Elliptic flow, eccentricity and eccentricity fluctuations

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for the PHOBOS collaboration

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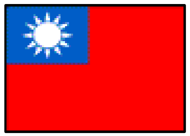
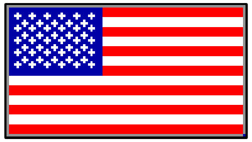
XXXVI International Symposium on Multiparticle Dynamics
September 02-08, 2006, Paraty, Rio de Janeiro, Brazil

PHOBOS collaboration

Burak Alver, Birger Back, Mark Baker, Maarten Ballintijn, Donald Barton, Russell Betts, Richard Bindel, Wit Busza (Spokesperson), Zhengwei Chai, Vasundhara Chetluru, Edmundo García, Tomasz Gburek, Kristjan Gulbrandsen, Clive Halliwell, Joshua Hamblen, Ian Harnarine, Conor Henderson, David Hofman, Richard Hollis, Roman Hołyński, Burt Holzman, Aneta Iordanova, Jay Kane, Piotr Kulinich, Chia Ming Kuo, Wei Li, Willis Lin, Constantin Loizides, Steven Manly, Alice Mignerey, Gerrit van Nieuwenhuizen, Rachid Nouicer, Andrzej Olszewski, Robert Pak, Corey Reed, Eric Richardson, Christof Roland, Gunther Roland, Joe Sagerer, Iouri Sedykh, Chadd Smith, Maciej Stankiewicz, Peter Steinberg, George Stephans, Andrei Sukhanov, Artur Szostak, Marguerite Belt Tonjes, Adam Trzupek, Sergei Vaurynovich, Robin Verdier, Gábor Veres, Peter Walters, Edward Wenger, Donald Willhelm, Frank Wolfs, Barbara Wosiek, Krzysztof Woźniak, Shaun Wyngaardt, Bolek Wysłouch

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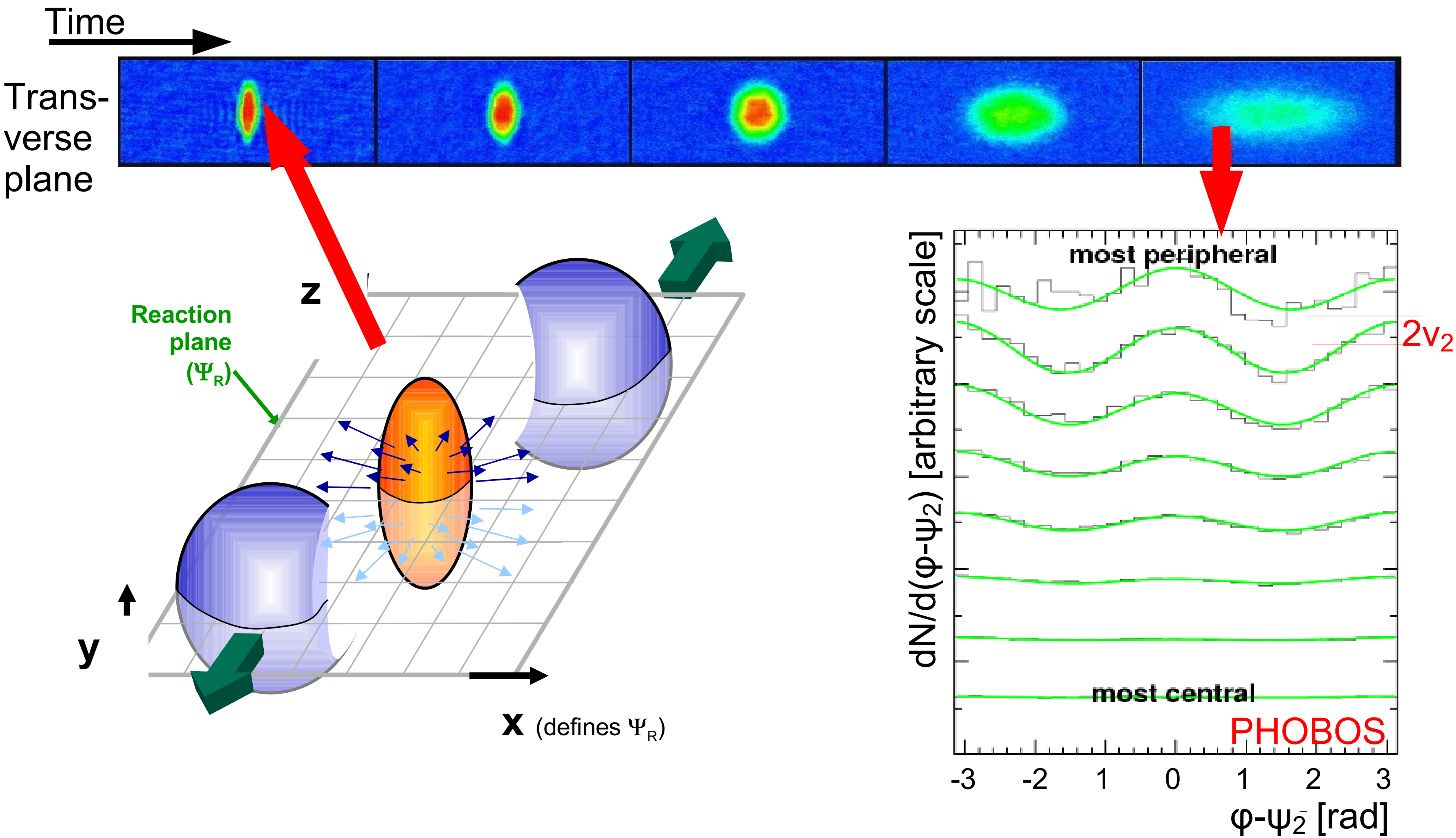
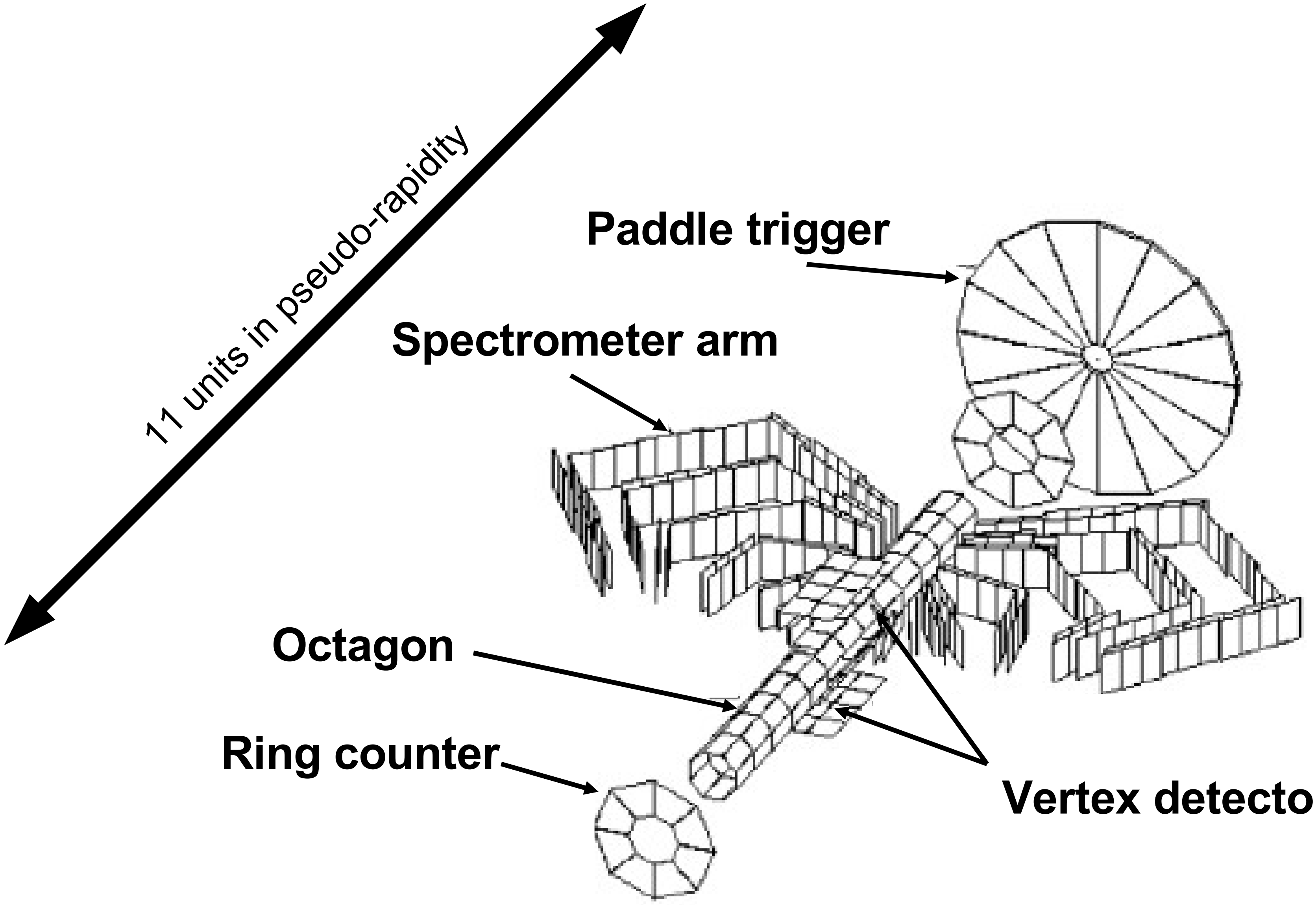
Elliptic flow



Non-central collision:
Initial state eccentricity

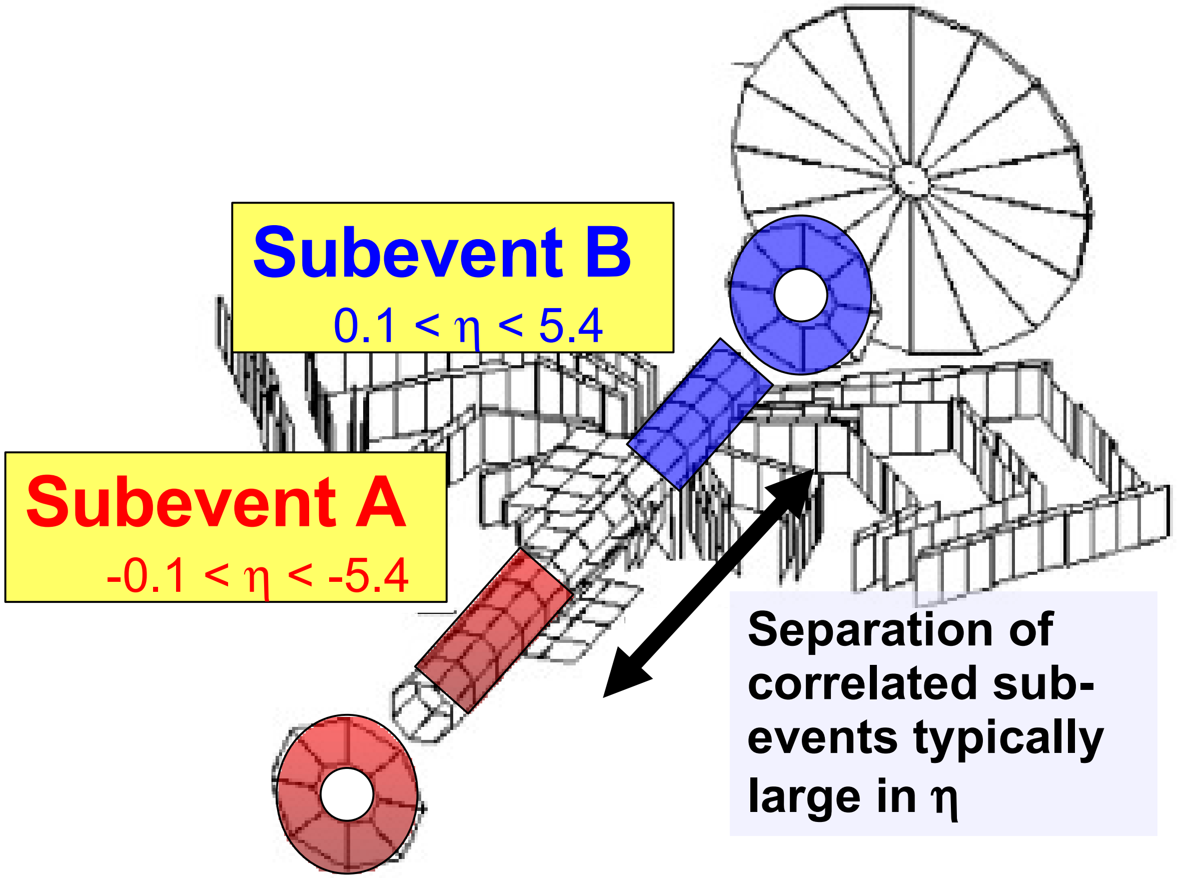
Momentum space anisotropy
$v_2 = \langle \cos(2\phi - 2\Psi_R) \rangle$

Flow measurement in PHOBOS



Flow measurement in PHOBOS

- Reaction-plane / Subevent technique
  - Correlate reaction plane determined from azimuthal pattern of hits in one part of the detector with information from other parts of the detector



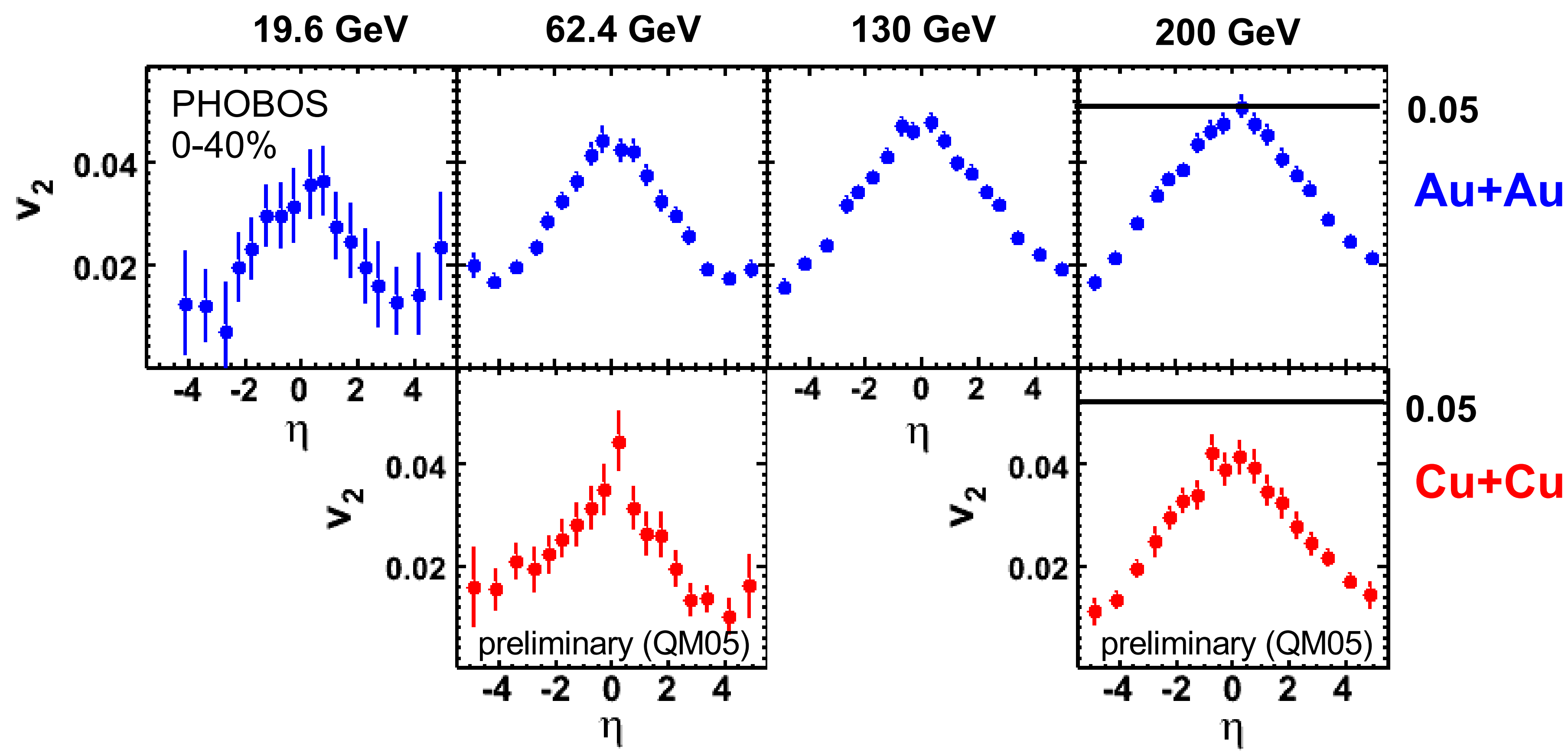
$$\tan(2\psi_A) = \frac{\langle \sin(2\phi) \rangle_A}{\langle \cos(2\phi) \rangle_A}$$

$$v_2^{obs} = \langle \cos(2\phi - 2\psi_A) \rangle_B$$

$$v_2 = \frac{\langle v_2^{obs} \rangle_{events}}{\sqrt{\langle \cos(\psi_A - \psi_B) \rangle_{events}}}$$

A.Poskanzer, S.Voloshin, nucl-ex/9805001

Elliptic flow (v_2)



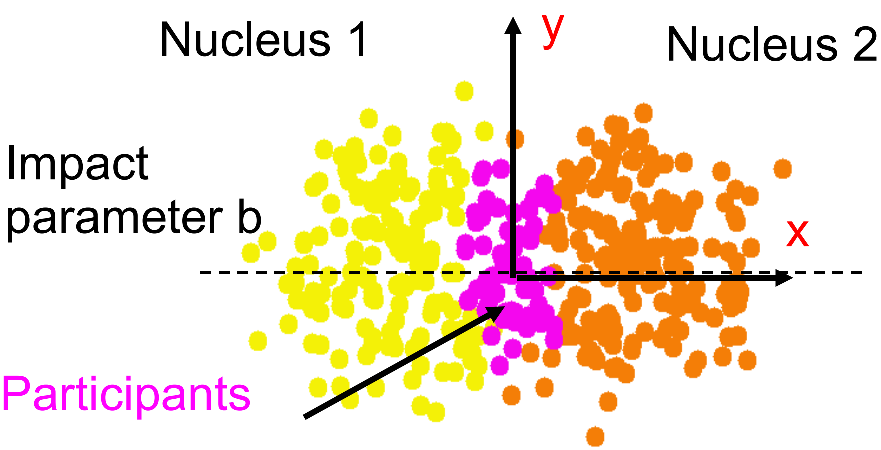
Cu+Cu about 20% lower than Au+Au

Au+Au: PRL 94 122303 (2005)
Cu+Cu: nucl-ex/0510031 (preliminary)

Relating system size and eccentricity

- Glauber Monte Carlo
  - Radial distribution of nucleons (in nucleus) drawn from Wood-Saxon distribution
  - Isotropic angular distribution
  - Separate by impact parameter
  - Nucleons travel on straight-line paths and interact inelastically when

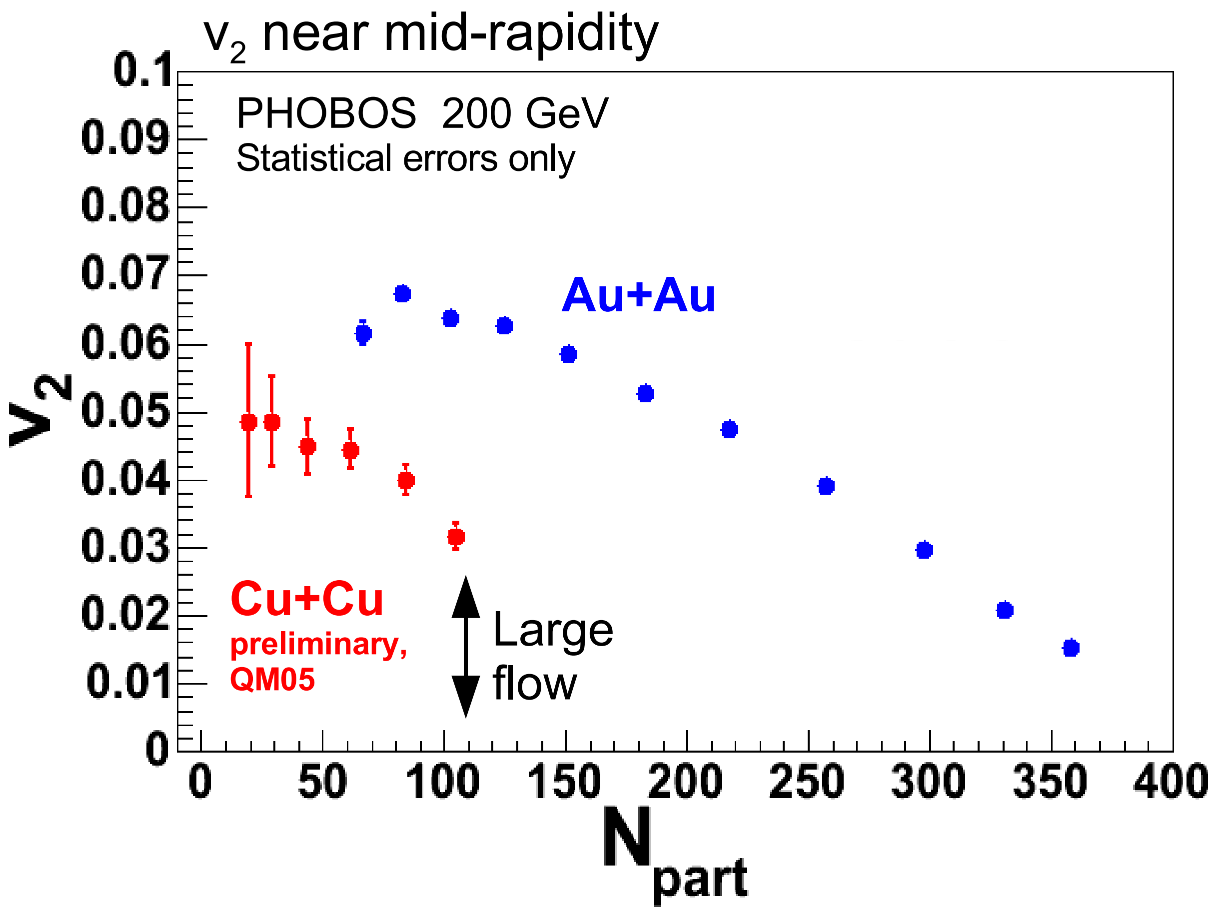
$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}<\sqrt{\sigma_{NN}/\pi}$$

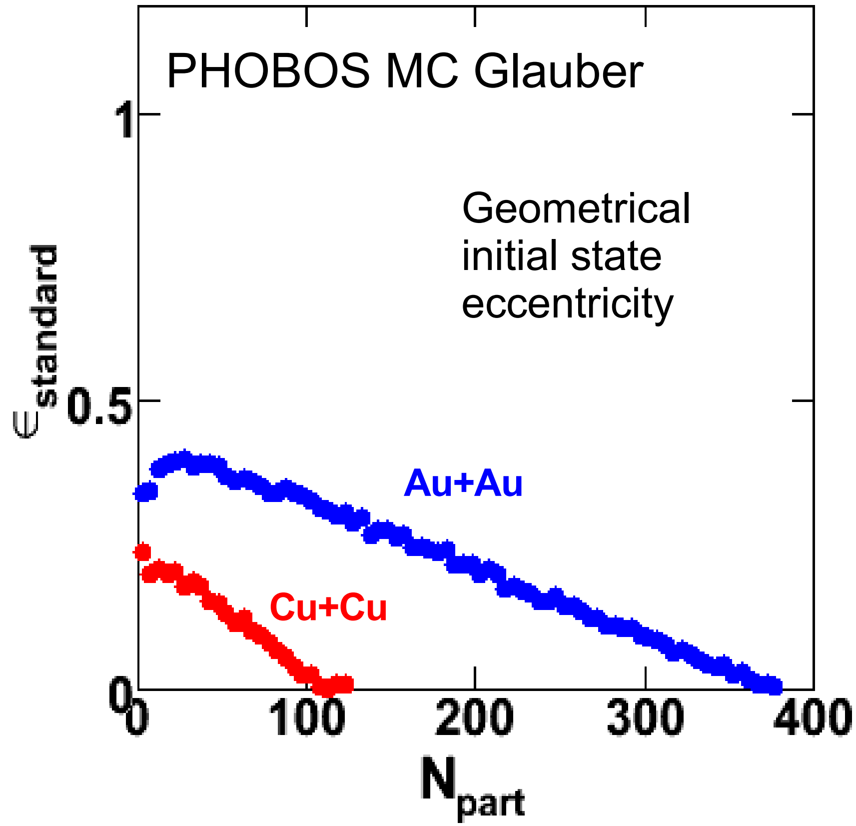


- Centrality of collision
  - #Participants
    - Nucleons that interact at least once
  - Related to cross section and impact parameter range
- Eccentricity of collision zone
  - Given by participants position distributions

Eccentricity: $\epsilon_{std}=\frac{\sigma_y^2-\sigma_x^2}{\sigma_y^2+\sigma_x^2}$

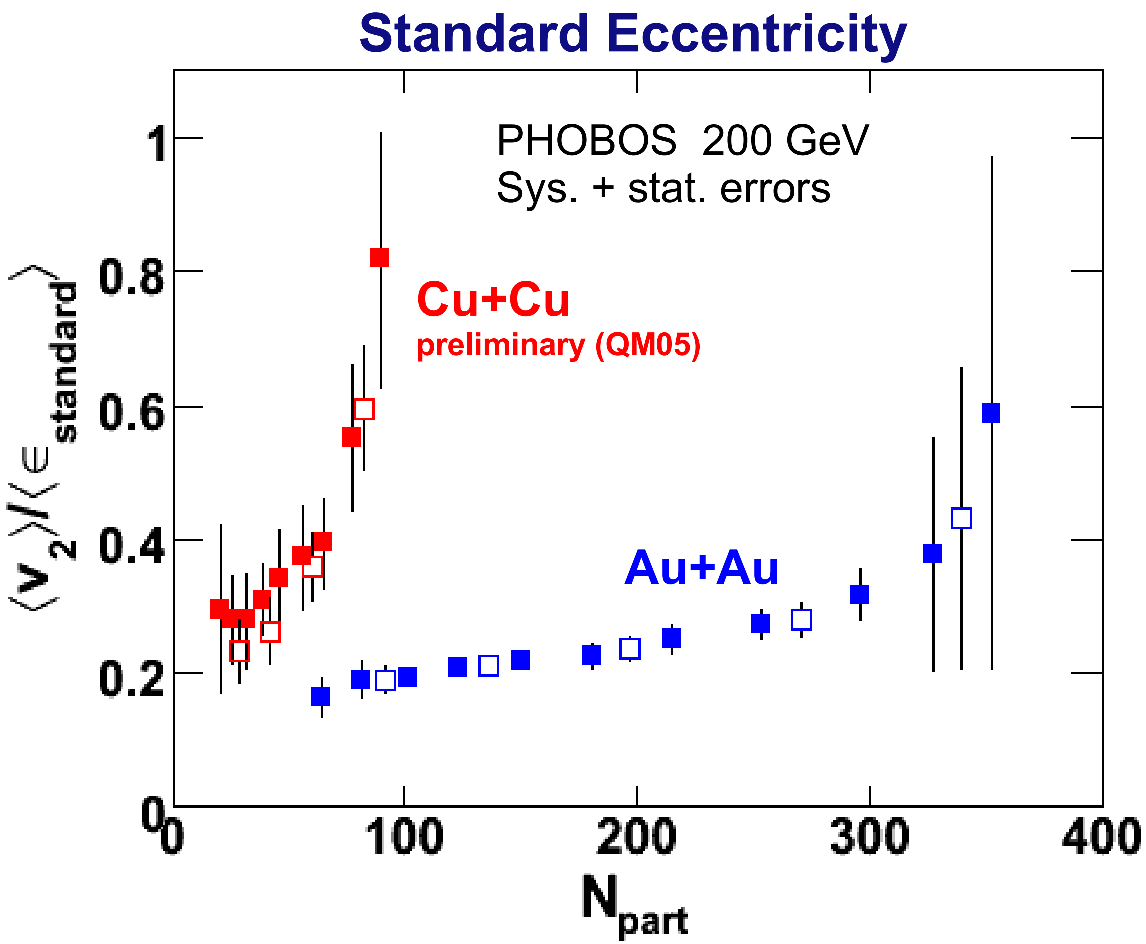
Elliptic flow in Cu+Cu and Au+Au





Au+Au: PRC 72, 051901 (2005)
Cu+Cu: prel. QM05, nucl-ex/0510042

Scaled elliptic flow vs N_{part}

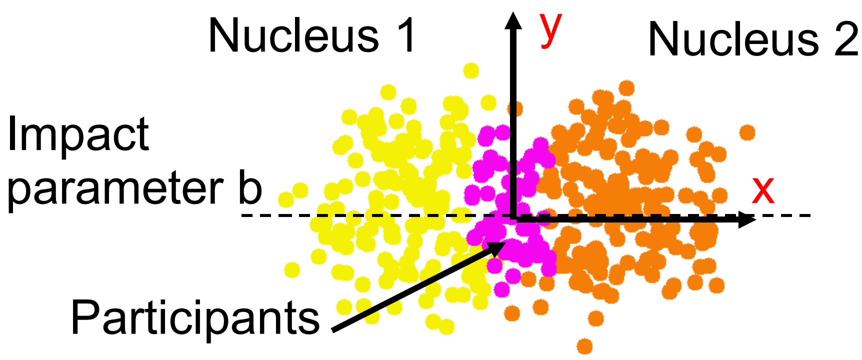


No agreement between Cu+Cu and Au+Au scaled by $\epsilon_{standard}$

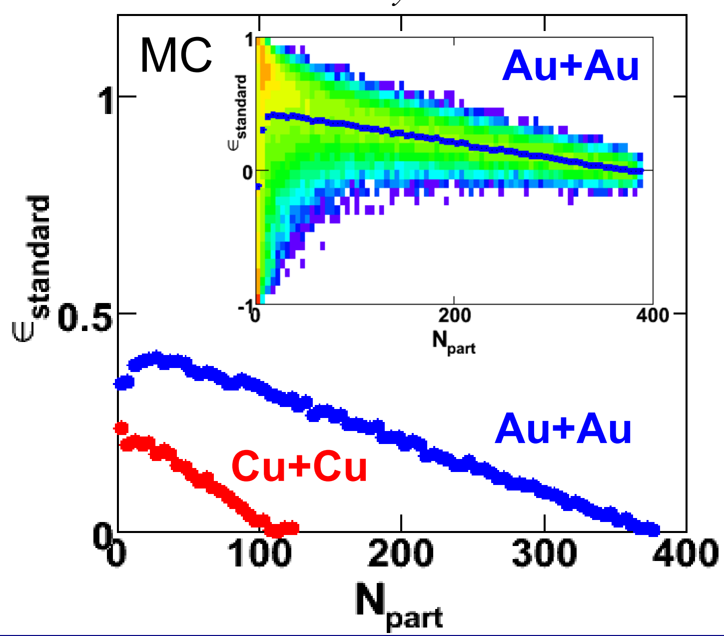
Au+Au: PRC 72, 051901 (2005)
Cu+Cu: prel. QM05, nucl-ex/0510042

Standard eccentricity calculation

Standard Eccentricity

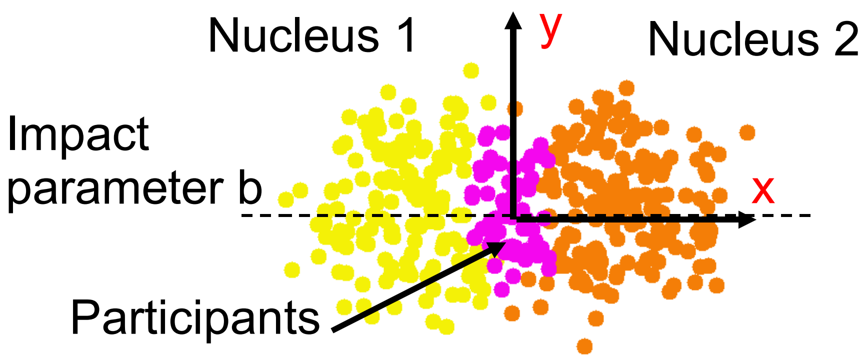


$$\epsilon_{std} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$

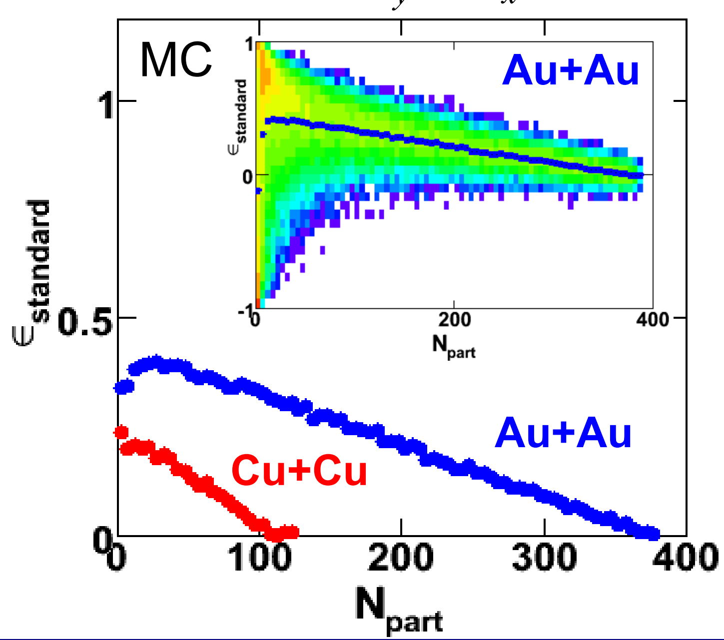


Participant eccentricity calculation

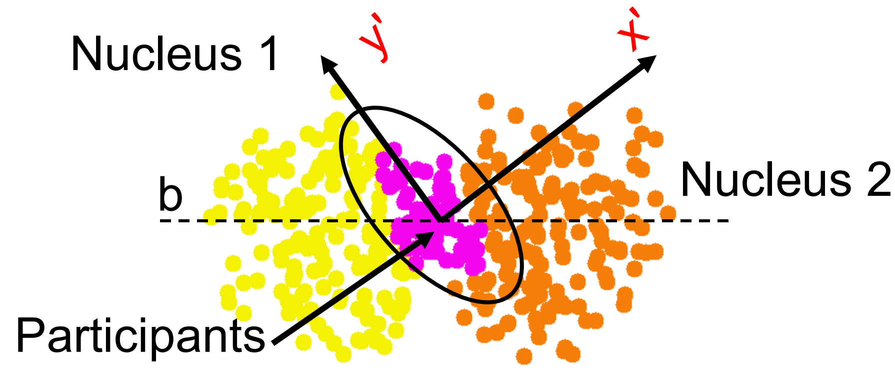
Standard Eccentricity



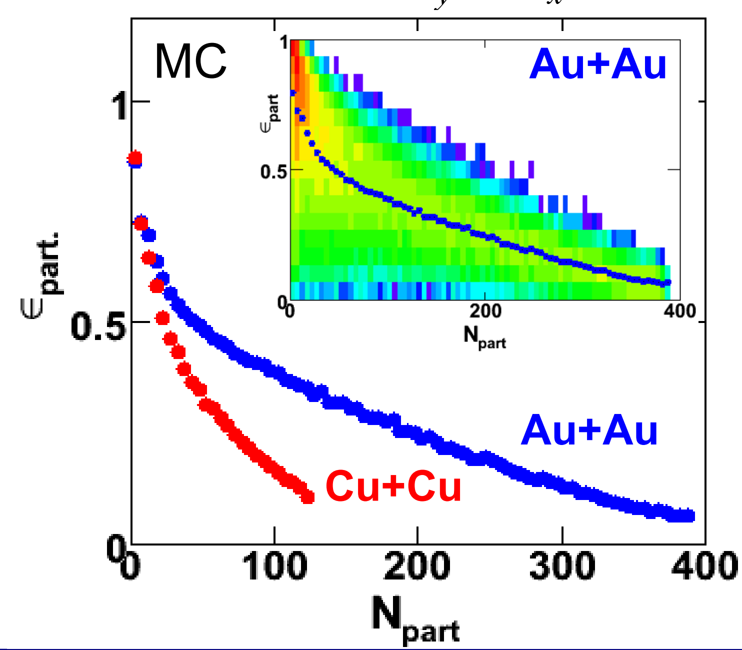
$$\epsilon_{std} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$



Participant Eccentricity



$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 - 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$

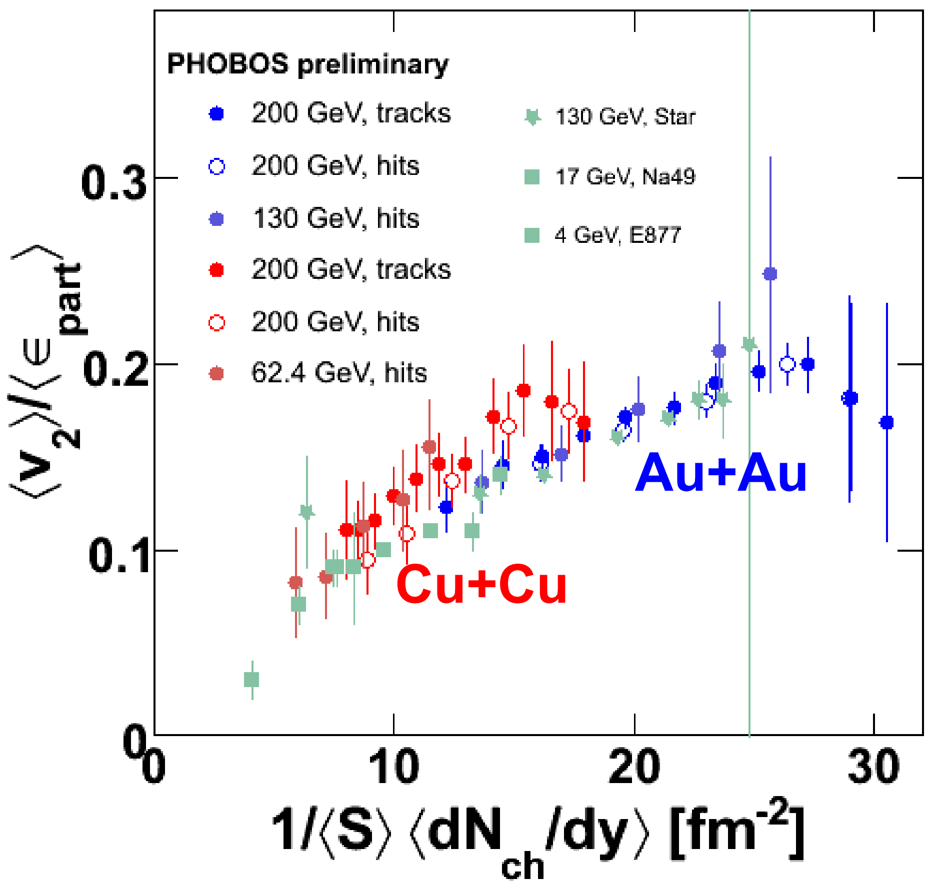


Participant eccentricity scaling

- Caution: We used $\epsilon_{participant}$ for PHOBOS data. Important for Cu-Cu, less critical for Au-Au.
- Scale $v_2(\eta)$ to $\sim v_2(y)$ (10% lower)
- Scale $dN/d\eta$ to be $\sim dN/dy$ (15% higher)
- STAR and AGS Au+Au and CERN Pb+Pb results are not scaled by $\epsilon_{participant}$
- S is overlap area (MC Glauber)

STAR, PRC 66 034904 (2002)
Voloshin, Poskanzer, PLB 474 27 (2000)
Heiselberg, Levy, PRC 59 2716, (1999)
Au+Au: PRC 72, 051901 (2005)
Cu+Cu: prel. QM05, nucl-ex/0510042

Participant Eccentricity

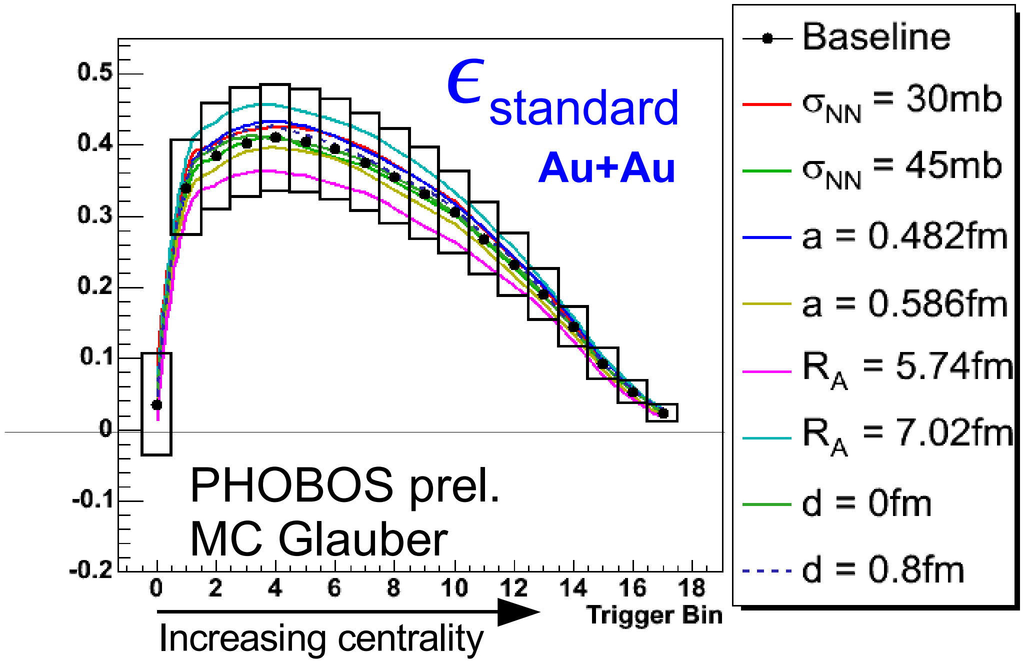


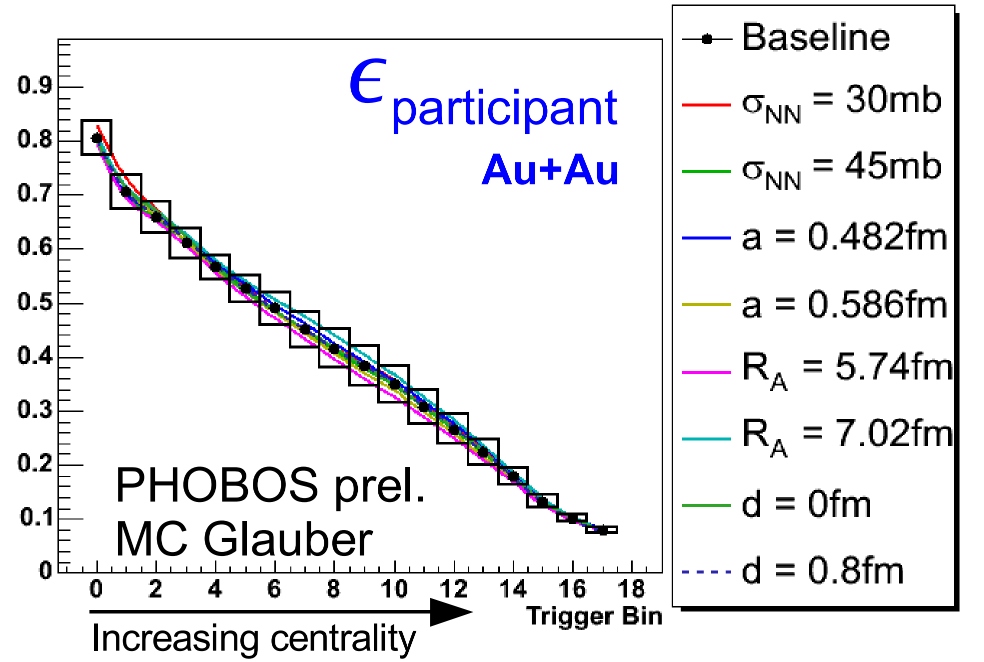
Approximate scaling between Cu+Cu and Au+Au

Since QM 2005

- Examine properties of participant eccentricity
  - Study robustness wrt to geometry parameters
  - Connection to v_2 measurements and hydro
    - Work in collaboration with Uli Heinz
- Measure higher moments in v_2

Robustness with geometry variables





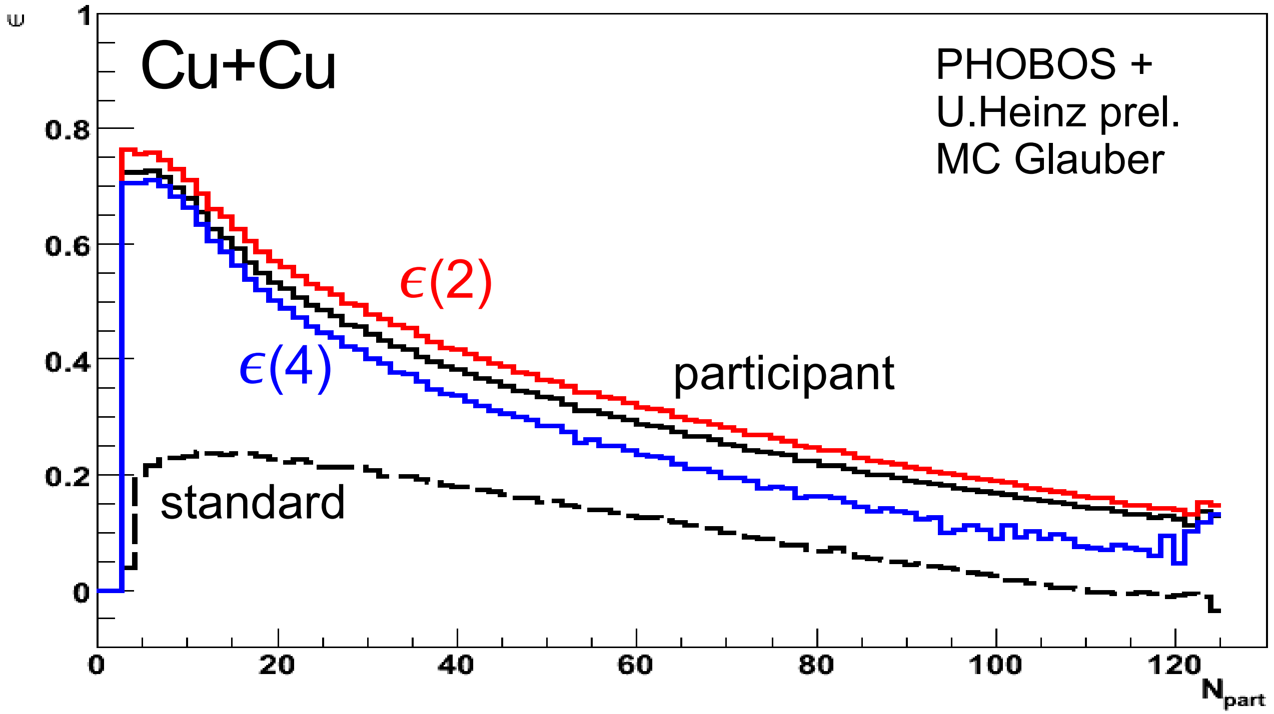
- Variation of
  - Nucleon-nucleon cross section (30-45mb)
  - Nuclear radius (±10% from the nominal value)
  - Skin depth (0.482-0.586fm)
  - Minimum separation distance between nucleons (d=0-0.8fm)

$$\rho(r) = \frac{\rho_0}{1 + \exp((r - R)/a)}$$

$\epsilon_{participant}$ even slightly more robust than $\epsilon_{standard}$

Moments of eccentricity

- If one measures v_2 with two-particle correlations
  $$v_2(2)=\sqrt{\langle \cos 2(\phi_1-\phi_2)\rangle}=\sqrt{\langle v_2^2\rangle}$$
- If v_2 fluctuates prop. to ϵ_{part}
  - Scaling with
    $$\epsilon(2)=\sqrt{\langle \epsilon_{part}^2\rangle}$$
- For 4-cumulant method
  $$v(4)=[2\langle v^2\rangle^2-\langle v^4\rangle]^{1/4}$$
  - Scaling with
    $$\epsilon(4)=[2\langle \epsilon^2\rangle^2-\langle \epsilon^4\rangle]^{1/4}$$



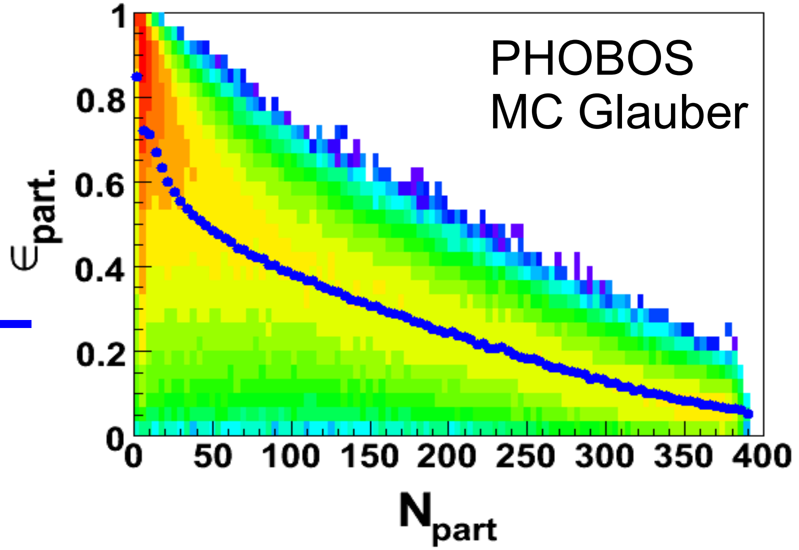
$\epsilon\{2\}$ rather similar to ϵ_{part}, unlike $\epsilon\{4\}$ that is between ϵ_{std} and ϵ_{part}

M.Miller, R.Snellings, nucl-ex/0312008
R.Bhalerao, J.Ollitrault, nucl-th/0607009

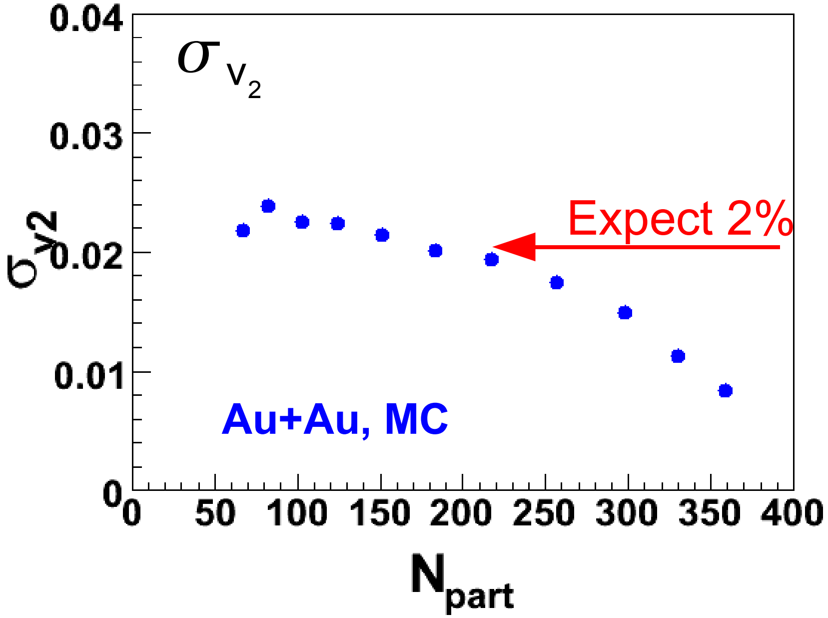
Expected elliptic flow fluctuations

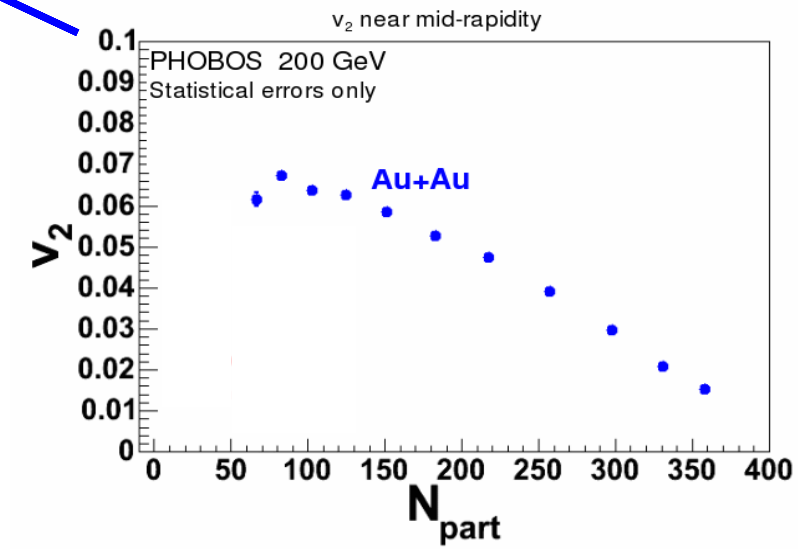
Assuming $v_2 \propto \epsilon_{part}$, the participant eccentricity model predicts

$$\sigma_{v_2} = \frac{\sigma_{\epsilon_{part}}}{\epsilon_{part}} v_2$$



Expected σ_{v2} from fluctuations in ϵ_{part}





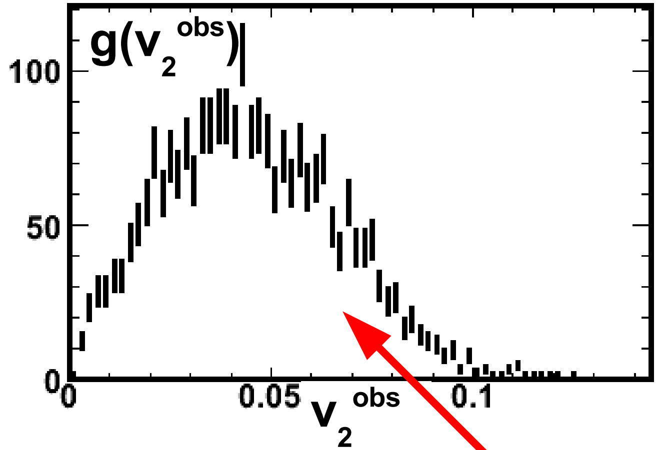
Measuring elliptic flow fluctuations

- Ongoing analysis on 200GeV Au+Au
  - NO DATA will be shown
  - Overview of the measurement
    - Studies on fully simulated MC events
  - Details are in nucl-ex/0608025

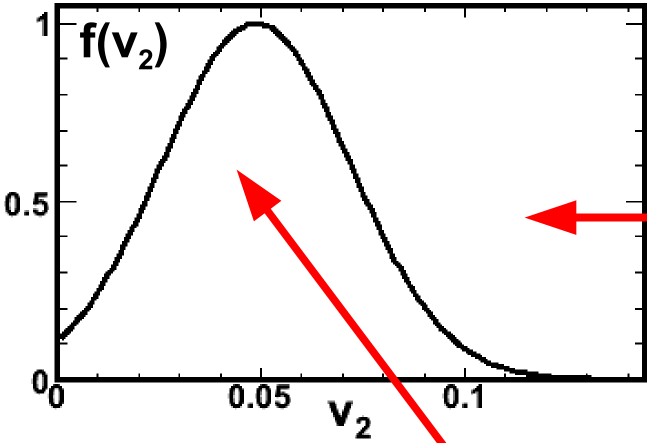
B.Alver et.al. (PHOBOS), nucl-ex/0608025

Measuring elliptic flow fluctuations

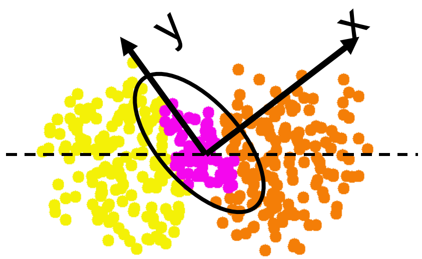
Observed v2 distribution



True v2 distribution

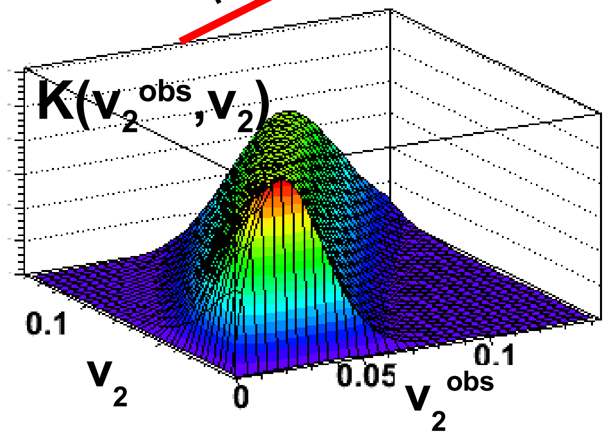


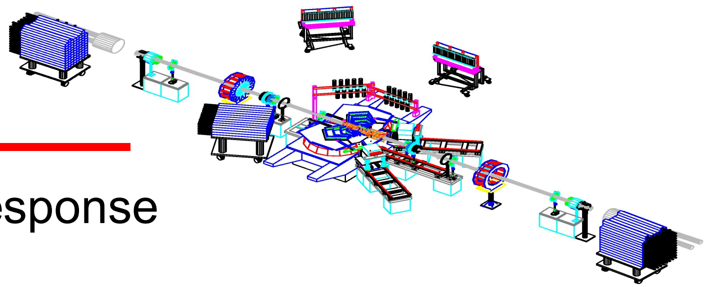
Participant eccentricity



$$g(v_2^{obs}) = \int_0^{\infty} K(v_2^{obs}, v_2) f(v_2) dv_2$$

Kernel

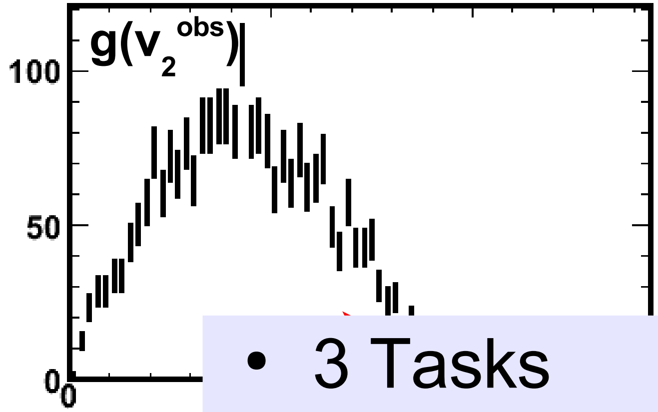




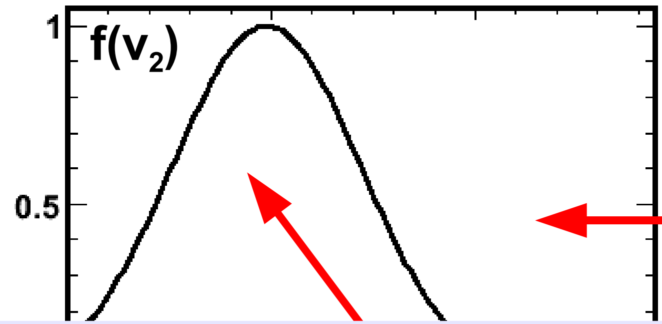
Detector response

Measuring elliptic flow fluctuations

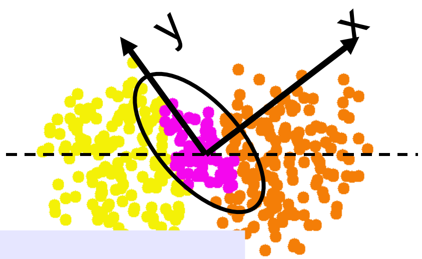
Observed v2 distribution



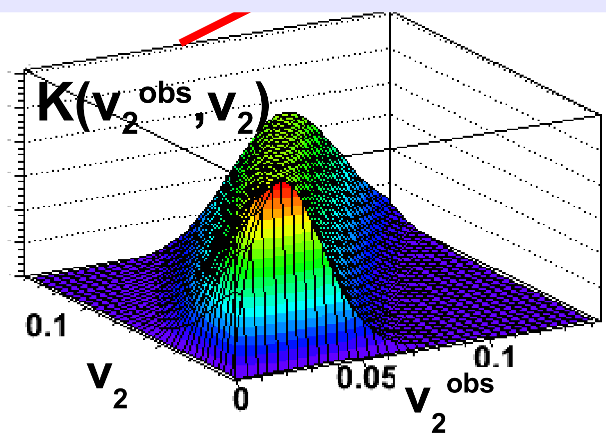
True v2 distribution

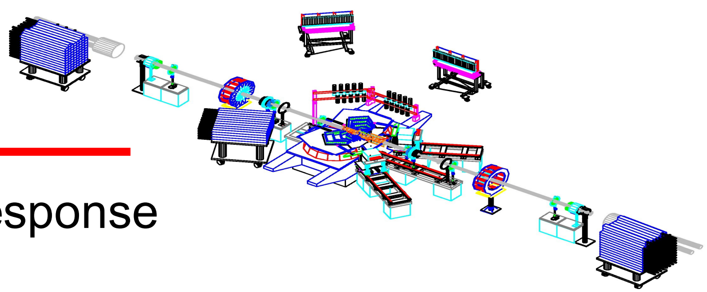


Participant eccentricity



- 3 Tasks
  - Measurement of v_2^{obs} event-by-event: $g(v_2^{obs})$
  - Construction of the kernel: $K(v_2^{obs},v_2)$
  - Extraction of dynamical fluctuations: $f(v_2)$





Detector response

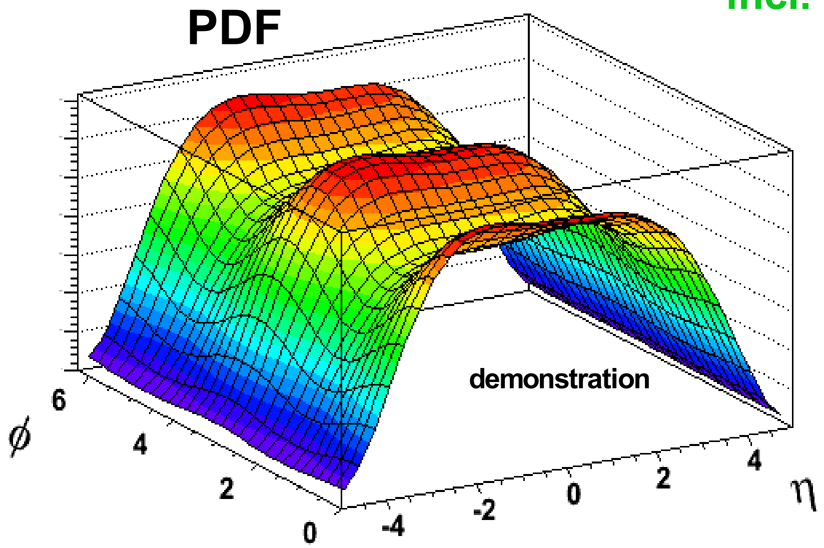
Event-by-event measurement of v_2^{obs}

- Probability Distribution Function (PDF) for hit positions:

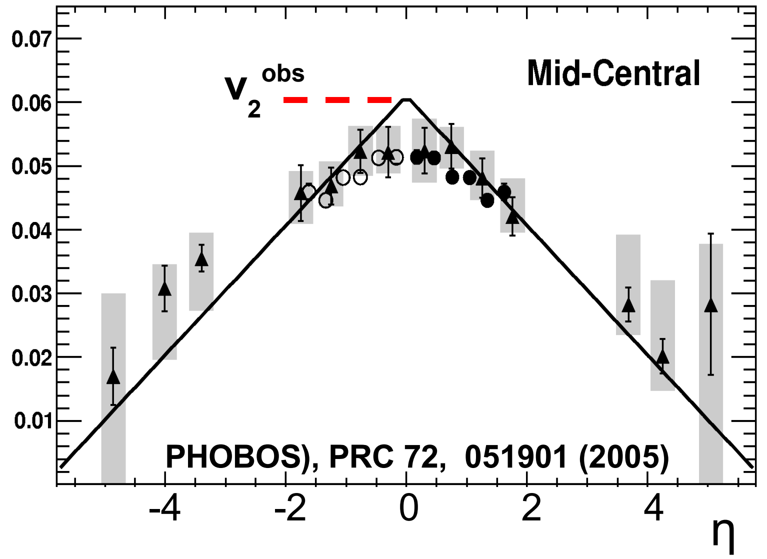
$$P(\eta,\phi;\ v_2^{obs},\phi_0)=p(\eta)[1+2v_2(\eta)\cos(2\phi-2\phi_0)]$$

Normalization incl. acceptance

Probability of hit in (φ,η)



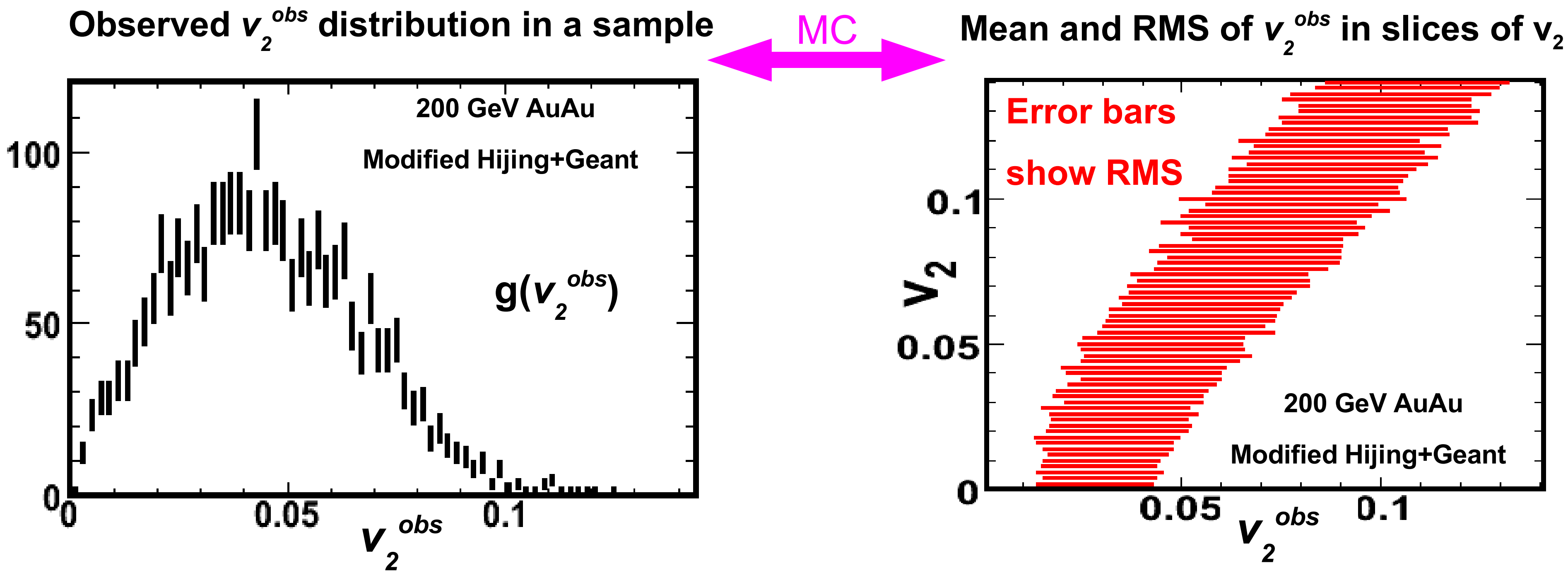
$$v_2(\eta)=v_2^{obs}\left(1-\frac{|\eta|}{6}\right)$$



- Maximize the likelihood function

$$L(v_2^{obs},\phi_0)=\prod_{i=1}^{n}P(\eta_i,\phi_i;\ v_2^{obs},\phi_0)$$

Determining the kernel



- Construction of kernel: "Measure" v_2^{obs} distribution in bins of v_2 in MC
- 2 small complications
  - Kernel really depends on multiplicity (centrality): $K(v_2^{obs}, v_2, n)$
    - n = number of hits on the detector
    - Determine v_2^{obs} distribution in bins of v_2 and n.
  - Deal with low statistics in bins by determining smooth functions

Extracting dynamical fluctuations

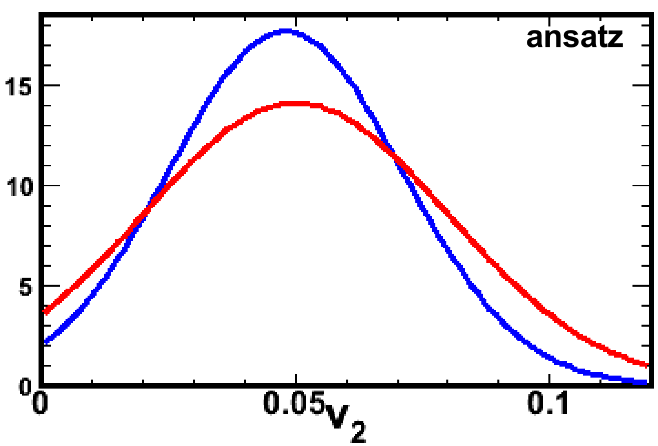
$$g(v_2^{obs}) = \int_0^\infty K(v_2^{obs}, v_2)\, f(v_2)\, dv_2$$

known

Gaussian **Ansatz**:

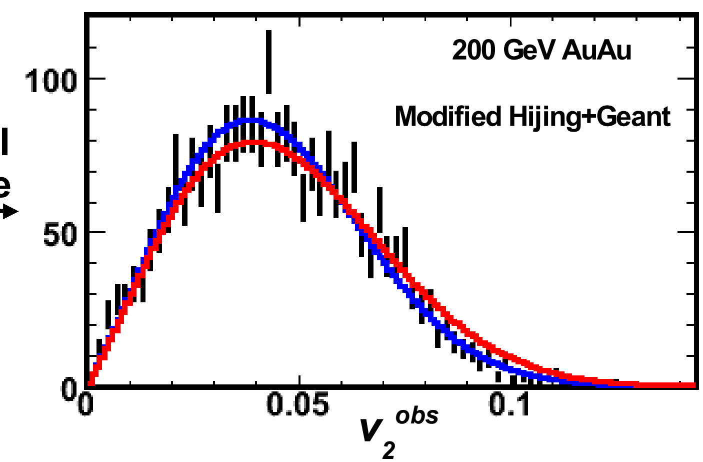
$$f(v_2) = \exp\left[\frac{-(v_2 - \langle v_2 \rangle)^2}{2\sigma_{v_2}^2}\right]$$

Trials for f(v_2)



Use kernel + integrate

Comparison with sample



Compare expected g(v_2^{obs}) for Ansatz with measurement

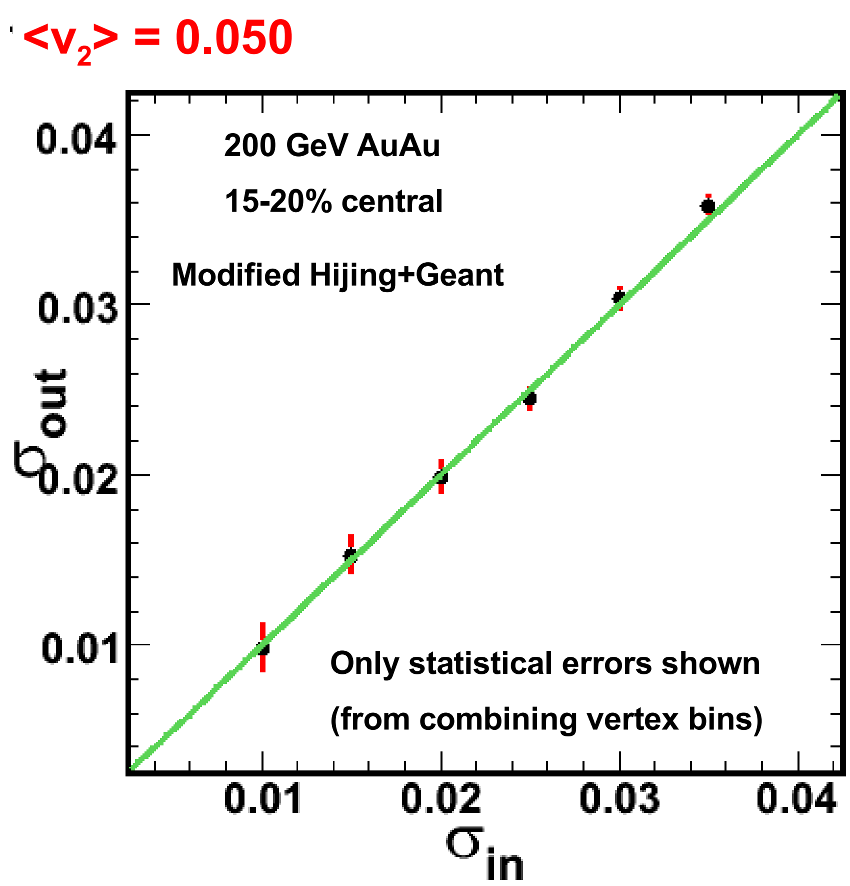
Minimum $\chi^2 \rightarrow \langle v_2 \rangle$ and σ_{v2}

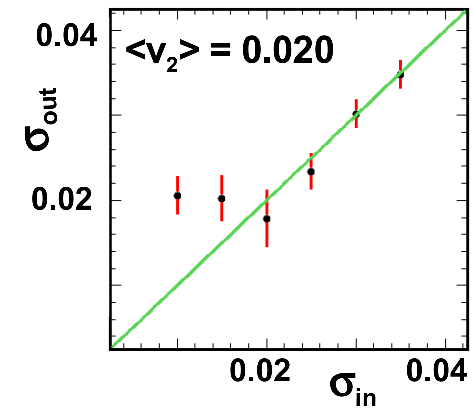
Verification of analysis method

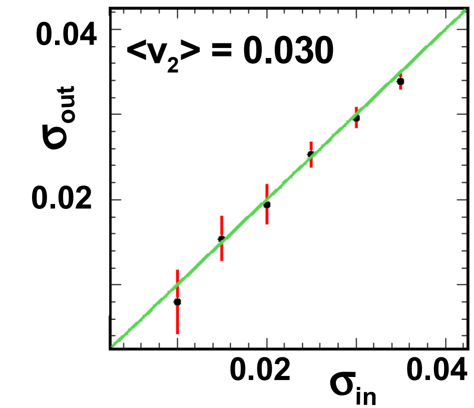
- Run analysis on Modified Hijing
  - Shape $v_2(\eta) = v_2(0) \cdot (1-|\eta|/6)$
    - Same as the assumption in our pdf
  - Analysis done in 10 collision vertex bins
    - Final results are averaged
- Kernel
  - 0-40% central events used to construct the kernel
- Input sample
  - 15-20% central events used as sample
  - $v_2(0)$ given by a Gaussian distribution
    - Same as our Ansatz

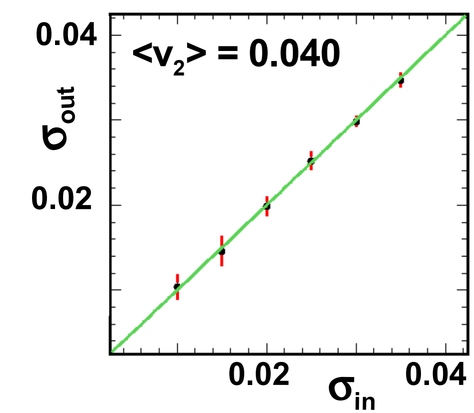
Verification of analysis method

- Run analysis on Modified Hijing
  - Input fluctuations are successfully reconstructed



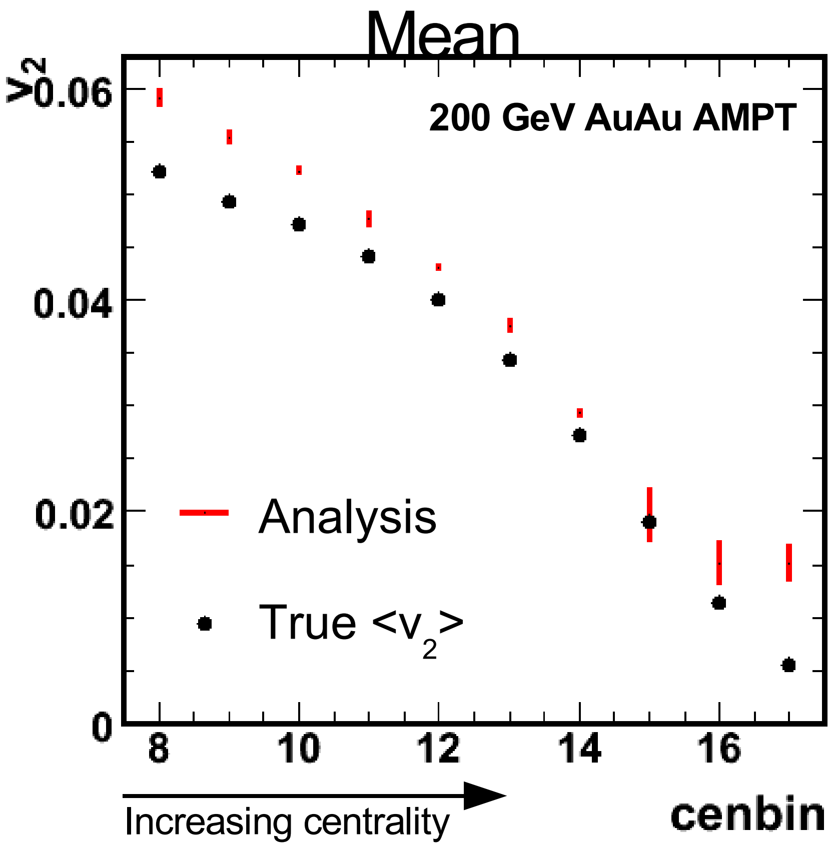


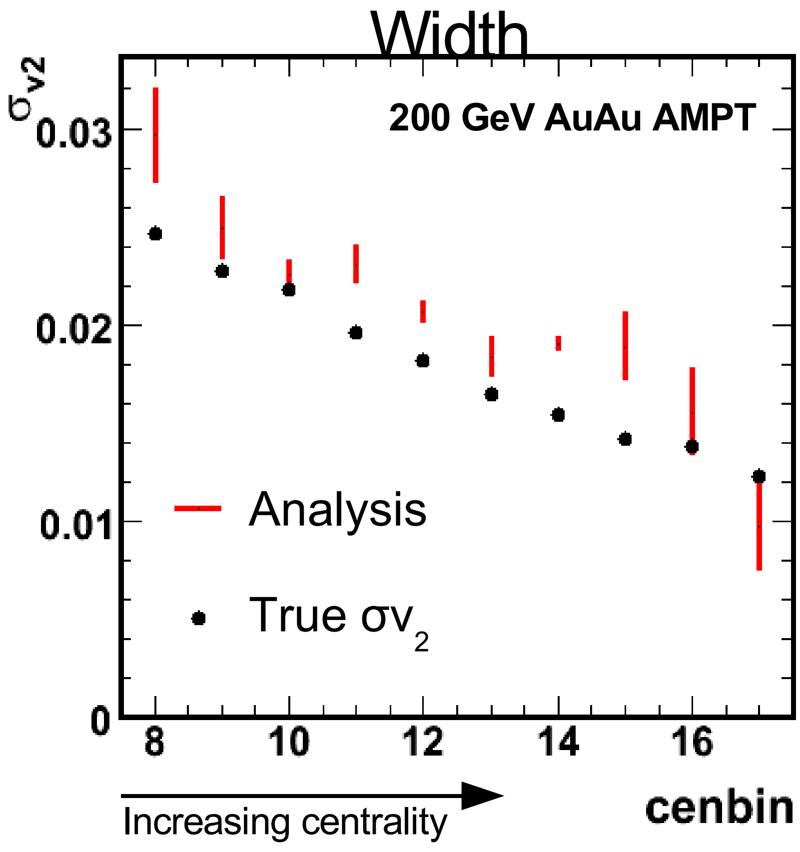




Systematic verification with AMPT

- Run analysis on AMPT
  - Kernel is constructed from Hijing as before
  - Input fluctuations are successfully reconstructed

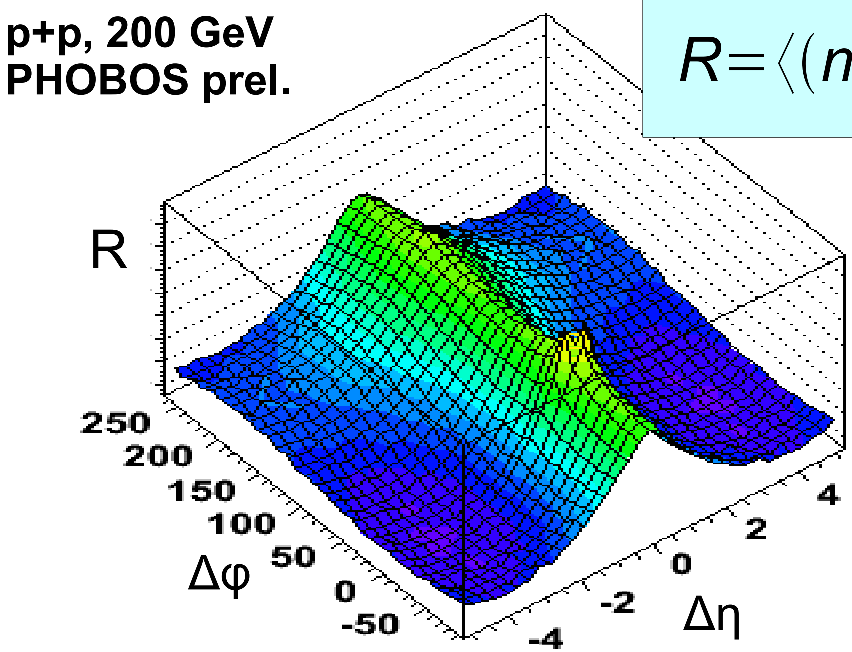




Non-flow contributions to flow fluctuations

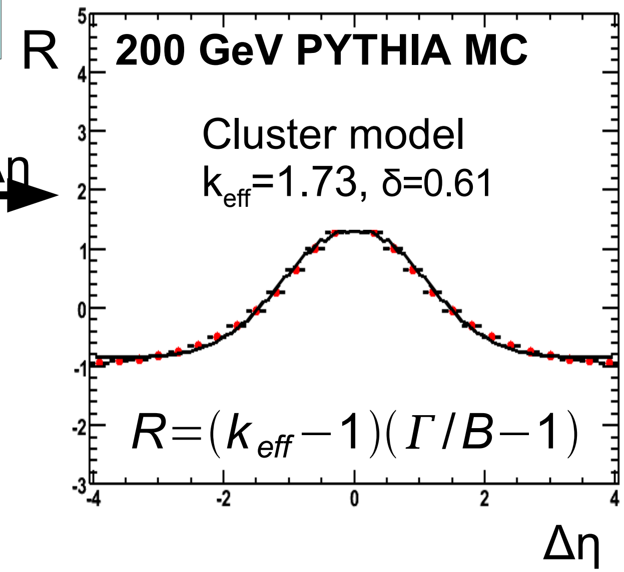
- Non-flow correlations mimic dynamical fluctuations and will contribute to the width of the v_2 distribution
  - The resolution of our method depends on the kernel
    - Modified Hijing: particle multiplicity defines the resolution
    - Data (AMPT): clusters flow and therefore the cluster multiplicity determines the resolution
  - The fluctuations we measure are real (present at particle level) but might not be the ones we are after
- Kernel could compensate for non-flow effects if they are correctly described by the MC used to construct it
  - Construct and tune MC on data
  - Two-particle correlation measurements can be used as input to disentangle the different contributions

Two-particle angular correlations in p+p



$$R = \langle (n-1)\left(\frac{F_n}{B_n} - 1\right) \rangle$$

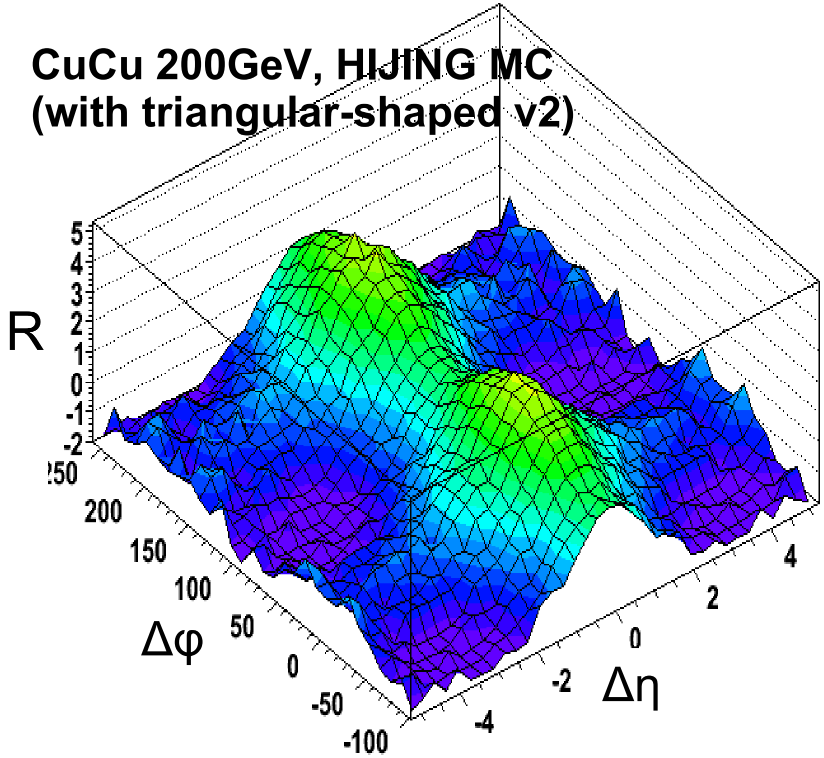
Projection onto Δη
(altern. onto Δφ)



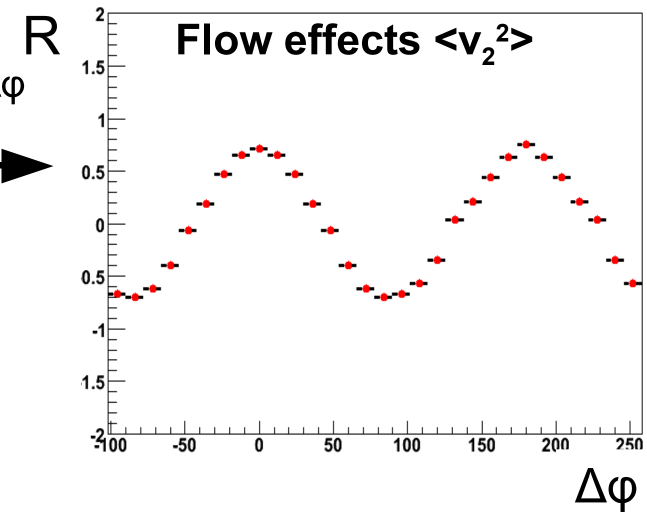
- Construction of R, event-by-event, weighted by event multiplicity
  - Full φ and large |η|≤3 coverage (|η|≤5 for future studies)
  - Single hit in silicon layer instead of particle information
    - Need special care for secondary contamination
  - Study soft physics (No trigger particle)
    - Clusters

Will be published soon

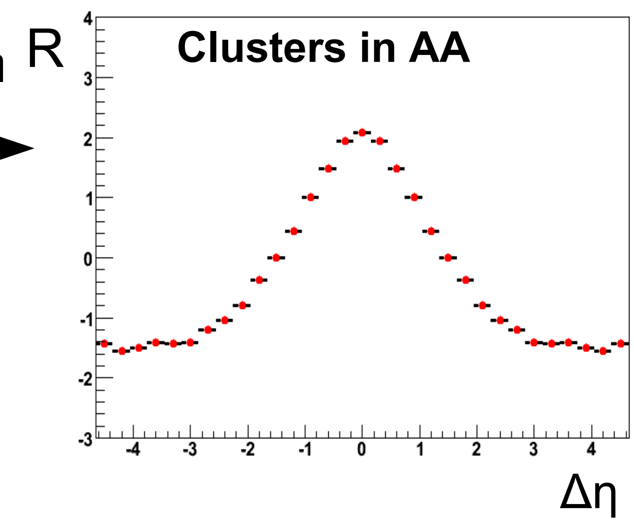
Two-particle angular correlations in A+A



Projecting onto Δφ from 0<|Δφ|<180



Projecting onto Δη from 0<|Δη|<6



Comprehensive study of two-particle correlations in p+p, d+A and A+A will help distangle different effects in HI systems

Summary and Perspectives

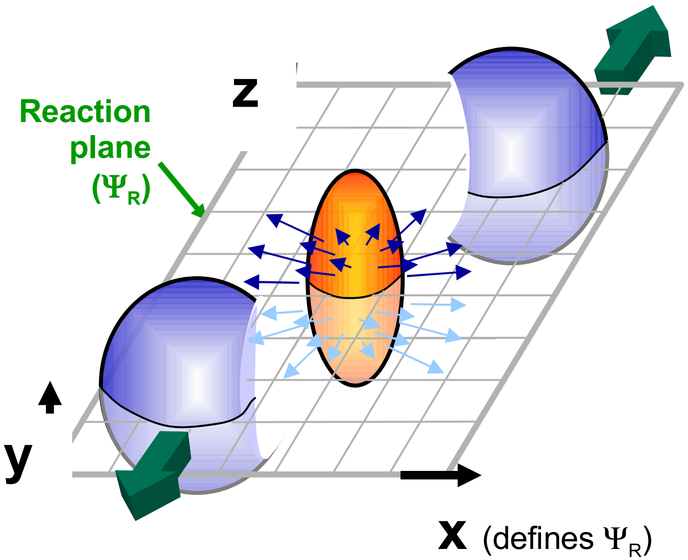
- Mean v_2 measurements
  - Large elliptic flow in Cu+Cu compared to Au+Au
- Eccentricity fluctuations
  - Participant eccentricity connects elliptic flow in small system to initial geometry fluctuations
  - Robustness studies wrt to changes in Glauber geometry
  - Work in progress with U.Heinz to investigate/check further properties
- Elliptic flow fluctuations
  - A method to measure elliptic flow fluctuations has been developed
  - Fluctuations in MC simulations are successfully reconstructed
  - Study systematic uncertainties due to MC/DATA differences
- Two-particle angular correlations
  - Properties of clusters in p+p, d+A, and A+A
  - Estimate non-flow contribution to elliptic flow fluctuations
    - Tune MC

Backup

Anisotropic flow

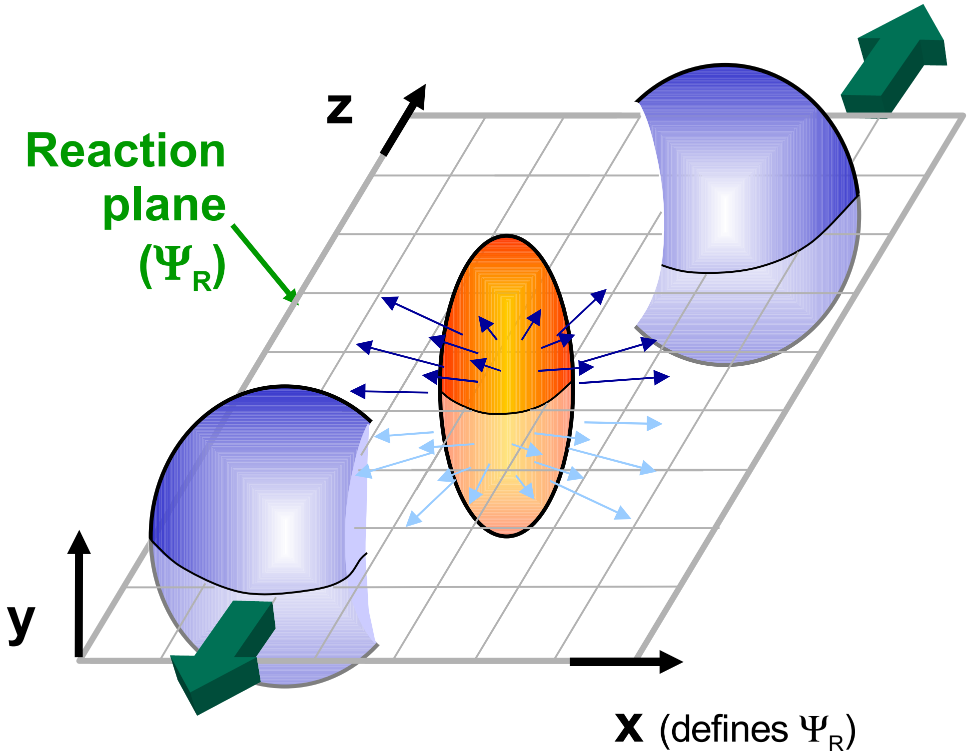
$$E\frac{d^3N}{d^3p}=\frac{1}{2\pi}\frac{d^2N}{p_t\,dp_t\,dy}\left[1+2\sum_{n=1}^{\infty}v_n(p_t,y)\cos(n\phi-n\Psi_R)\right]$$

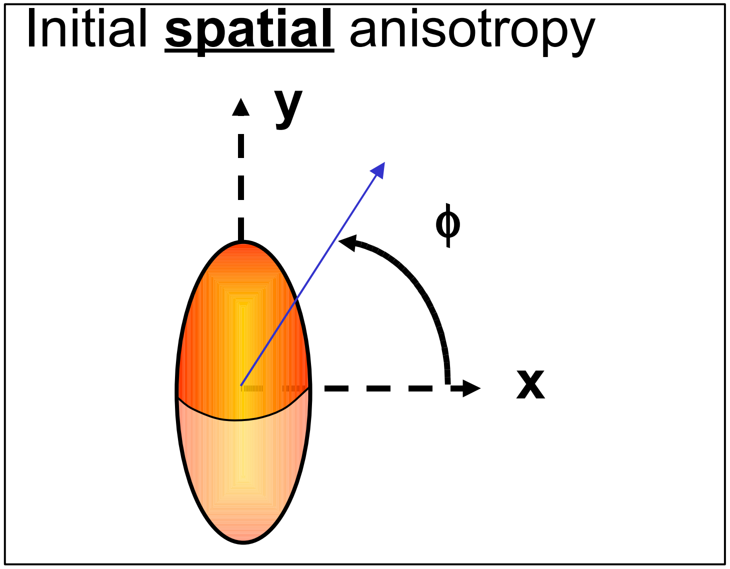
$$v_n=\langle\cos(n\phi-n\Psi_R)\rangle$$

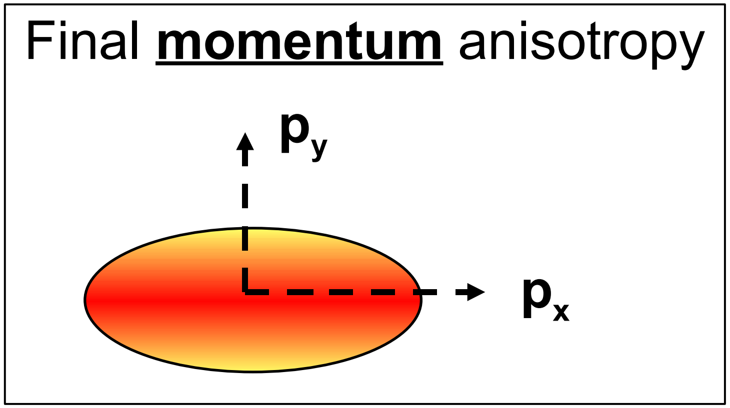


- Anisotropic flow ≡ azimuthal correlations with reaction plane
  - Consequence of thermalization
  - Final-state reinteractions
  - Typically described by hydro-dynamics (but flow does not necessary imply it)
- Non-flow ≡ contribution to v_n from azimuthal correlations between particles (HBT, resonances, jets)

Direct (v_1) and elliptic (v_2) flow







$$dN/d(\phi - \Psi_R) = N_0 (1 + 2v_1 \cos(\phi - \Psi_R) + 2v_2 \cos(2(\phi - \Psi_R)) + \dots)$$

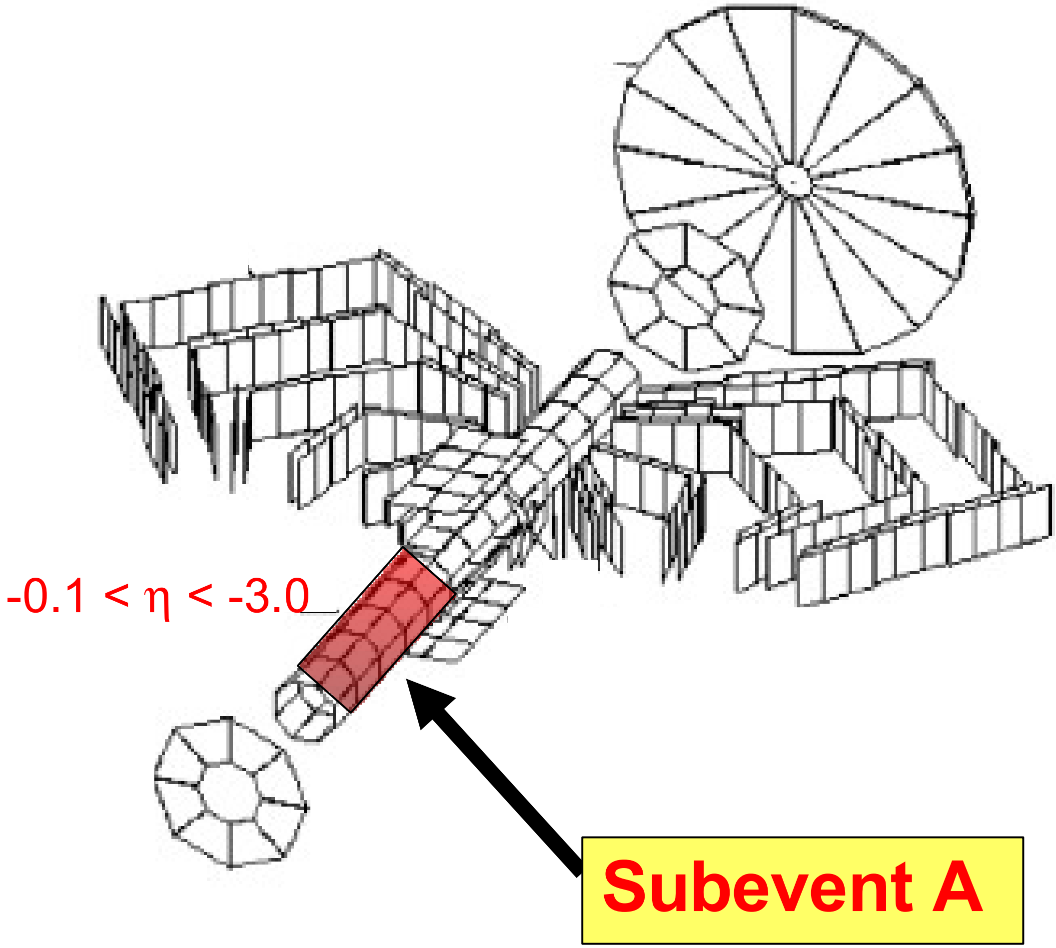
Flow measurement in PHOBOS

- Reaction-Plane / Subevent technique
  - Correlate reaction plane determined from azimuthal pattern of hits in one part of the detector with information from other parts of the detector
    - Hits
    - Tracks

$$\tan(2\psi_A) = \frac{\langle \sin(2\phi) \rangle_A}{\langle \cos(2\phi) \rangle_A}$$

$$v_2^{obs} = \langle \cos(2\phi - 2\psi_A) \rangle_B$$

$$v_2 = \frac{\langle v_2^{obs} \rangle_{events}}{\sqrt{\langle \cos(\psi_A - \psi_B) \rangle_{events}}}$$



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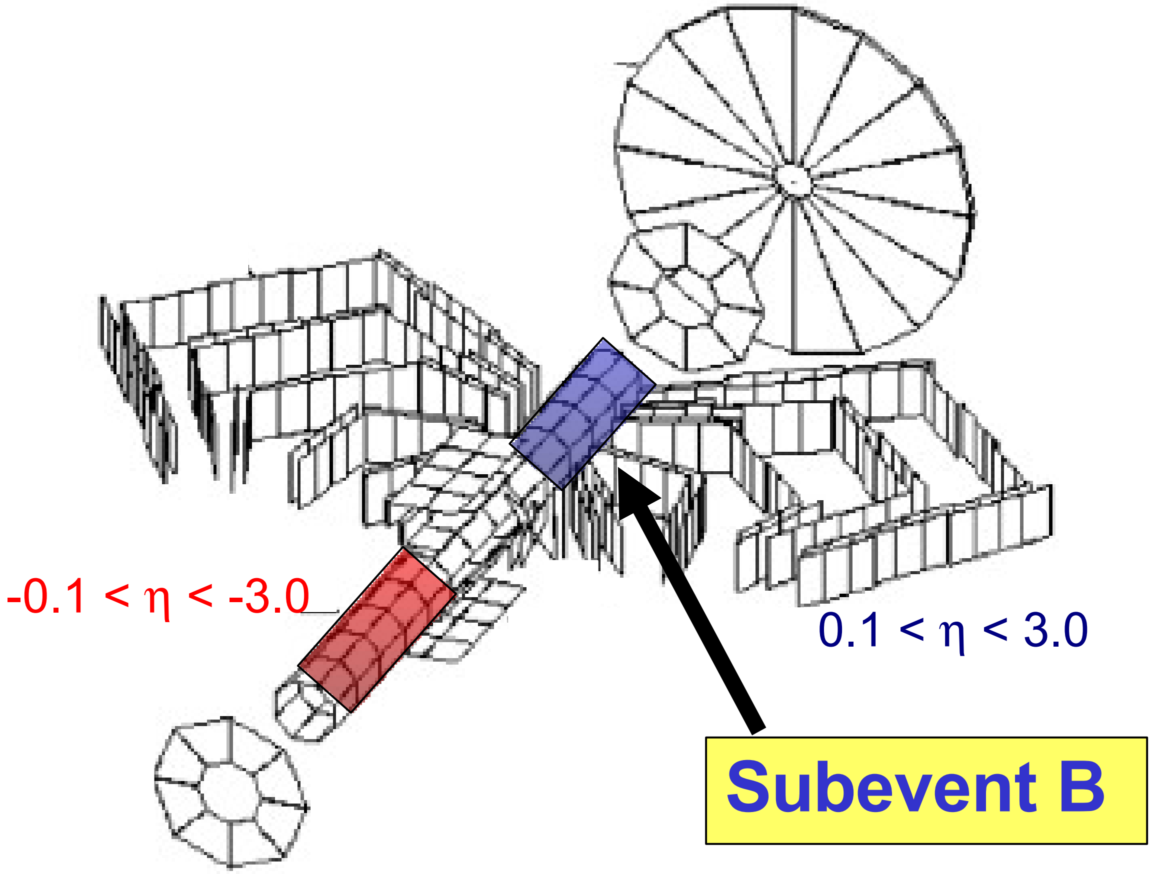

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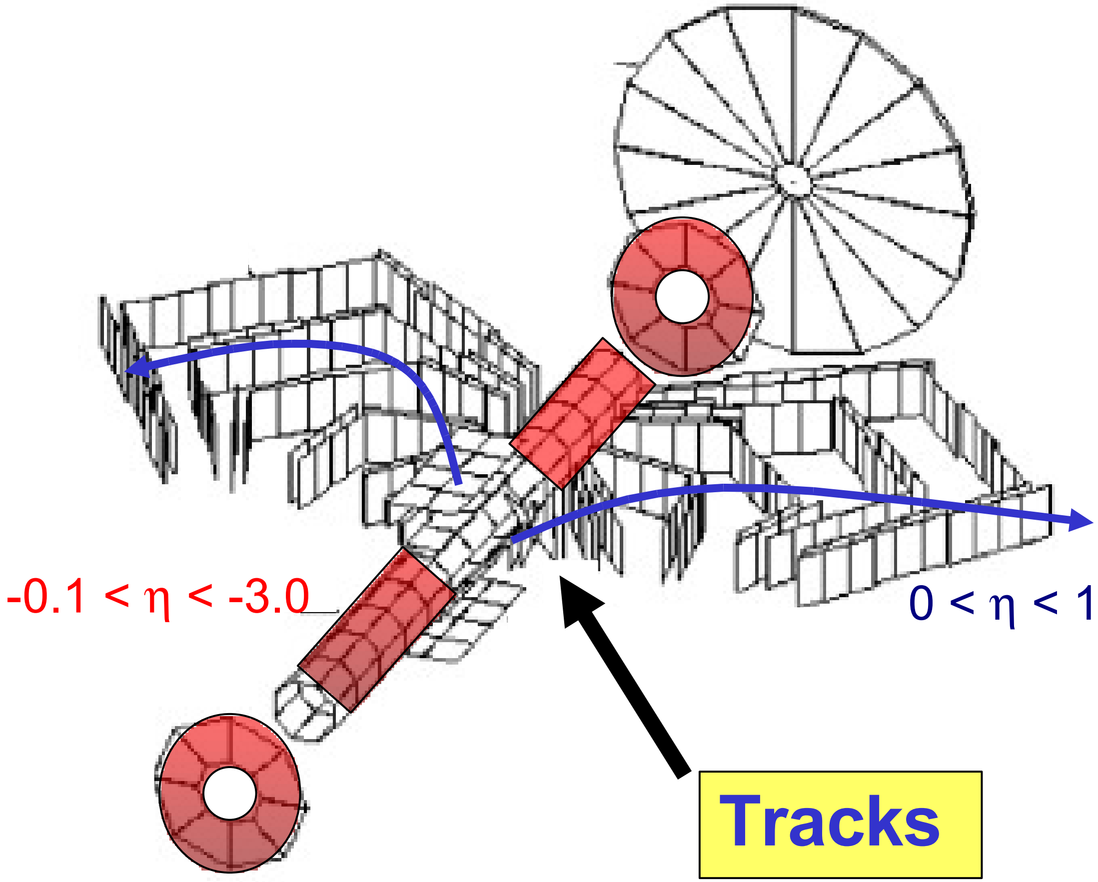

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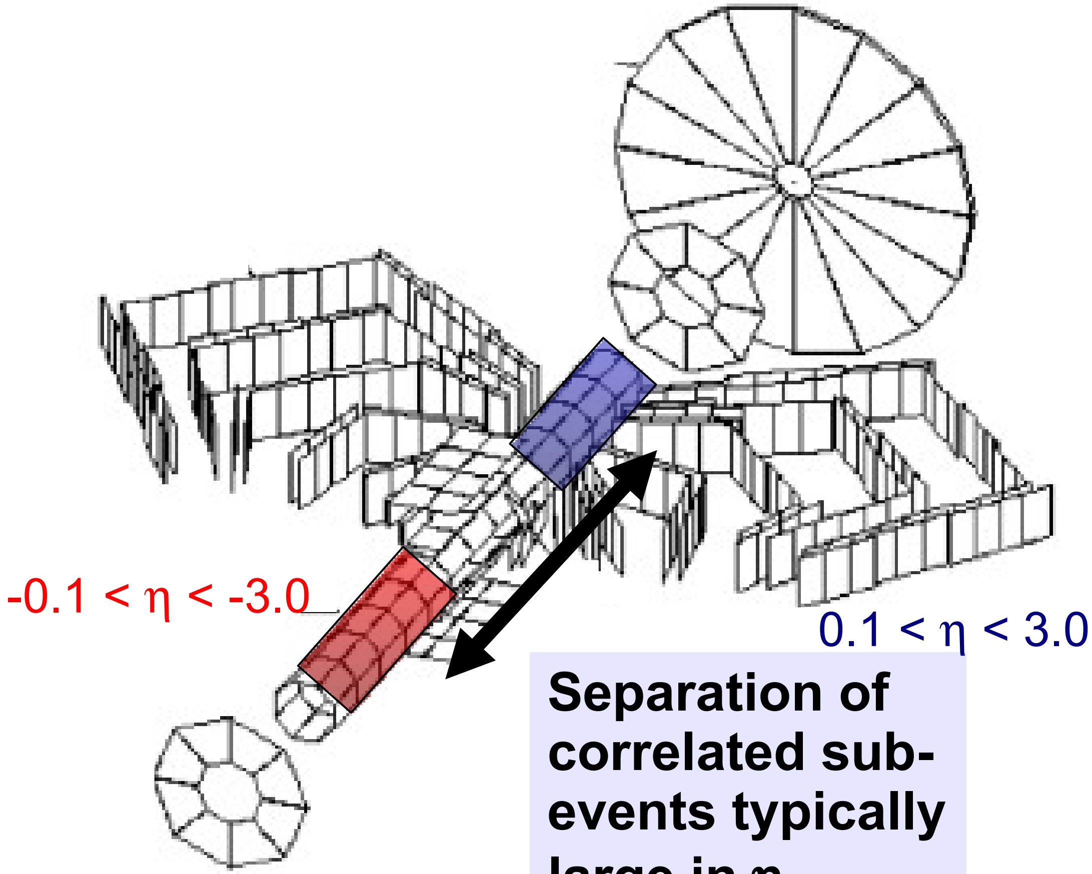
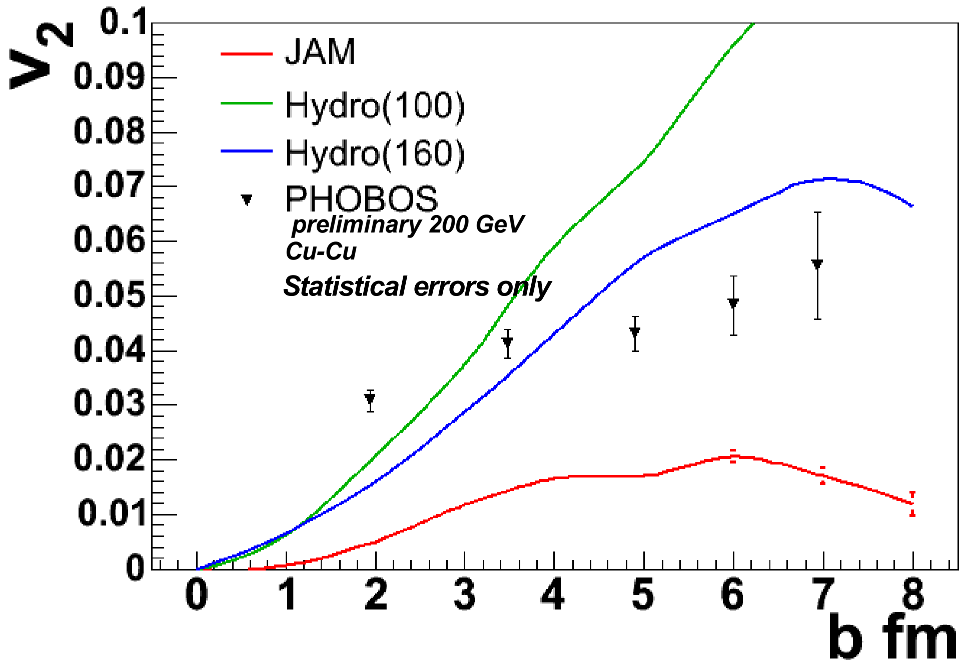
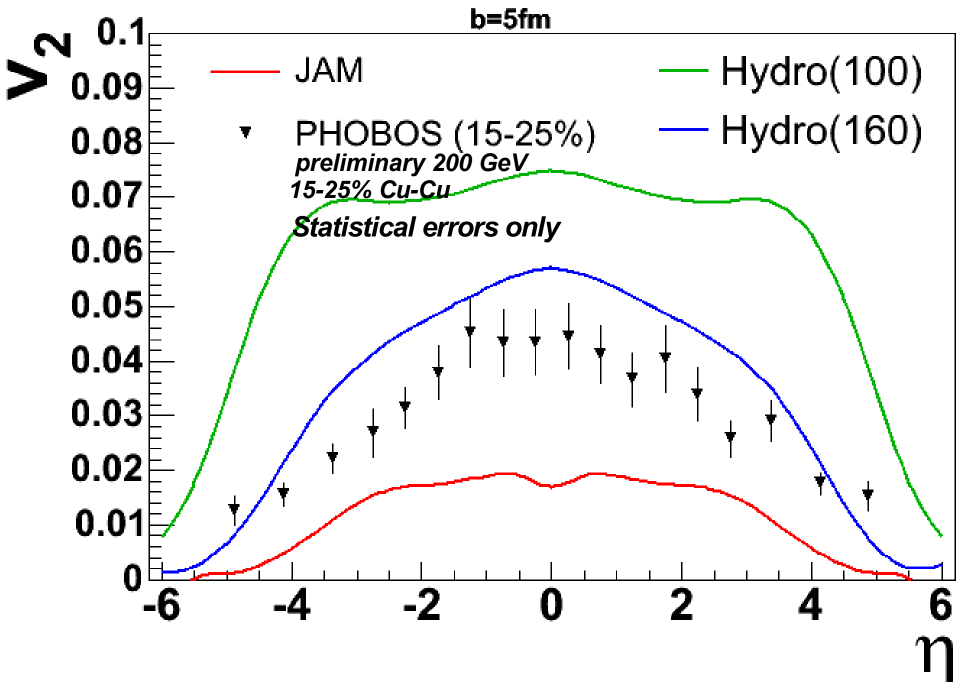


Separation of correlated sub-events typically large in η

Elliptic flow – Cu+Cu results



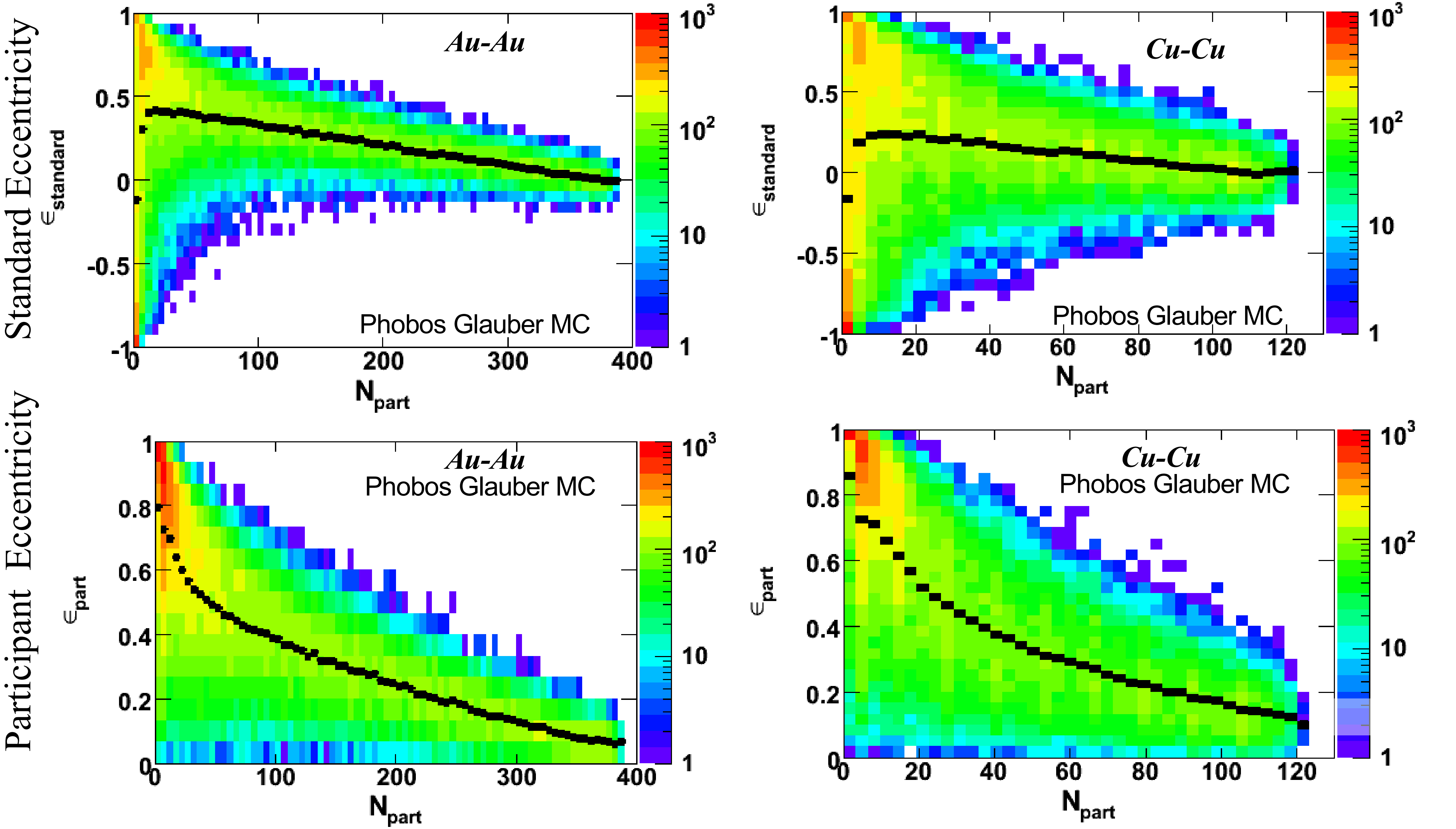


Cu+Cu more like Hydro than JAM hadron string cascade model

Models from Hirano et al., nucl-th/0506058; Here JAM uses a 1 fm/c formation time. Hydro (160) has kinetic freezeout temperature at 160 MeV

Standard and participant eccentricity

Mean eccentricity shown in black

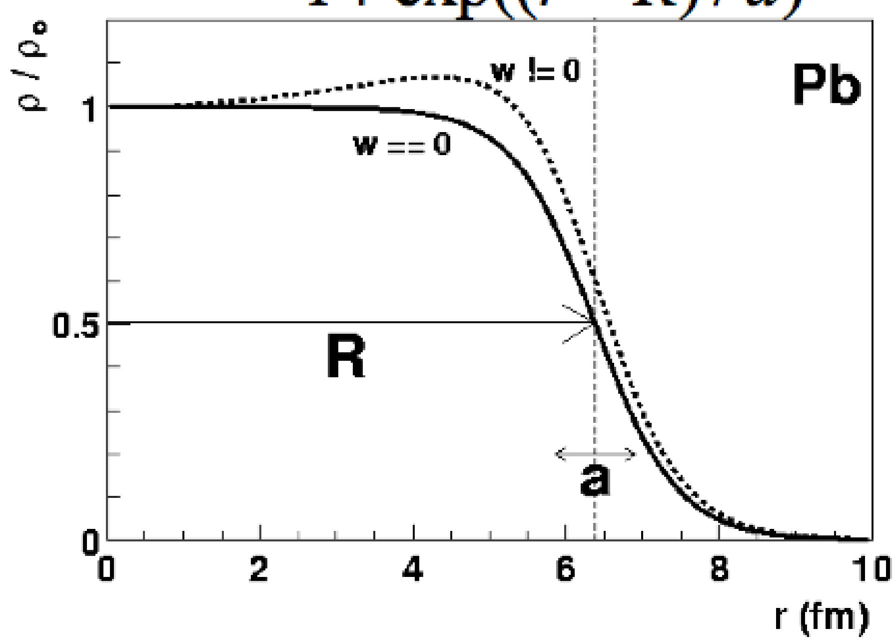


Greater fluctuations in Cu+Cu. Positive fluctuations lead to non zero mean.

Glauber Parameters Changed

Systematic Source	Standard	How Much We Vary
Nucleon-nucleon cross-section	42 mb (for 200GeV)	30 mb (<20GeV) 45 mb (>200GeV)
Nuclear skin depth	0.535fm(Au)0.596fm(Cu)	±10%
Nuclear radius	6.38fm (Au)4.2fm (Cu)	±10%
Minimum nucleon separation (center-to-center)	0.4fm (like HIJING)	0fm 0.8fm

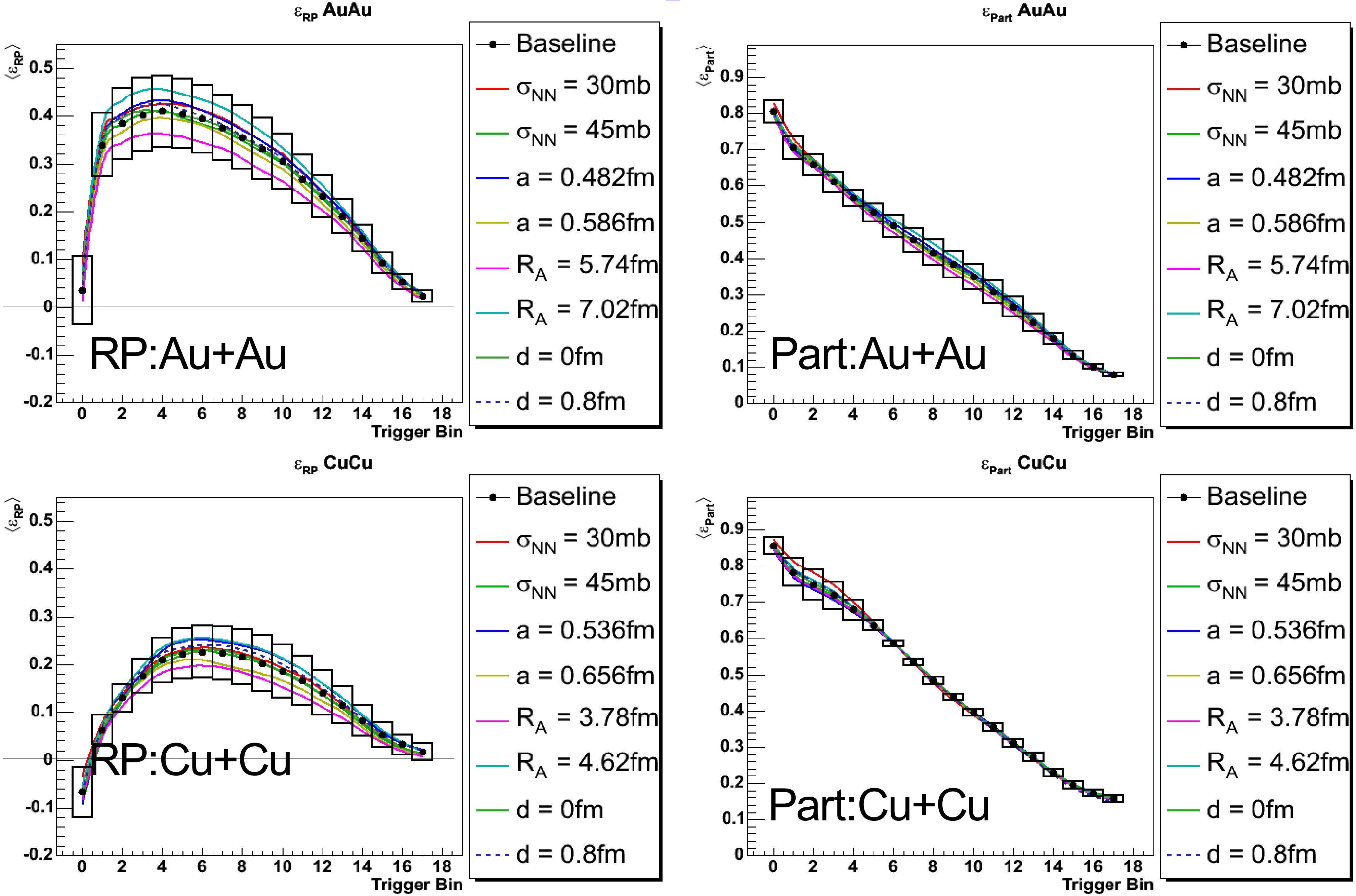
$$\rho(r) = \frac{\rho_0 \left(1 + wr^2 / R^2\right)}{1 + \exp((r - R)/a)}$$



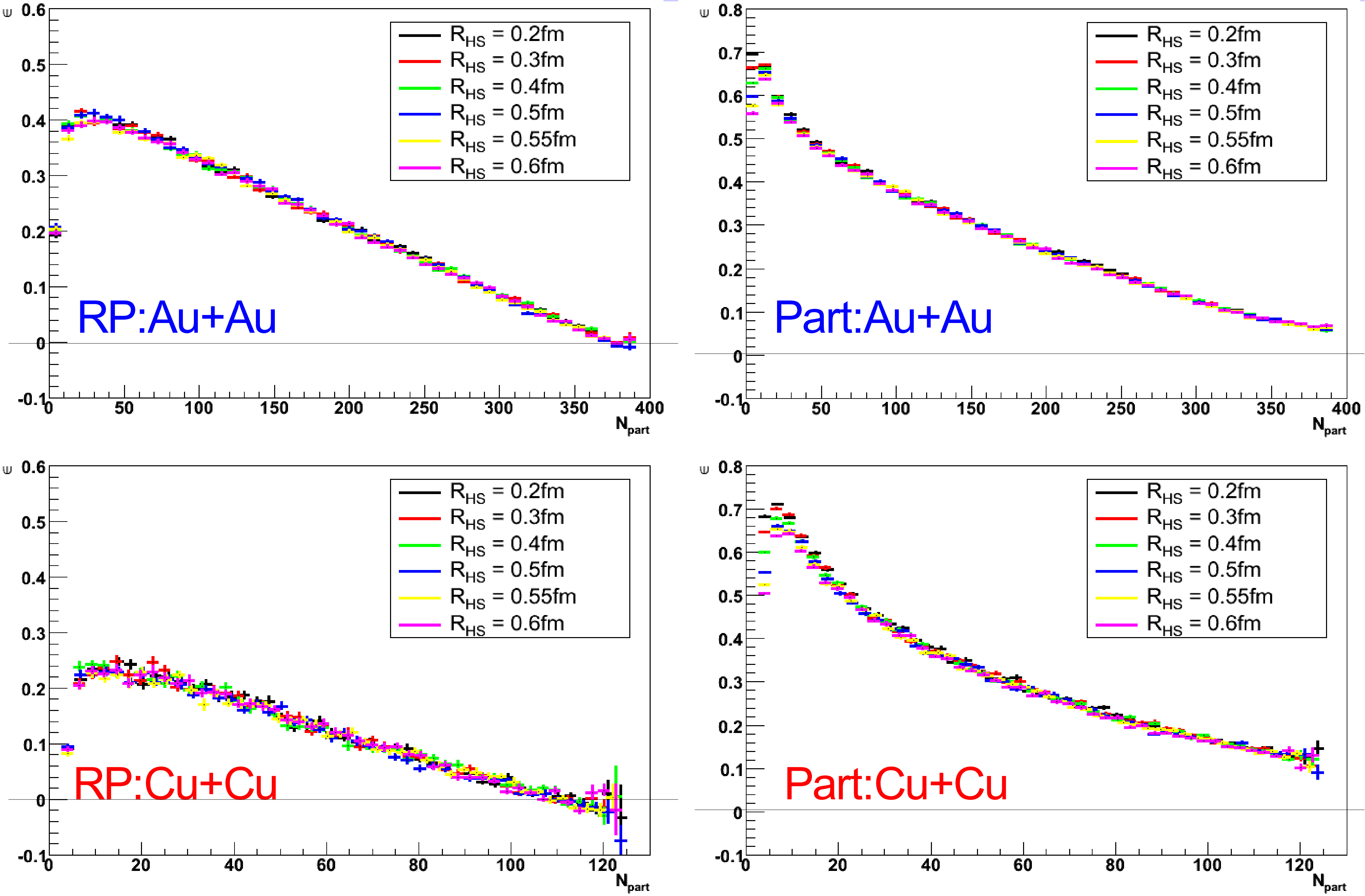
Nucleus	A	R	a	w
C	12	2.47	0	0
O	16	2.608	0.513	-0.051
Al	27	3.07	0.519	0
S	32	3.458	0.61	0
Ca	40	3.76	0.586	-0.161
Ni	58	4.309	0.516	-0.1308
Cu	63	4.2	0.596	0
W	186	6.51	0.535	0
Au	**197**	**6.38**	**0.535**	**0**
Pb	208	6.68	0.546	0
U	238	6.68	0.6	0

H. DeVries, C.W. De Jager, C. DeVries, 1987

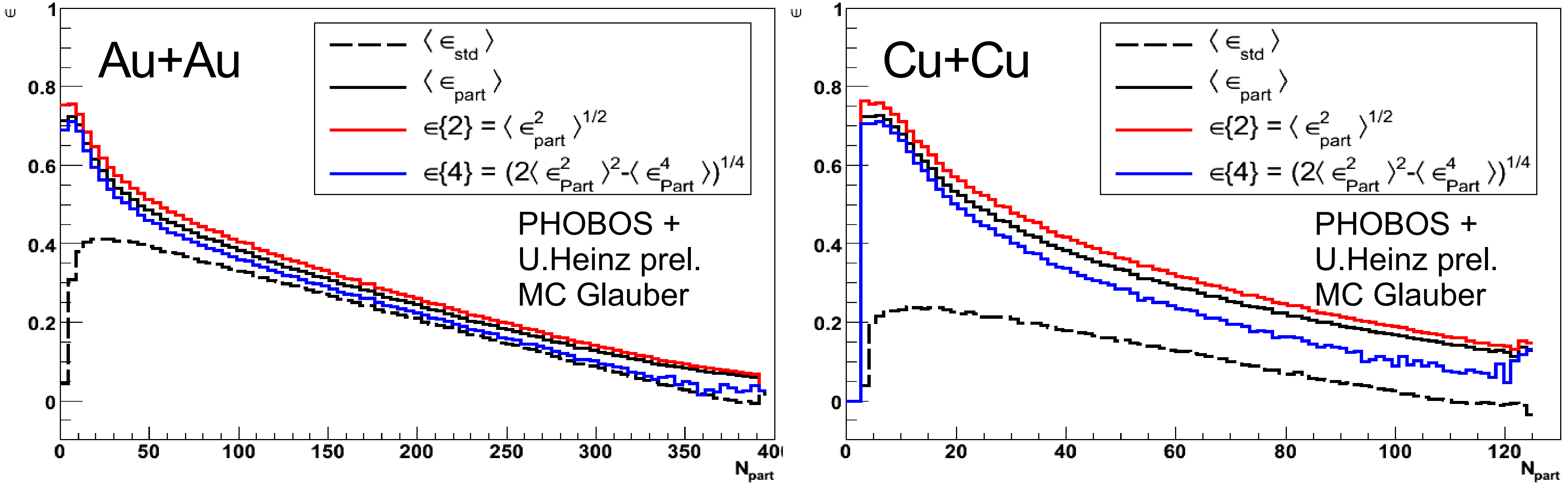
Robustness with geometry variables



Hard-sphere smearing (Npart weighted)



Moments of eccentricity

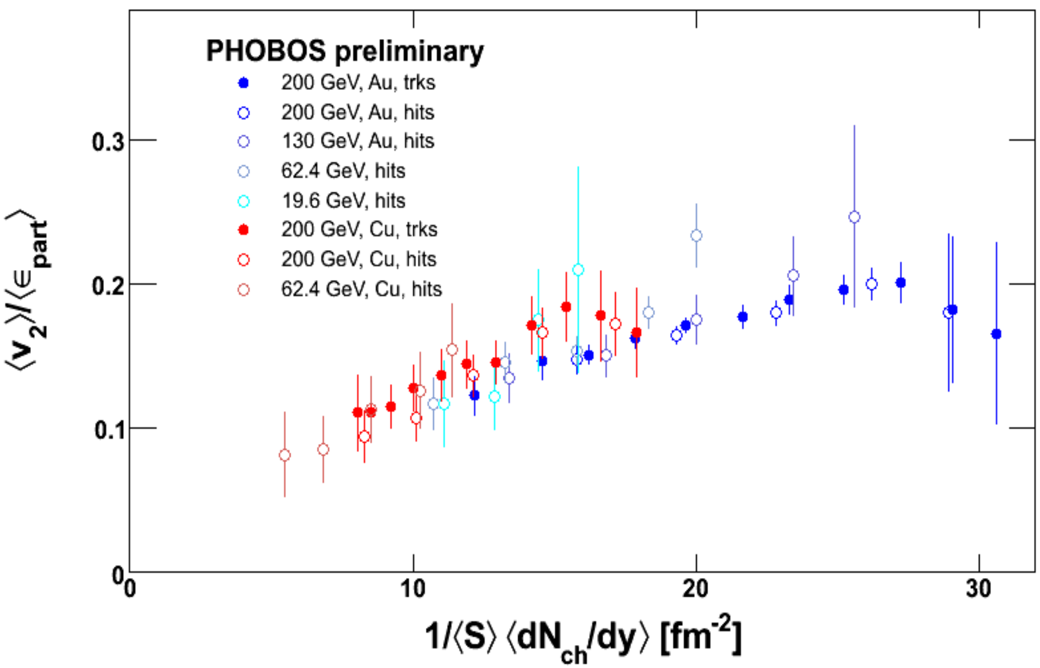


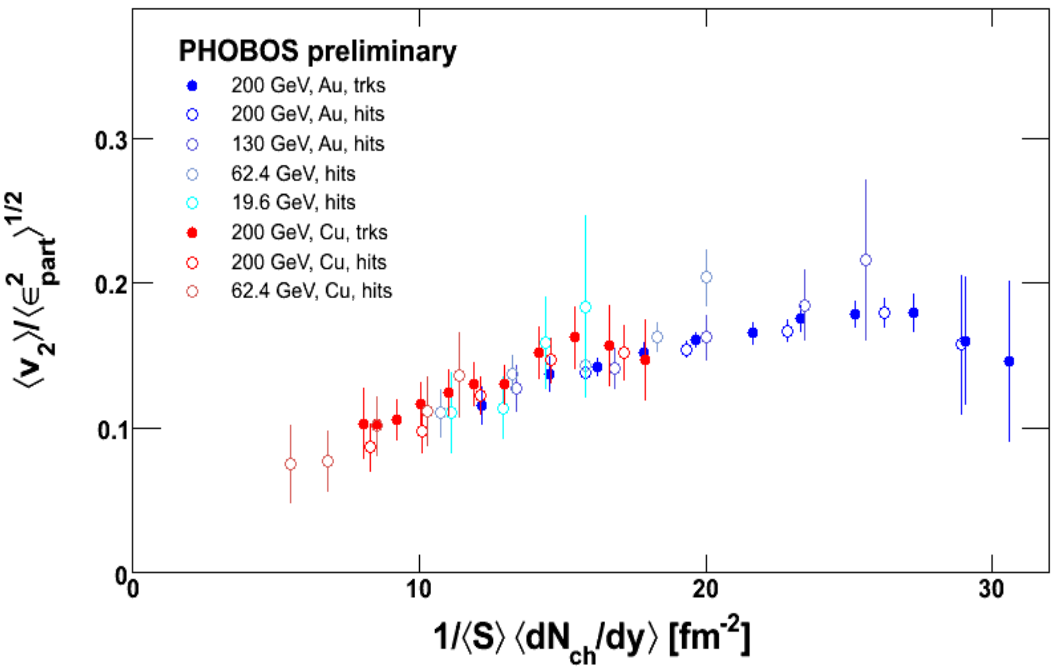
- If v_2 fluctuates proportional to ϵ_{part}, then use cumulants for v_2/ϵ ratio
  - $\epsilon\{2\}$ rather similar to ϵ_{part}, unlike $\epsilon\{4\}$ that is between ϵ_{std} and ϵ_{part}
- Outstanding question:
  To what extent is PHOBOS $v_2\{RP\}$ sensitive to higher moments in v2
  - Answer probably dependent on system

M.Miller, R.Snellings, nucl-ex/0312008
R.Bhalerao, J.Ollitrault, nucl-th/0607009

Voloshin plots wrt to participant eccentricity

- Open question (issue): v2{RP} ≈ v2{2} (claimed by Star, Bhalerao/Ollitrault)

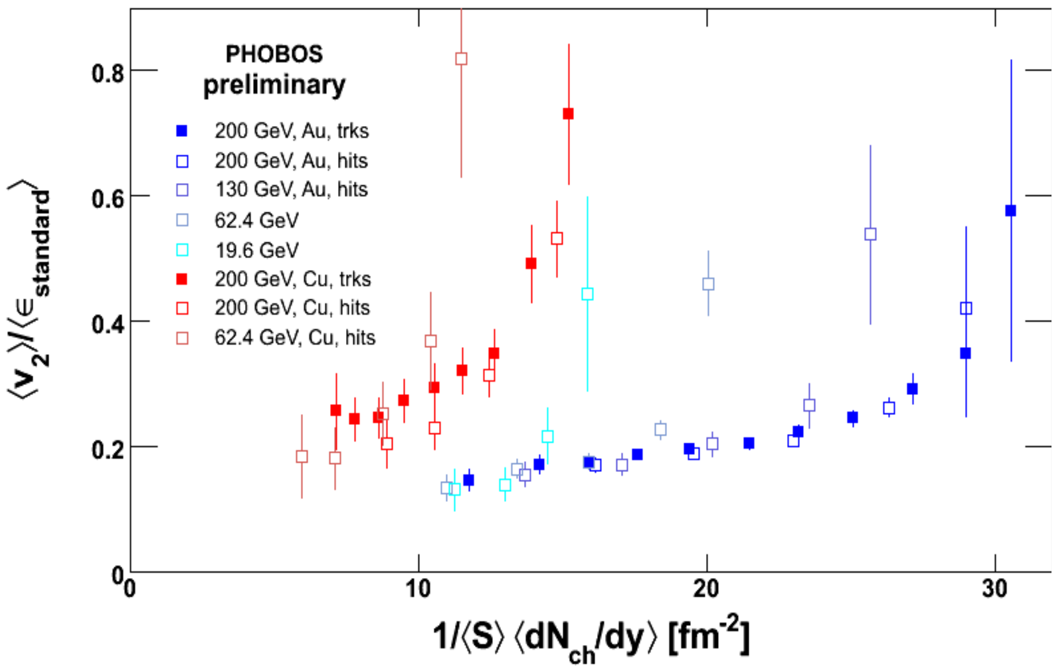


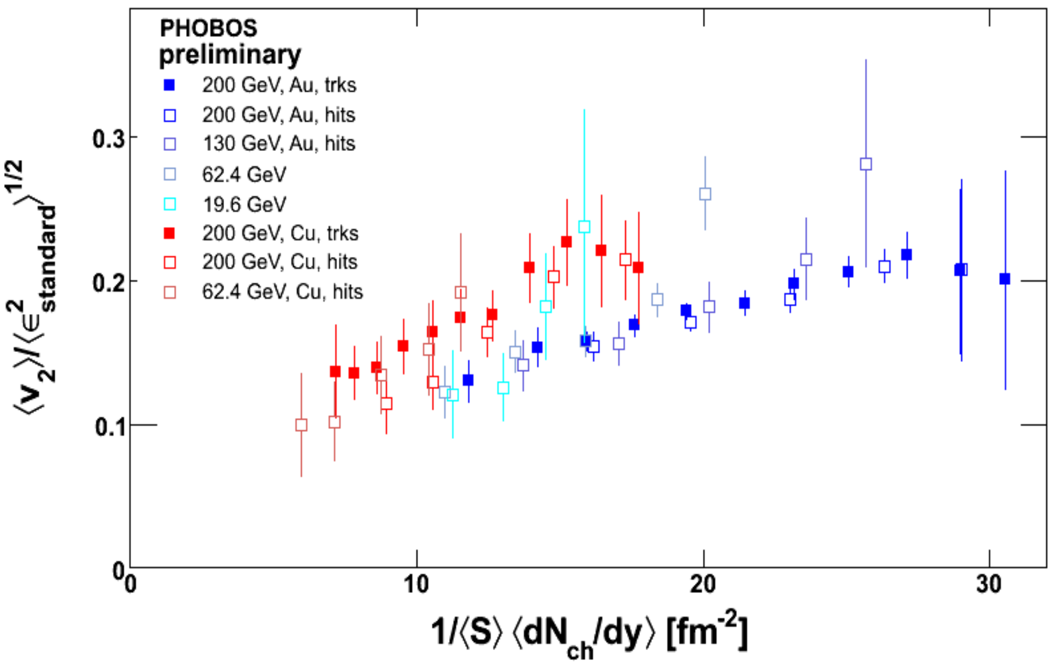


- Quantitative "improvement" of the scaling

Voloshin plots wrt to reaction-plane eccentricity

- Open question (issue): v2{RP} ≈ v2{2} (claimed by Star, Bhalerao/Ollitrault)

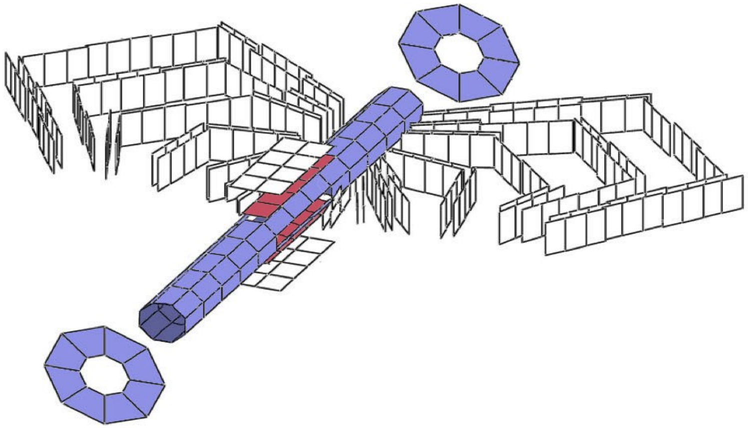


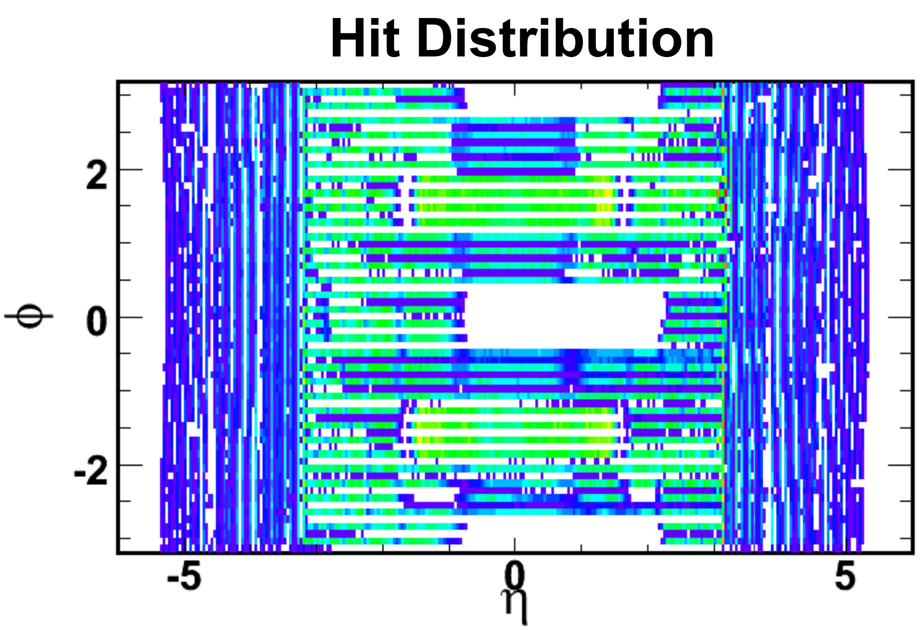


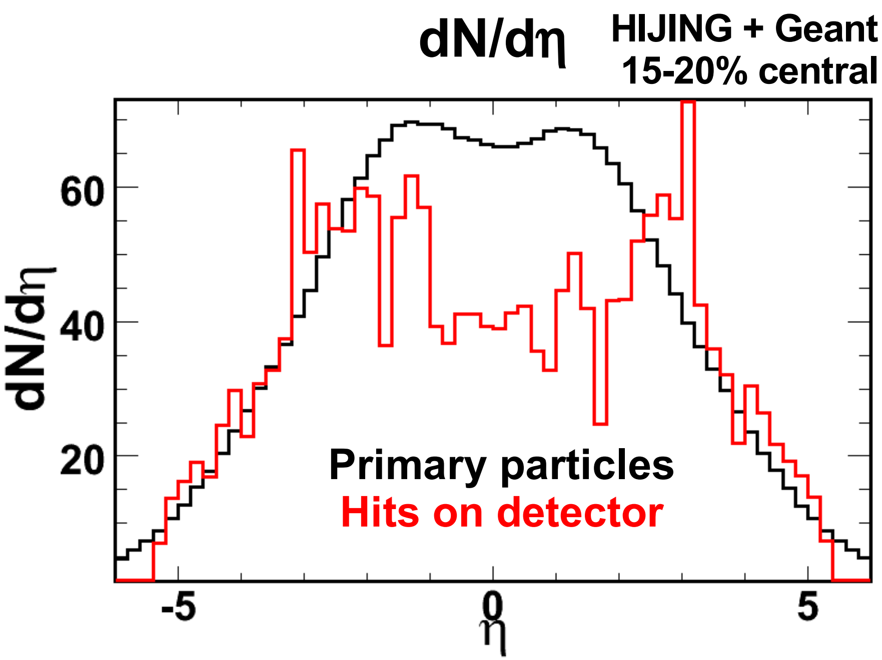
- Qualitative “improvement” of the scaling ?

Event-by-event measurement of v_2^{obs}

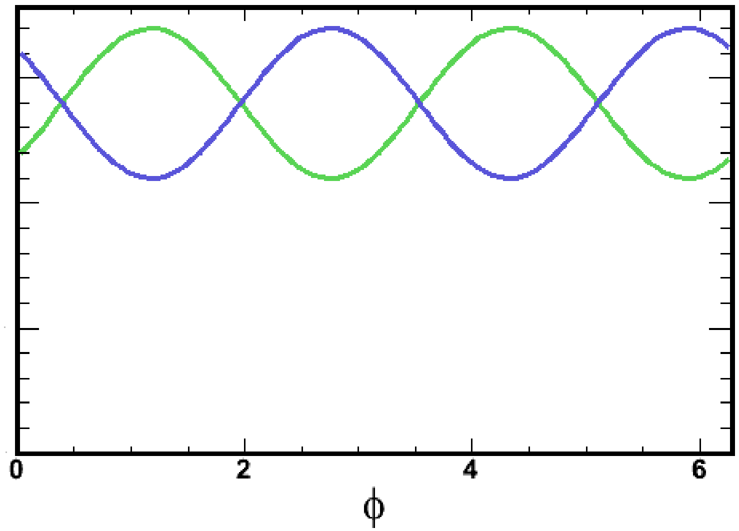
- PHOBOS Multiplicity Array
  - -5.4<η<5.4 coverage
  - -Holes / granularity differences
- Usage of all available information in event to determine single value for v_2^{obs}

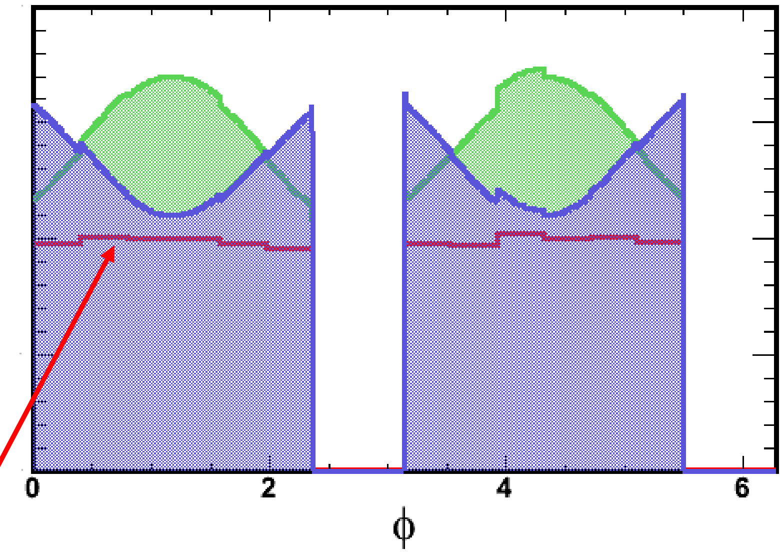






Likelihood fit normalization





Acceptance

$$s(u, \phi_0|\eta) = \int A(\eta, \phi) \cdot [1 + 2\, u\, (1 - |\eta|/6)\, cos(2(\phi - \phi_0))]\, d\phi\, d\eta$$

$$L(u, \phi_0) = \prod_{i=1}^{n} \frac{1}{s(u, \phi_0|\eta_i)} [1 + 2\, u\, (1 - |\eta_i|/6)\, cos(2(\phi_i - \phi_0))]$$

Event-by-event measurement of v_2^{obs}

$$L(v_2^{obs}, \phi_0) = \prod_{i=1}^{n} p(\eta_i)\left[1 + 2 v_2^{obs} \left(1 - \frac{|\eta_i|}{6}\right) \cos(2\phi - 2\phi_0)\right]$$

- Maximize likelihood to find "most likely" value of v_2^{obs} *(and ϕ_0)*
- Comparing values of v_2^{obs} and ϕ_0
  - PDF folded by acceptance must be normalized to the same value for different v_2^{obs} and ϕ_0's

$$p(v_2^{obs}, \phi_0; \eta) = 1 / \int_{\eta} A(\eta, \phi)\left[1 + 2 v_2^{obs} \left(1 - \frac{|\eta|}{6}\right) \cos(2\phi - 2\phi_0) d\eta d\phi\right]$$

Acceptance

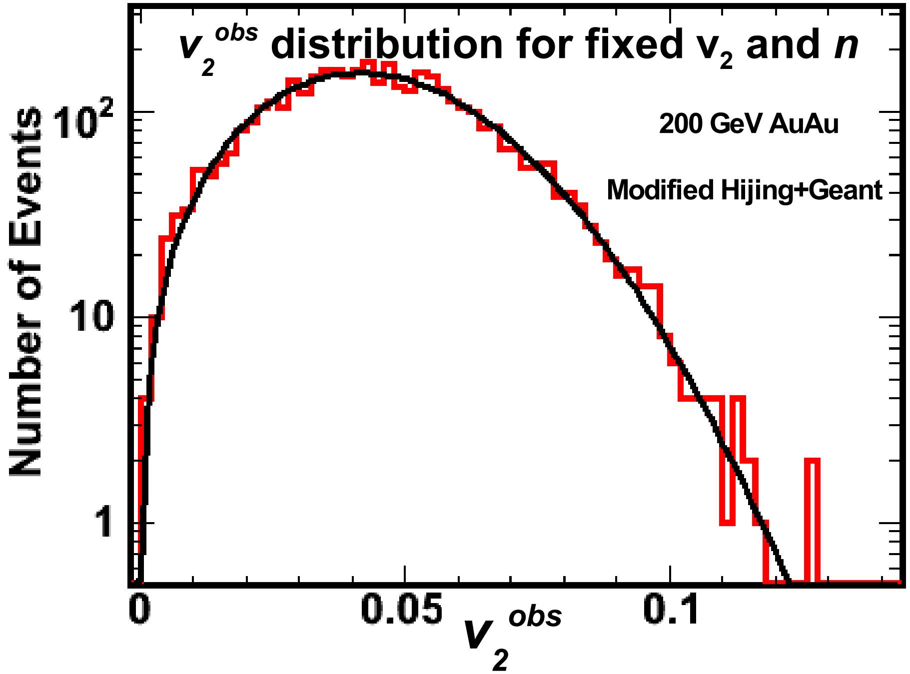
Determining the kernel

- In a single bin of v_2 and *n*

$$K(v_2^{obs};\, v_2, n) = v_2^{obs} \exp\left(\frac{-(v_2^{obs} - a)^2}{2b^2}\right)$$

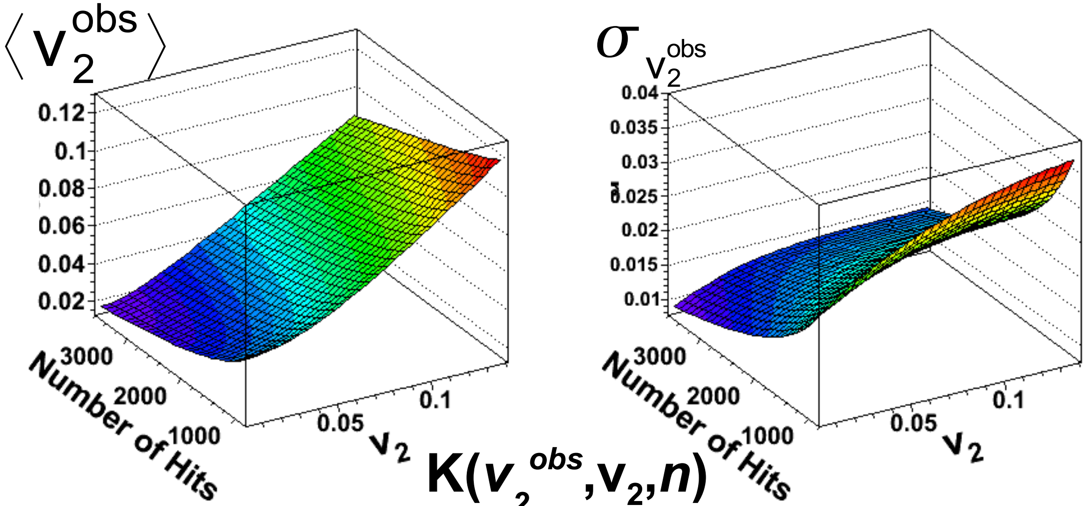
  - Correspondance

$$(a, b) \Leftrightarrow (\langle v_2^{obs} \rangle, \sigma_{v_2^{obs}})$$



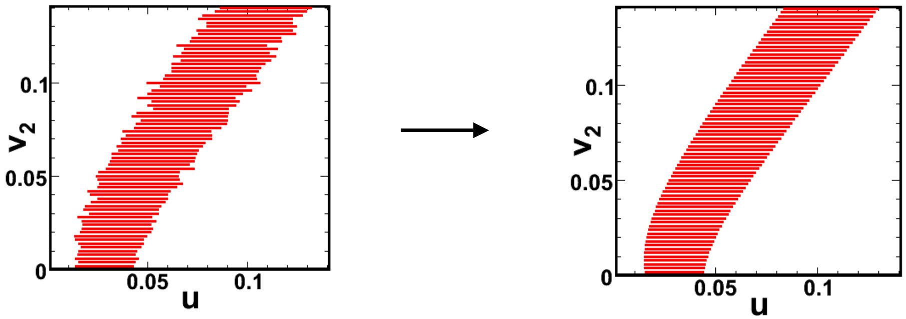
- Determine $\langle v_2^{obs} \rangle$ and $\sigma_{v_2^{obs}}$ in bins of v_2 and n and determine (a,b)
- Fit smooth functions
- Integrate out multiplicity

$$K(v_2^{obs};\, v_2) = \int K(v_2^{obs};\, v_2, n)\, N(n)\, dn$$



$K(v_2^{obs}, v_2, n)$

Calculating the kernel: Functions observed to fit

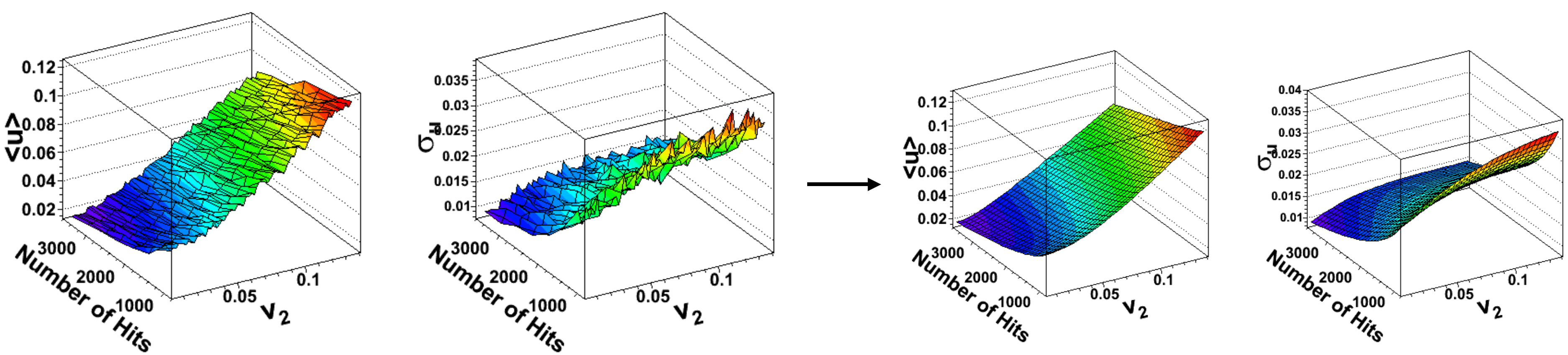


$$< u > (v_2)|_n = \sqrt{m_1 \cdot v_2^2 + m_2}$$

$$\sigma_u(v_2)|_n = \frac{1}{r_1 + \frac{2}{3}\, e^{r_2 v_2}}$$

$$< u > (v_2, n) = \sqrt{(M_1 n^2 + M_2 n + M_3) \cdot v_2^2 + (M_4 * n + M_5)}$$

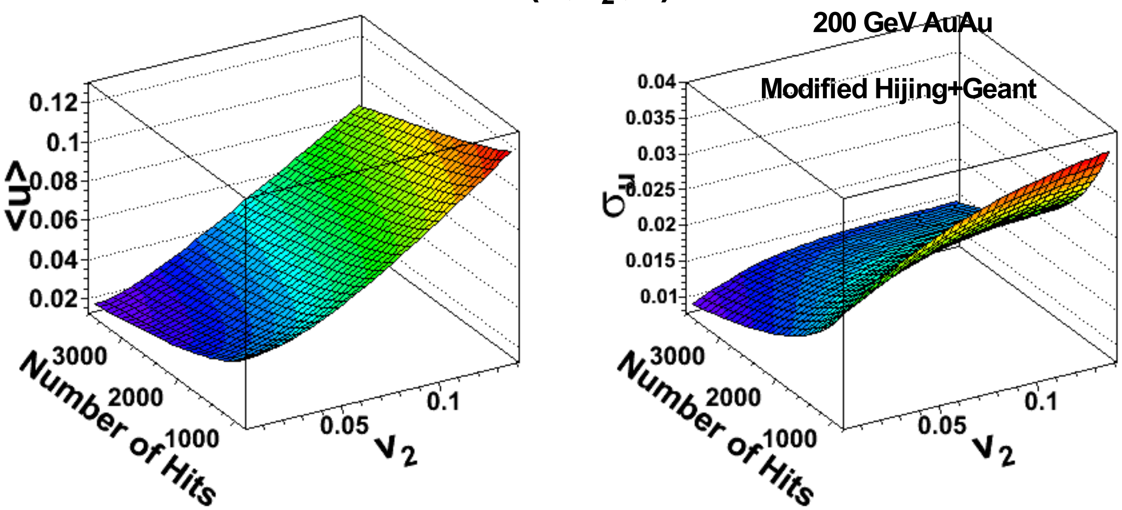
$$\sigma_u(v_2, n) = \frac{R_1}{(R_2\sqrt{n} + 1) \cdot (1 + \frac{2}{3}\, e^{(R_3 n + R_4) v_2})}$$



Determining the kernel

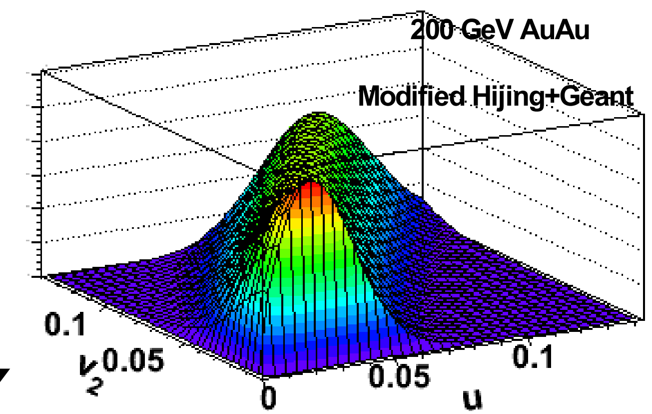
- Multiplicity dependence can be integrated out

K(u,v_2,n)

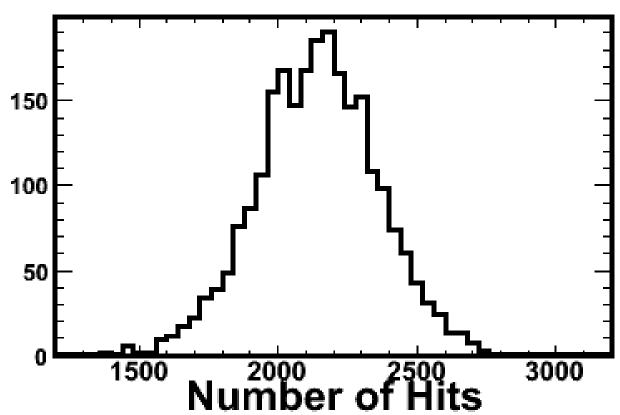


$$K(v_2^{obs};\, v_2) = \int K(v_2^{obs};\, v_{2,} n)\, N(n)\, dn$$

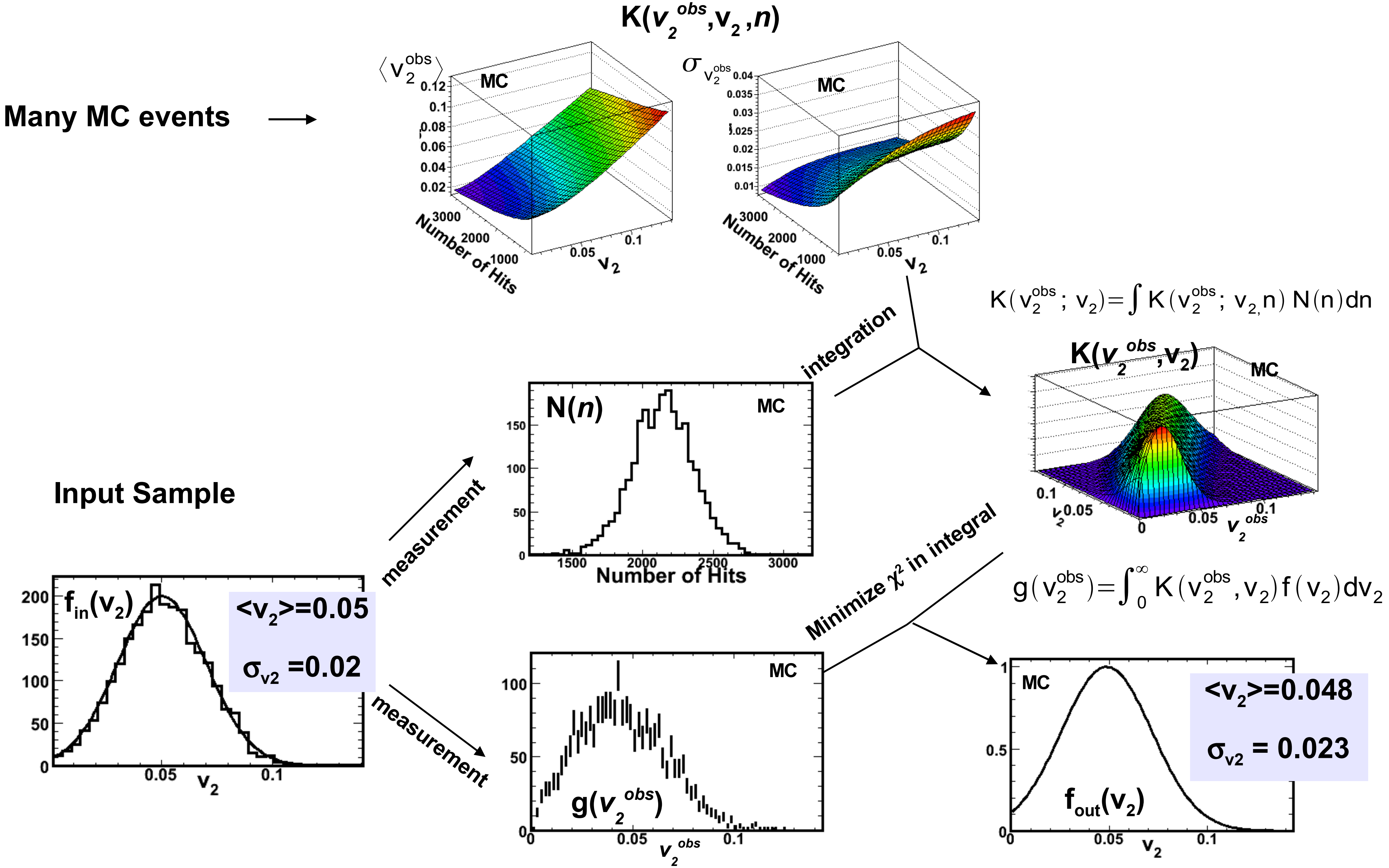
K(v_2^{obs},v_2)



N(n) = Number hits distribution in sample



Summary of flow fluctuation measurement method



$$K(v_2^{obs};\, v_2) = \int K(v_2^{obs};\, v_2, n)\, N(n)\, dn$$

$$g(v_2^{obs}) = \int_0^{\infty} K(v_2^{obs}, v_2)\, f(v_2)\, dv_2$$