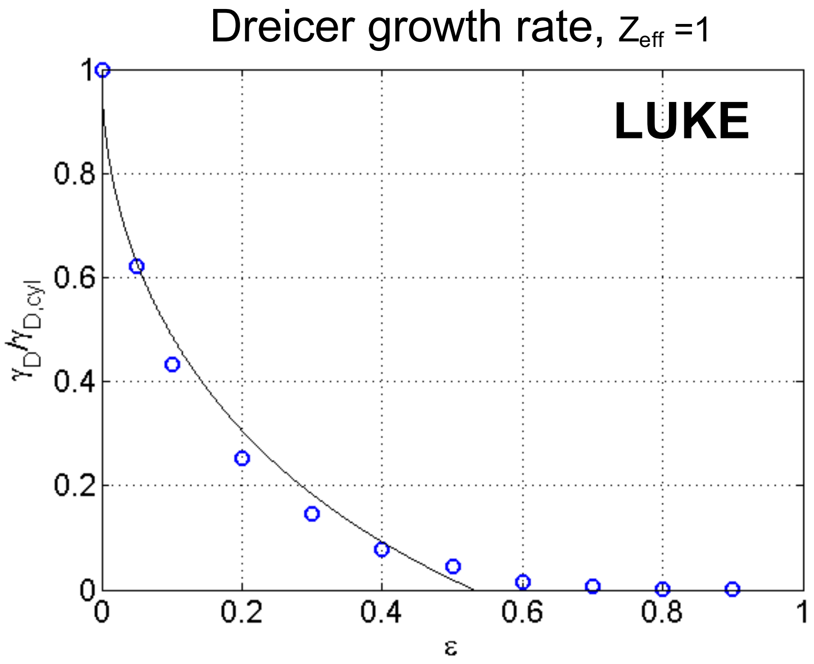
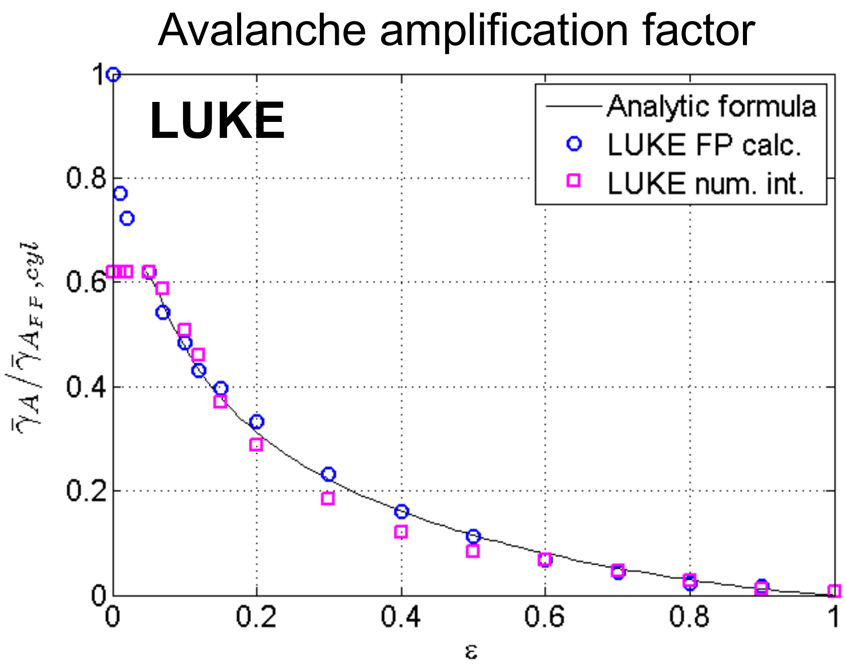
REM-2015

Growth rate: $\frac{\partial n_r}{\partial t} = n_e\left(\gamma_D + \gamma_A\right)$

$\gamma_A = n_r \overline{\gamma}_A$





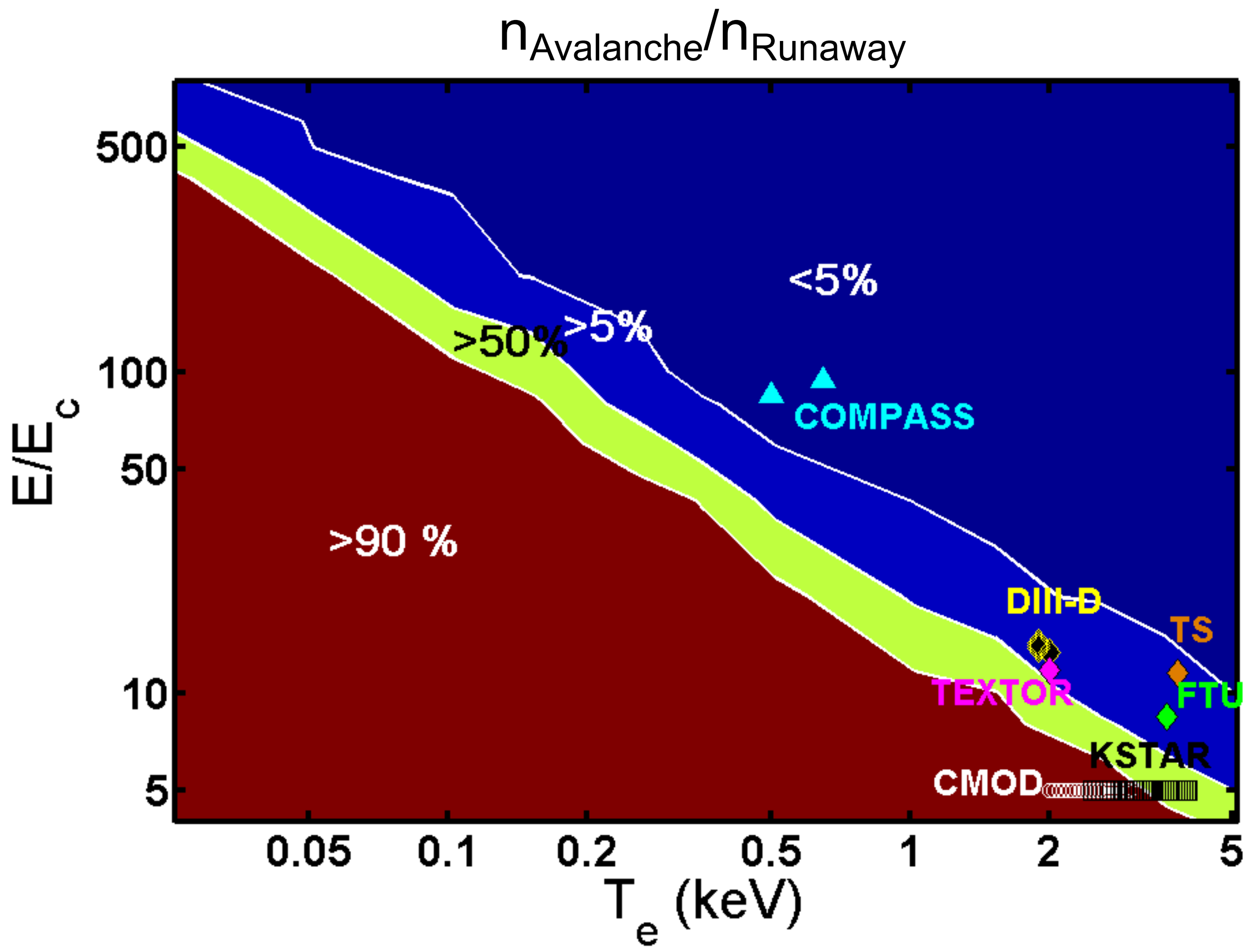
Runaway rate strongly reduced due to trapped electrons!

Agrees with predictions by ARENA code *[Eriksson & Helander, Comp. Phys. Comm. **154** (2003)]*
and CQL3D code *[Harvey & McCoy, IAEA (1992)] [Decker & Peysson, EUR-CEA-FC-1736, Euratom-CEA, 2004]*

Comparison to RE onset from tokamaks

Experiments show that E/Ec >3 is required to generate REs

[Granetz et al., Phys of Plasmas ***21*** *(2014)]*



[Nilsson et al., Plasma Phys. and Controlled Fusion ***57*** *(2015)]*

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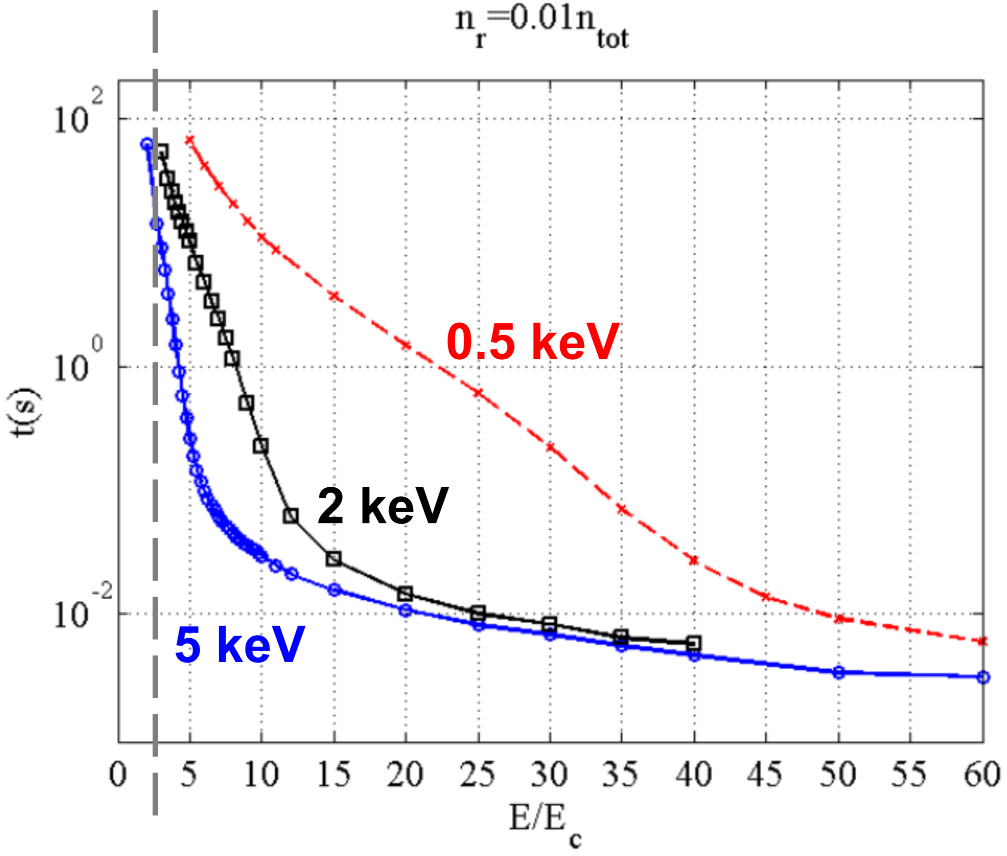
Experimental RE onset from Granetz's compilation + COMPASS and Tore Supra

Near critical field – time and temperature



Time required for 1% of initial Maxwellian electrons to run away.

*[Nilsson et al., Plasma Phys. and Controlled Fusion **57** (2015)]*

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REM-2015

No RE-discharge:
T_e ~ 3 keV, E/E_c ~2.5, t =10 s

a 10 s discharge is not enough for runaways to form in E/E_c~2.5 in a 3 keV plasma

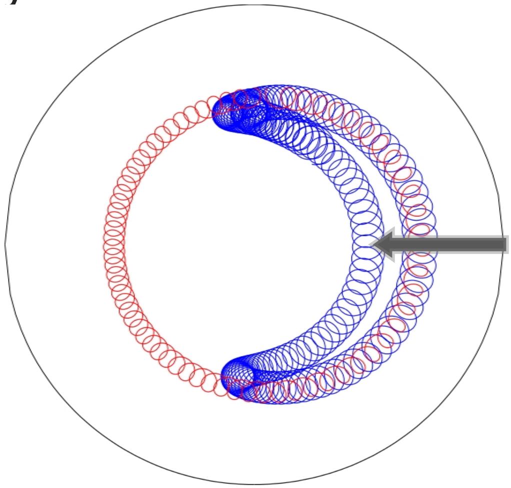
Trapped electron runaway effect
Ware pinch effect

- Knock-on electrons emerge highly magnetized → trapping off magnetic axis
- Conservation of canonical angular momentum → trapped electron Ware pinch towards the magnetic axis

$$\frac{dr}{dt} = -\frac{E_\phi}{B_\theta}$$

[Ware, Phys. Rev. Letters ***25*** *(1970)]*

- Trapped electrons can pinch inwards where they untrap and run away
- Calculated by LUKE 2



E. Nilsson
REM-2015

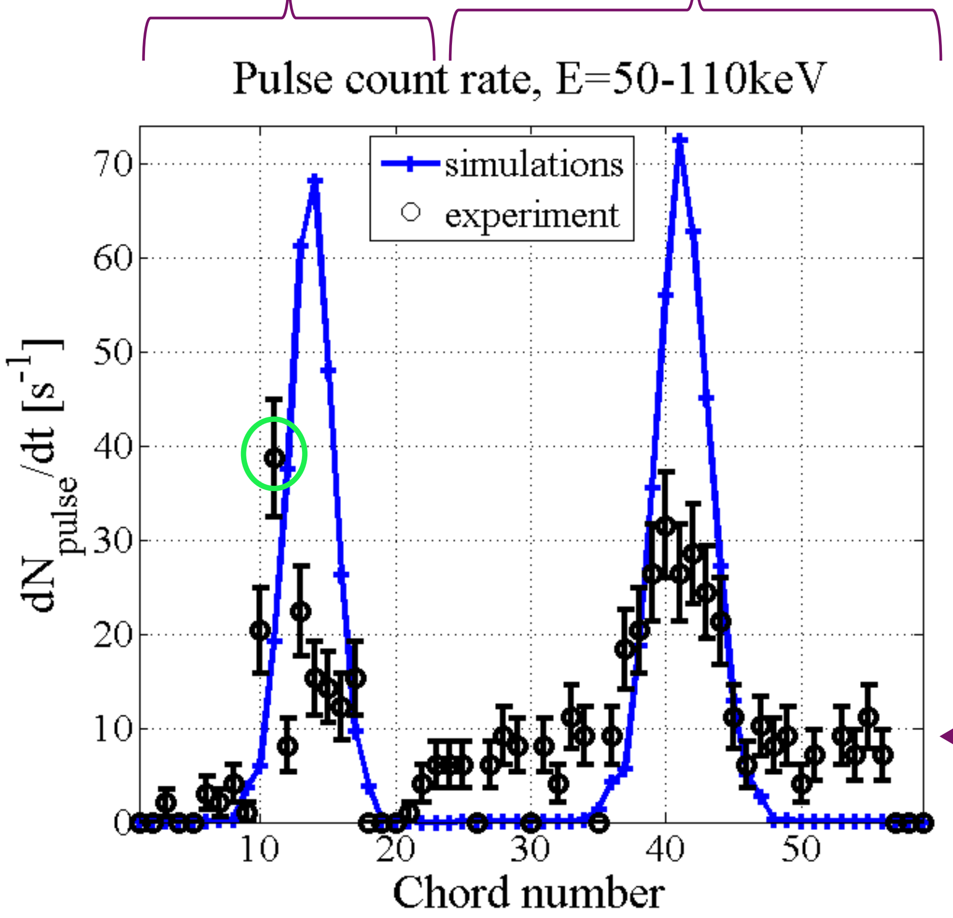
[E. Nilsson, J. Decker, N. Fisch and Y. Peysson, JPP 2015]

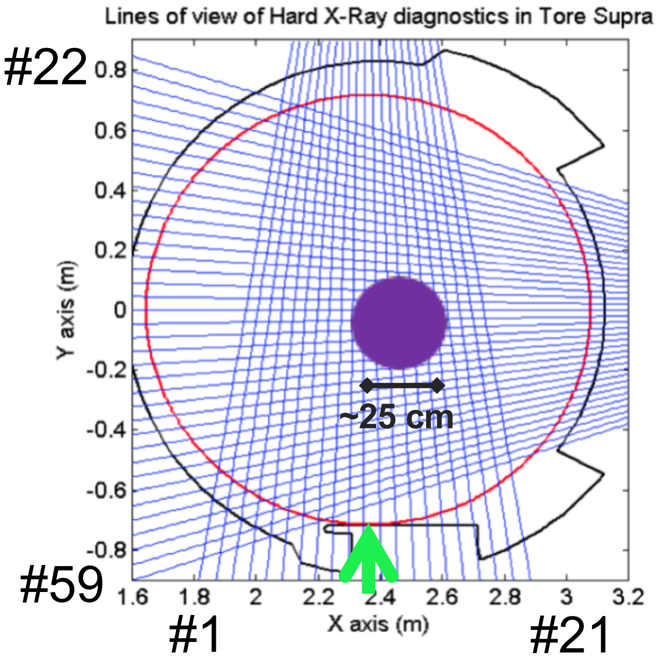
Fast electron bremsstrahlung emission from RE (Tore Supra)

- HXR tomographic system
- **R5X2: Synthetic diagnostic for bremsstrahlung emission**

*[Peysson & Decker, Phys. of Plasmas **15** (2008)]*

Vertical cam, 1-21 ***Horizontal cam, 22-59***





RUNAWAY DISCHARGE:
Emission profile reproduced at the end of the current flattop

X-rays backscattered by the tokamak inner wall

*[Peysson et al., Nucl. Fusion **33** (1993)]*

E. Nilsson, REM-2015

A drift-diffusion model for runaway transport in stochastic magnetic field

Clustering of particles leads to a reduced diffusion coefficient

Replacing $\tau_{\rm orb}$ with τ_c leads to a diffusion coefficient of form

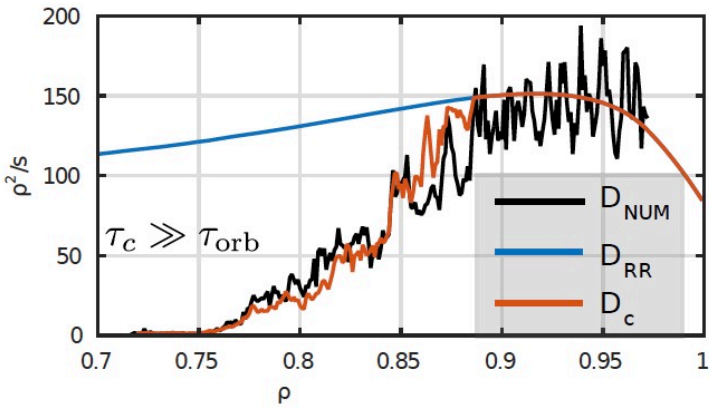
$$D_c \equiv C\frac{\sigma_{\rm orb}^2}{2\tau_c}\tilde{b}_{\rm pert}^2$$

which agrees well with numerical values.

At the edge where there are no islands,

$$\tau_c \approx \tau_{\rm orb}\,,$$

and D_c reduces to the Rechester-Rosenbluth result



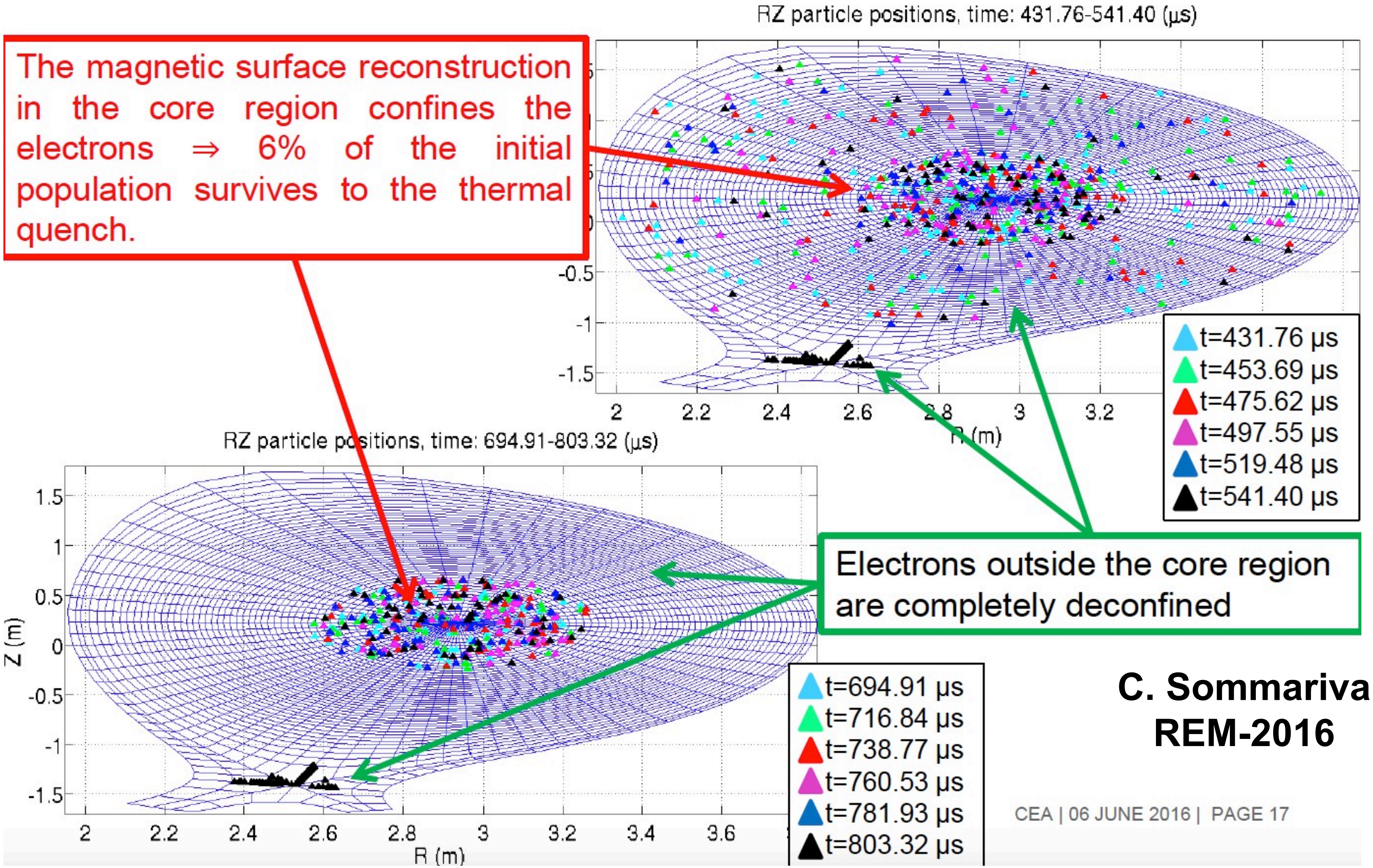
τ_c evaluated with orbit-following simulations. (independent from the coefficient evaluation)

Provides an alternative way to find D.

1) RE dynamics in mixed stochastic/coherent structures
2) Full topology is needed to get realistic RE transport description

K. Sarkimaki, REM-2016

Modeling runaway electron dynamics in realistic fields



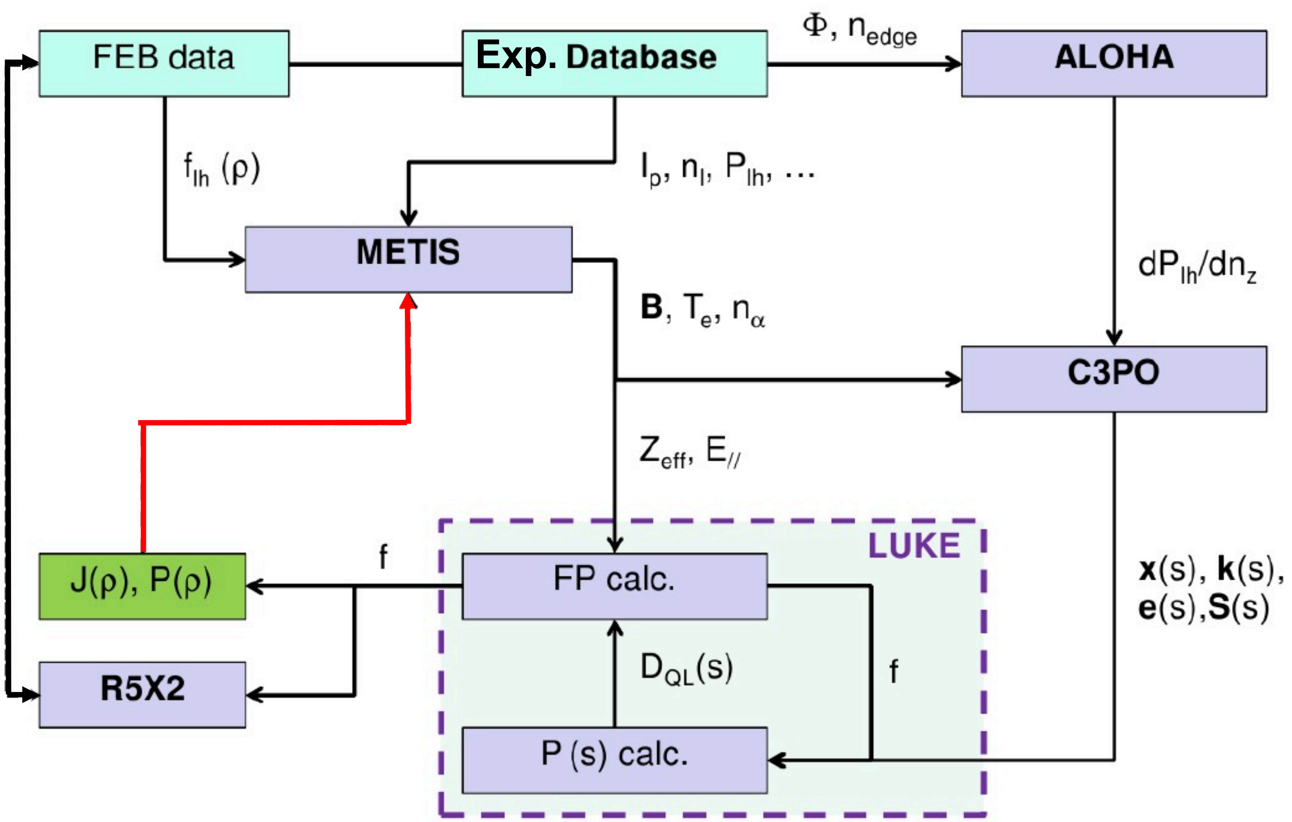
The magnetic surface reconstruction in the core region confines the electrons ⇒ 6% of the initial population survives to the thermal quench.

Electrons outside the core region are completely deconfined

C. Sommariva
REM-2016

The tools for self-consistent modeling of RE

- GO (tokamak simulator 0D) + CODE (2D momentum space, relativistic Fokker-Planck solver)
- METIS (tokamak simulator 1D radial) + LUKE (2D momentum space + 1D radial relativistic bounce-averaged Fokker-Planck solver)
- ITM



- METIS – LUKE link is done
- Stability of the convergence between j and $E_{||}$ to be improved *(scheduled for 2016)*
- Effect of RF waves on RE can be studied

RE modelling within ITM framework

G. Pokol
REM-2016

Runaway growth rates benchmarked

```
actuator_runaways = "ON", cpos = {core = {coreimpur = {cpo = "c
```

```
| | | |  +description - Source from runaway processes
| | |  +name -
| |  +species
| | |  +type
| | | |  +id = electron
| | | |  +flag - 1
| | | |  +description - Electron
| |  +gyro_type - 1
| |  +fast_filter
| | |  +method
| | | |  +id -
| | | |  +description -
| | |  +energy_sep - []
| |  +profiles_1d
| | |  +geometry
| | | |  +rho_tor - [0.0,0.013346608477615823,0.0400398254328474
| | | |  +rho_tor_norm - [0.0,0.00510204081632653,0.0153061224489
```

Conclusions

- A large amount of work has been done since the beginning of the ERP on runaway electron physics *(experimental, theoretical, modeling)*
- Good collaboration level between participants to ERP. Good connections with external labs also *(PSFC, GA, SWIP,…).*
- A very large number of publications and communications have been written in 2015 and are submitted in 2016, with oral presentations. Very active community !
- Interesting RE experiments has been performed, some particularly well suited for comparison with quantitative modeling (crucial role of diagnostics).
- Synthetic diagnostic for synchrotron radiation is available for CODE and in preparation for LUKE.
- Selfconsistent modeling of RE discharges with GO-CODE already operational. METIS-LUKE will be available in 2016 → *simulations of RE for JET*

Challenges

- Detailed RE kinetic modeling will take more than the 3 years of the ER project !
  - Analysis quantitatively the radiation synchrotron emission (angular dependence of the SR spectrum)
  - Describe the anomalous transport of RE in existing Fokker-Planck codes
  - Add bremsstrahlung reaction force in existing Fokker-Planck codes
  - Role of plasma shape
  - Effect of initial fuelling
  - Finite-width orbit effects → self-consistent Ware pinch effect
  - Effect of RF waves on disruptive RE dynamics (EC wave)
  - …

Publications, communications and thesis

Articles

« Kinetic modelling of runaway electron avalanches in tokamak plasmas », E. Nilsson, J. Decker, Y. Peysson, R.S. Granetz, F. Saint-Laurent and M. Vlainic, *Plasma Physics and Controlled Fusion* **57**, 095006 (2015).

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« Runaway electrons in non-disruptive scenarios in the Tore Supra tokamak », E. Nilsson, Y. Peysson, J. Decker, J. F. Artaud, T. Aniel, M. Irishkin, D. Mazon and F. Saint-Laurent. Submitted to *Nuclear Fusion*, (2015).

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E. Hirvijoki, J. Decker, A. Brizard and O. Embréus, *J. Plasma Phys.*, 81, 475810504 (2015)

Publications, communications and thesis

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« Effect of bremsstrahlung radiation emission on distributions of runaway electrons in magnetized plasma », O. Embréus, A. Stahl, S. Newton, G. Papp, E. Hirvijoki and T. Fülop, submitted to *Phys. Plasmas* (2015)

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« Status of research toward the ITER disruption mitigation system », E. Hollman et al, including T. Fülop and G. Papp, *Phys. Plasmas,* **22**, 021802 (2015)

« Numerical characterization of bump formation in the runaway electron tail », J. Decker, E. Hirvijoki, O. Embreus, Y. Peysson, A. Stahl, I. Pusztai, T. Fülop accepted to *Plasmas Phys. Contr. Fusion* (2015)

Publications, communications and thesis (2015)

Conference contributions:

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« Non-monotonic features in the runaway electron tail. » I. Pusztai, E. Hirvijoki, J. Decker, O. Embréus, A. Stahl and T. Fülop, *42nd EPS Conference on Plasma Physics* (2015). **[ORAL]**

« Towards self-consistent runaway electron modelling », G. Papp, A. Stahl, Drevlak, T. Fülop, E. Lauber and G. Pokol,, *42nd EPS Conference on Plasma Physics* (2015). **[POSTER]**

« Numerical calculation of ion runaway distributions. » ,O. Embréus, S. Newton, A. Stahl, E. Hirvijoki and T. Fülop: *42nd EPS Conference on Plasma Physics* (2015). **[POSTER]**

« Kinetic modelling of runaway electron dynamics », A. Stahl, O. Embréus, E. Hirvijoki, G. Papp, M. Landreman, I. Pusztai and T. Fülop,, *IAEA Energetic Particle meeting* (2015) **[POSTER]**

« Coupled kinetic-fluid runaway simulations », G. Papp, A. Stahl, T. Fülop, Drevlak, G. Pokol, E. Lauber and A. Fehér,, *IAEA Energetic Particle meeting* (2015) **[ORAL]**

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« Reaction of runaway electron distributions to radiative Processes » A. Stahl et al,, *APS conference Savannah* (2015) **[POSTER]**

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« Numerical calculation of ion runaway distributions », S. Newton et al,, *APS conference Savannah* (2015) **[POSTER]**

Thesis report:
"Dynamics of runaway electrons in tokamak plasmas" by E. Nilsson
PhD thesis (Ecole Polytechnique Paris, France, September, 2015)
https://pastel.archives-ouvertes.fr/tel-01212017/

"Relativistic runaway electron simulations in 3D background" by Konsta Särkimäki
PhD thesis (Aalto University, Helsinki, Finland, 2015)