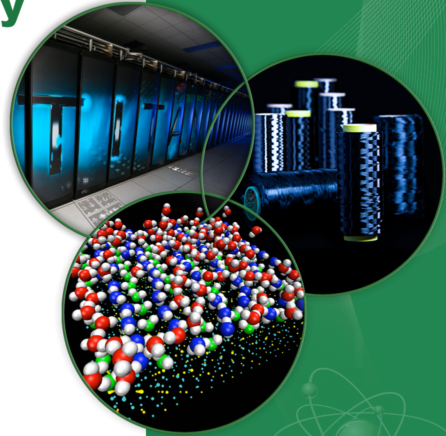
Full-orbit effects in the dynamics of runaway electrons in toroidal geometry

D. del-Castillo-Negrete
L. Carbajal

D.A. Spong, L.R. Baylor,
S.K. Seal



Theory and Simulation of Disruptions

Workshop

PPPL

July 21, 2016

MODELING RUNAWAY ELECTRONS (RE) DYNAMICS

- The dynamics of RE spans a huge range of time scales, from the gyro-period $t \sim 10^{-11}$sec to the observational time scales $t \sim 10^{-3} \rightarrow 1$sec.
- This, among other reasons, motivates the development of reduced models which, starting from the exact dynamics, lead to tractable physically insightful models.
- On the other hand, the full-orbit (Lorentz-force model) fully resolves the gyro-motion and provides 6D information.
- Computer power limitations should not be a reason for not using this model.
- The next level of description is provided by the 4D guiding center model that eliminates the gyro-motion degree of freedom.
- Although this approximation is remarkably good to study transport in tokamaks it might face limitations in the study of RE due to relativistic motion and synchrotron radiation.

MODELING RUNAWAY ELECTRONS DYNAMICS

- Bounce-average approximations eliminate spatial degrees of freedom and lead to 2D phase space Fokker-Planck models.
- This approach has lead to remarkably deep physical insights.
- However, the elimination of spatial information, does not allow to access the role of confinement neither the spatial variations of the magnetic field.
- The ultimate level of approximation is provided by 0D test particle models that eliminate all the moments of the Fokker-Planck model (except for the first one) and reduce the dynamics to two coupled ordinary differential equations following the mean momentum degrees of freedom.
- To the previous limitations, test particle models add the neglect of momentum space diffusion (second and higher order moment dynamics).

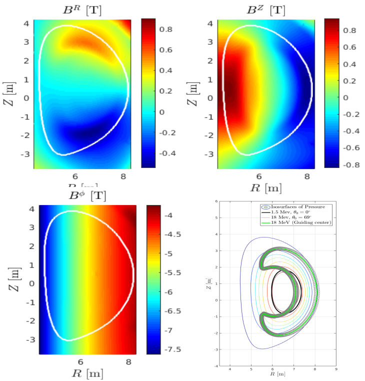
MODELING RUNAWAY ELECTRONS DYNAMICS

Some disclaimers:

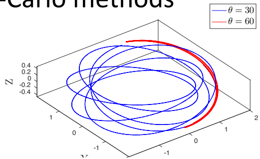
- I am not against reduced models! They are indeed deeply insightful.
- Just because it is computational tractable to do the full 6D problem one should not embark in these calculations without the physics guidance provided by experiments and reduced models.
- The goal of full-orbit simulations is to complement reduced models not to disproved them.
- The information provided by full orbit simulations should help improve reduced models and get closer to predictive simulations.

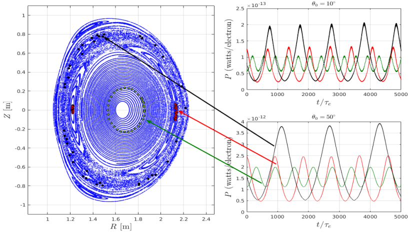
KORC: KINETIC ORBIT RUNAWAY ELECTRONS CODE

- **Lorentz** force **relativistic** full orbit equations of motion
- Fast, small scale **gyro-motion** fully resolved
- **General 3-Dimensional**, integrable or chaotic magnetic fields
- Accurate **synchrotron radiation damping** calculation
- **Collisional effects** incorporated using Monte-Carlo methods



3D SIESTA
MHD
Equilibrium
magnetic fields





KORC: KINETIC ORBIT RUNAWAY ELECTRONS CODE

NUMERICAL ACCURACY:

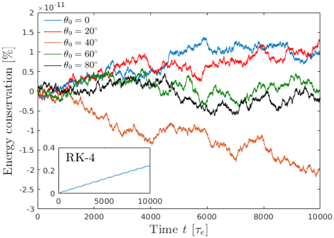
Long term integration ~10^{-3} sec resolving fast gyro-motion ~10^{-10} sec requires accurate and stable method

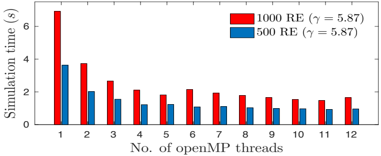
- Implemented modified **relativistic leapfrog** (MRL) method
- The MRL provides **long-term stability**
- **Energy is conserved** up to machine precision.
- **Operator splitting** for radiation damping

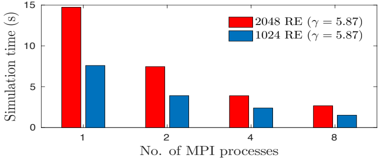
PARALLELIZATION:

Very large number of particles N ~10^6 needed to reduce noise

- Parallelized using **open MP** & **MPI**.
- First studies of KORC in a single HPC node show good **strong scaling**.







RE FULL ORBIT MODEL DETAILS

- Relativistic equations $\frac{d}{dt}\mathbf{p} = \mathbf{F_L} + \mathbf{F_R} + \mathbf{D}$ where $\mathbf{p} = \gamma m\mathbf{v}$, $\mathbf{F}_L$ is the Lorentz force, $\mathbf{F}_R$ is the radiation reaction force, and $\mathbf{D}$ denotes collisional effects.
- Modeling $\mathbf{F}_R$ is not trivial. The Abraham-Lorentz-Dirac force is inconsistent and should not be used directly.
- The correct account of radiation reaction is described by the Landau-Lifshitz model $\mathbf{F}_R = \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3$

$$\begin{aligned}
\mathbf{f}_1 &= \frac{2q^3}{3mc^3 4\pi\epsilon_0}\gamma\left[\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{E} + \mathbf{v}\times\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{B}\right] \\
\mathbf{f}_2 &= \frac{2q^4}{3m^2 4\pi\epsilon_0 c^3}\left[\mathbf{E}\times\mathbf{B} + \mathbf{B}\times(\mathbf{B}\times\mathbf{v}) + \frac{1}{c^2}\mathbf{E}(\mathbf{v}\cdot\mathbf{E})\right] \\
\mathbf{f}_3 &= -\frac{2q^4}{3m^2 c^5 4\pi\epsilon_0}\gamma^2\mathbf{v}\left[(\mathbf{E} + \mathbf{v}\times\mathbf{B})^2 - \frac{1}{c^2}(\mathbf{E}\cdot\mathbf{v})^2\right]
\end{aligned}$$

- $\mathbf{f}_1$ can be safely neglected for RE and it is not included in KORC.
- In practice, the dominant terms are those highlighted.

RE FULL ORBIT MODEL DETAILS

- KORC can be run with any kind of magnetic fields.
- However, in this presentation we will use the following model

$$\mathbf{B} = \frac{1}{1+\eta\cos\theta}\left[B_0\,\hat{\mathbf{e}}_\zeta + B_\theta(r)\,\hat{\mathbf{e}}_\theta\right]$$

where B_0, is assumed constant, and

$$B_\theta(r) = \frac{r/\lambda}{1+(r/\lambda)^2}\,B\,, \qquad q(r) = q_0\left(1+\frac{r^2}{\lambda^2}\right)\,.$$

- Toroidal symmetry implies that (in the absence of symmetry breaking forces) the toroidal momentum is conserved.
- This invariant and the energy (without acceleration and radiation damping) are used to benchmark the accuracy.

LIMITATIONS OF GUIDING CENTER MODEL

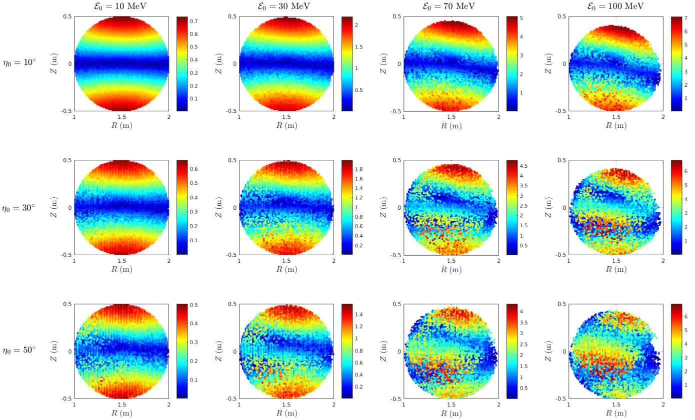
- Beyond to the $\rho/R \ll 1$ condition (where ρ is the gyro-radius) relativistic RE electrons might violate the guiding-center approximation due to the breakdown of the condition $d/R \ll 1$ where d is the distance traveled in the parallel direction during a gyro-period.
- Numerical simulations [Liu et.a;., 2016, Wang et.al., 2016] indicate that the second condition might be violated. In particular

  $$\Lambda(\Psi) = \frac{|\Psi(\mathbf{x_0}, t_0 + \tau_g) - \Psi(\mathbf{x_0}, t_0)|}{|\Psi(\mathbf{x_0}, t_0)|}$$

  might exhibit large variations, where τ_g is the gyro-period and Ψ denotes either $\mathbf{B}$ or B.
- Here we study this problem in detail focusing on the spatial distribution of RE and the pitch angle dependence for large ensembles of RE.

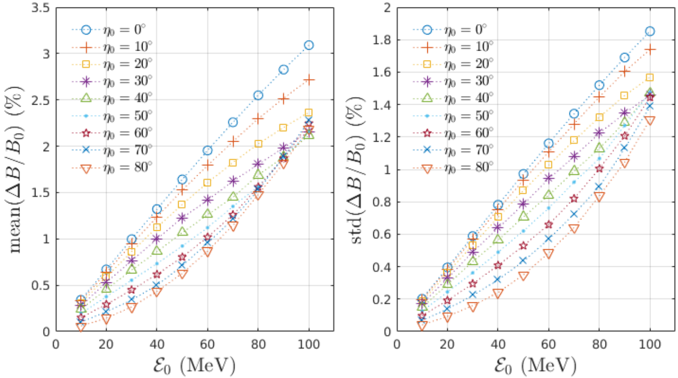
LIMITATIONS OF GUIDING CENTER MODEL

Spatial distribution of magnetic field **magnitude** variation



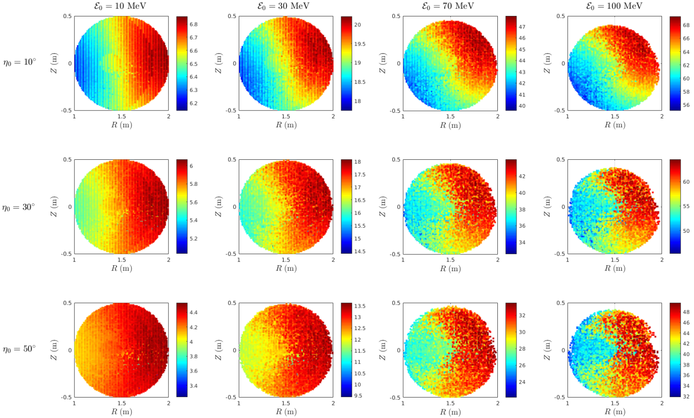
LIMITATIONS OF GUIDING CENTER MODEL

Statistics of magnetic field **magnitude** variation



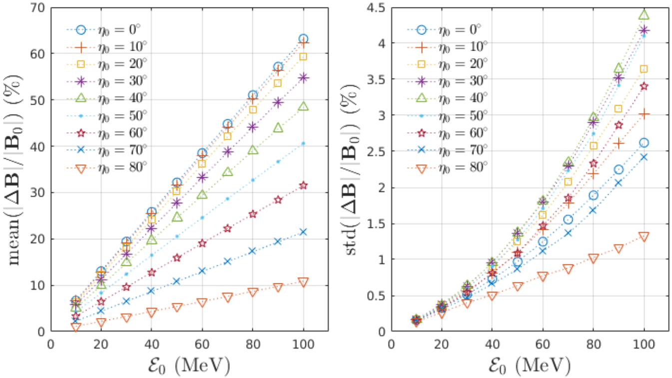
LIMITATIONS OF GUIDING CENTER MODEL

Spatial distribution of magnetic field **vector** variation



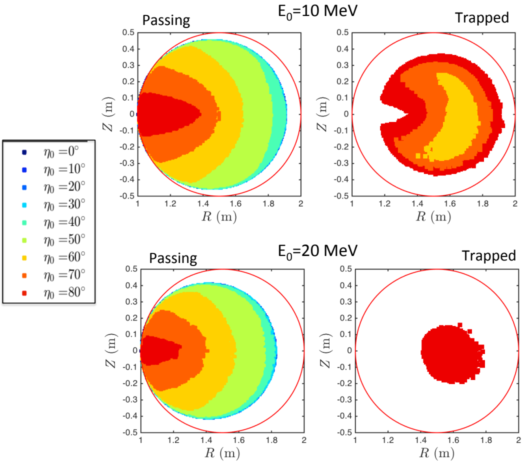
LIMITATIONS OF GUIDING CENTER MODEL

Statistics of magnetic field **vector** variation



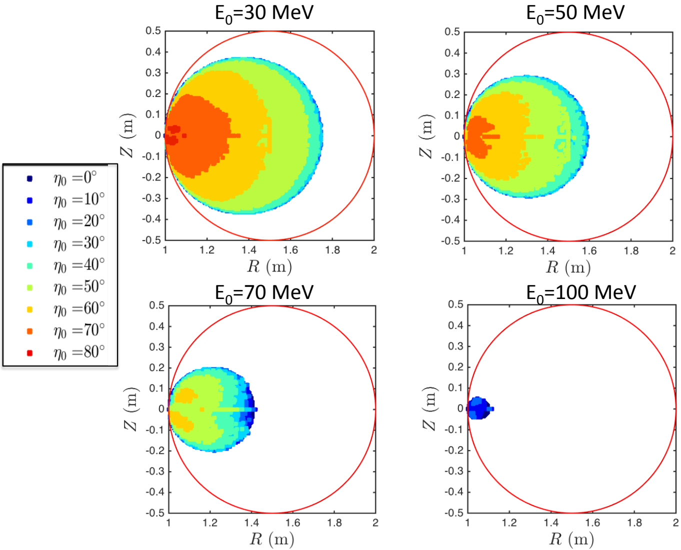
RADIAL CONFINEMENT OF RUNAWAY ELECTRONS

- The confinement of RE is affected by the radial drift of orbits [Knoepfel-Spong 1970; Guan et.al. 2010; Papp et.al. 2011].
- Here we focus on the pitch angle dependence using full-orbit simulations for large ensembles of particles.



RADIAL CONFINEMENT OF RUNAWAY ELECTRONS

Energy and pitch angle dependence of RE confinement due to neoclassical radial drift



E_0=30 MeV

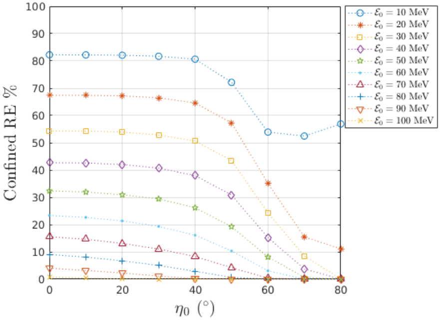
E_0=50 MeV

E_0=70 MeV

E_0=100 MeV

RADIAL CONFINEMENT OF RUNAWAY ELECTRONS

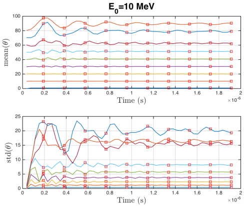
Energy and pitch angle dependence of RE confinement due to neoclassical radial drift

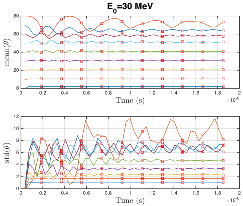


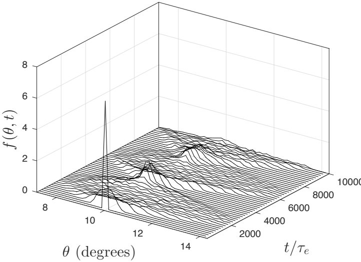
FULL ORBIT EFFECTS ON PITCH ANGLE DYNAMICS

Orbit induced collisionless pitch angle scattering

- Toroidal orbits can give rise to momentum transfer from parallel to perpendicular, even in absence of collisional pitch angle scattering [Liu et.a;., 2016, Wang et.al., 2016].
- This gives rise to a transitory time modulation "breathing" of the pitch angle probability distribution function.



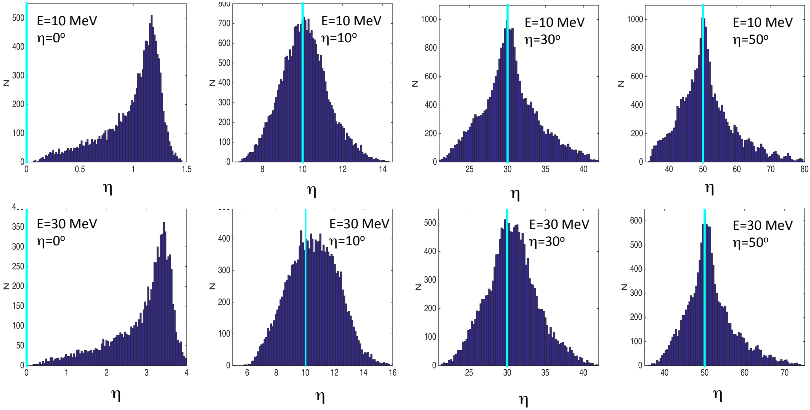




FULL ORBIT EFFECTS ON PITCH ANGLE DYNAMICS

Orbit induced collisionless pitch angle scattering

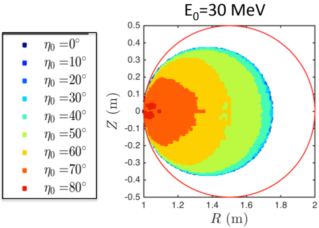
Pitch angle probability distribution function: long time steady state

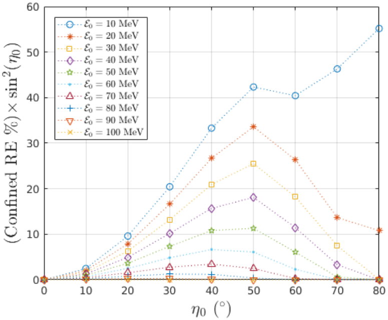


FULL ORBIT EFFECTS ON SYNCHROTRON RADIATION

Role of RE spatial confinement

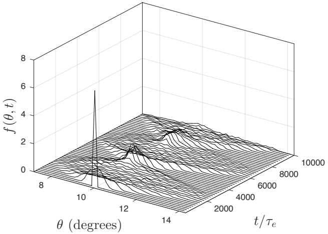
$$\mathcal{P} = \frac{\gamma p \sin^2 \eta}{\tau_r}, \qquad \tau_r = \frac{6\pi\varepsilon_0 (m_e c)^3}{e^4 B^2}$$

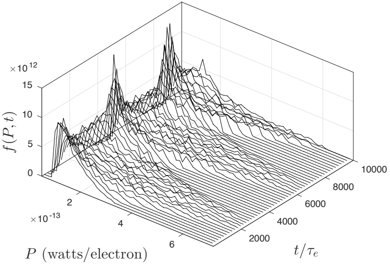


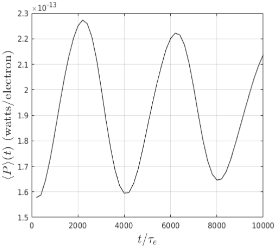


FULL ORBIT EFFECTS ON SYNCHROTRON RADIATION

Transient modulation due to orbit induced collisionless pitch scattering





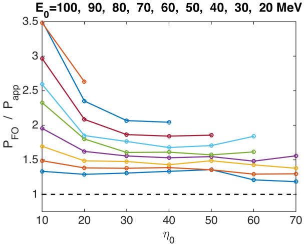


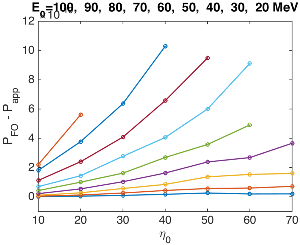
FULL ORBIT EFFECTS ON SYNCHROTRON RADIATION

The pitch angle-dependence of the radial confinement of RE has a direct impact in the total synchrotron radiation power

$$\mathcal{P} = \frac{\gamma p \sin^2 \eta}{\tau_r}, \qquad \tau_r = \frac{6\pi\varepsilon_0 (m_e c)^3}{e^4 B^2}$$

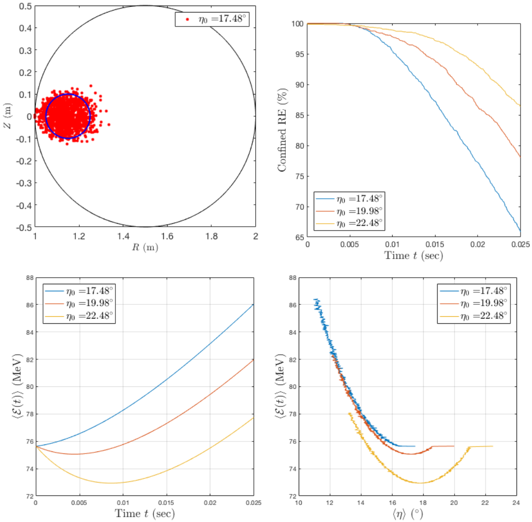
- In orbit-averaged models the power P_{app} is computed using the magnetic field at an averaged fixed position and the pitch angle η follows approximated equations of motion.
- In full-orbit calculations the power P_{FO} is computed by evaluating B and η using the exact equations of motion.





ELECTRIC FIELD AND RADIATION DAMPING

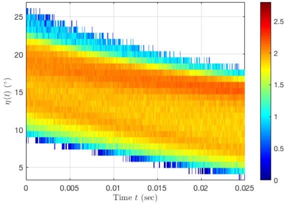
Acceleration and loss of confinement



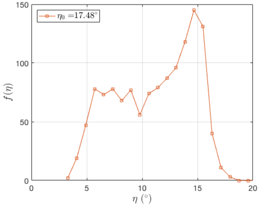
ELECTRIC FIELD AND RADIATION DAMPING

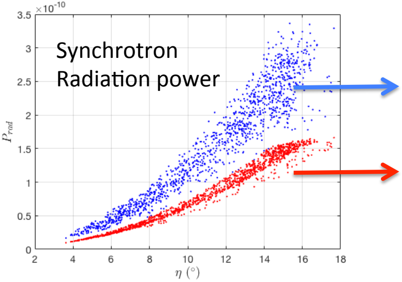
Pitch angle dynamics

Time evolution of pitch angle probability distribution function



Final time pitch angle pdf





Synchrotron Radiation power

Full orbit calculation

Orbit averaged approximation

CONCLUSIONS

- The magnetic field exhibits strong variations along the orbits of high energy RE questioning the validity of orbit-averaging.
- RE loss of confinement due to radial drift exhibits a dependence on pitch angle that impacts synchrotron radiation.
- In the absence of collisions, electric field, and radiation, the pitch angle exhibits orbit induced collisionless pitch angle scattering (CPAS).
- At short times CPAS exhibits oscillations that leads to modulation of synchrotron radiation.
- At long times the pitch angle exhibits non-Gaussian probability distributions due to CPAS.
- Reduced (orbit-averaged) models under-estimate synchrotron radiation.