A Stable and Accurate Method for Tetrahedral Elastic-Plastic Computations

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Albuquerque, NM

Sept 2-6, 2013
MULTIMAT 2013
SAND2013-7265C

Sandia National Laboratories

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

National Nuclear Security Administration

Acknowledgements

- Duke University: (algorithms for solids and fluid/solid coupling)
  - Prof. Gugliemo Scovazzi
  - Xianyi Zeng
- Sandia Labs: (interface to Trilinos/STK, multi-physics coupling)
  - Dave Hensinger
- Rensselaer Polytechnic Institute: (interface to FMDB library for adaptivity)
  - Prof. Mark Shephard
  - Ryan Molecke
  - Dan Ibanez

Contents

Motivation: Tetrahedral Meshes

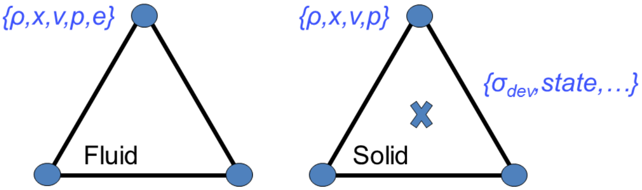
- Solid mechanics on hex meshes: mixed staggered Q1/Q0 formulation
  - continuous linear kinematic variables
  - discontinuous piece-wise constant stresses
  - requires various hourglass controls (e.g. Belytschko-Flanagan)
  - mesh generation very time consuming
- Tet meshes for solids:
  - use of automated fast meshing
  - ease of use for mesh adaptivity
  - ease of coupling with other physics (thermal, electromagnetic)

Overview of Recent Research

- Swansea: Bonet, Burton, Marriot, Hassan, (P1/P1-projection)
- Sandia: Dohrman, Key, Heinstein, Bochev, (P1/P1-projection)
- TU Munich-Sandia: Gee, Wall, Dohrman, (P1/P1+P1/P0-+proj.)
- LLNL: Puso, Solberg, (P1/P1+P1/P0-+proj.)
- RPI: Maniatty, Klaas, Liu, Shephard, Ramesh, (P1/P1-stabilized)
- Chorin's projection: Onate, Rojek, Taylor, Pastor (P1/P1)
- UPC Barcelona: Chiumenti, Cervera, Valverde, Codina (P1/P1-stabilized)
- UIUC: Nakshatrala, Masud, Hjelmstad, (P1/P1+bubble)
- Swansea II: Bonet, Gil, (P1/P1-stabilized)
- Berkeley/Pavia: Taylor, Auricchio, Lovadina, Reali, (Mixed enhanced)
- UCSD/University of Padua: Krysl, Micheloni, Boccardo (Mixed enhanced)
- Caltech: Thoutireddy, Ortiz, Molinari, Repetto, Belytschko (Composite Tets)

Finite Elements: Fluids and Solids

- Our approach to solids is an extension of the VMS-stabilized hydro approach [1]
- All variables are nodal except deviatoric stress and internal material state variables, which are based at quadrature points
- P1-based tets enable use of one-point quadrature (as in uniform gradient hexes)



[1] G. Scovazzi, J. Comput. Phys., Vol 231 (24), 2012, pp. 8029-8069

Governing Equations (Mixed Form)

- Solve for $\{d, v, \bar{\sigma}, p\}$ satisfying mass/momentum conservation, Cauchy stress decomposition, and velocity definition:

$$\rho J = \rho_0, \quad \rho \dot{v} = \nabla \cdot \sigma + \rho \cdot b, \quad \sigma = p I + \bar{\sigma}, \quad \dot{d} = v. \tag{1}$$

- Assume $\bar{\sigma}$ is a function of the kinematics (strains, strain rates), state variables, the history of $\bar{\sigma}$, etc.
- In the linear case we consider the **mixed** system for displacement (u) and pressure (p):

$$\begin{aligned} \rho \ddot{u} - \nabla \cdot \overline{\varepsilon}(u) - \nabla p &= f \\ p - \kappa \nabla \cdot u &= 0 \end{aligned}$$

where $\overline{\varepsilon}(u)$ is the deviatoric strain tensor.

Linear Elasticity: Static Case

- Stabilization for linear elasticity is very similar to Stokes flow
- Incompressible case:
  - P1/P0 locking (as in P1 displacement formulation)
  - P1/P1 checkerboard instability for pressure
- Solution for P1/P1: Hughes/Franca/Balestra stabilization (1986): enrich the velocity/displacement (u) with a residual-based term

$$u = u_h + u', \quad u' = -\tau \frac{h^2}{2\mu}(-\nabla p_h - \nabla \cdot \epsilon(u_h) - f)$$

- Stabilization derives from the additional pressure Laplacian
- This is now called Variational Multiscale (VMS) stabilization

Linear Elasticity: Dynamic Case

- The Hughes/Franca/Balestra stabilization extends naturally to time-dependent Stokes/Navier-Stokes flows
- We could not find an appropriate τ that worked for linear dynamics
- The issue appears to be the different character of the PDEs:
  - elliptic: Stokes (velocity/pressure)
  - elliptic: elasticity (displacement/pressure)
  - parabolic: time-dependent Stokes
  - hyperbolic: time-dependent elasticity
- Our solution is to formulate the pressure equation in rate form: which pairs naturally with the momentum equation:

$$\begin{aligned} \kappa^{-1}\dot{p} - \nabla \cdot v &= 0 \\ \rho\,\dot{v} - \nabla p &= \nabla \cdot \overline{\varepsilon}(u) + f \end{aligned}$$

Linear Elasticity: Dynamic VMS

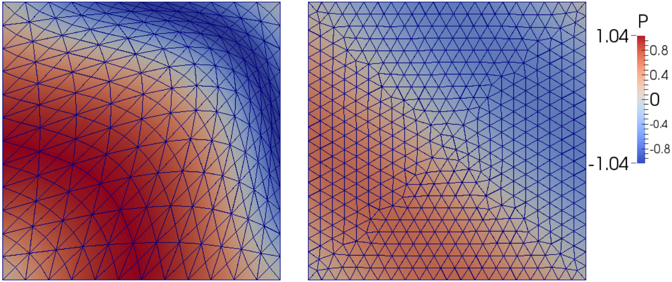
- The stabilization then is analogous to what is used for the linear acoustic wave equation
- We add in subgrid scales $\{v', p'\}$ defined using residuals

$$\begin{aligned} v = v_h + v', \quad v' &= -\tau\,\rho^{-1}(\rho\,\dot{v}_h - \nabla p_h - \nabla\cdot\bar{\epsilon}(u_h) - \rho\cdot b) \\ p = p_h + p', \quad p' &= -\tau(\dot{p}_h - \kappa\nabla\cdot v_h) \end{aligned}$$

- The resulting pressure Laplacian and velocity div-div terms provide stabilization
- The use of residuals provides consistency and thus accuracy

Linear Elasticity: Verification

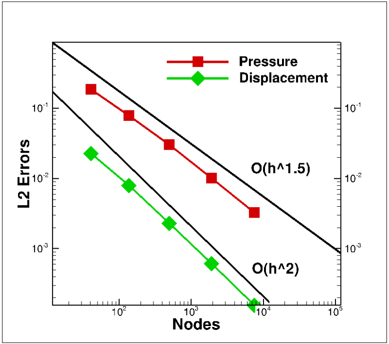
- We verified the linear elastic case under various options:
  - static/dynamic,
  - quad/tri/hex/tet,
  - compressible/nearly incompressible,
  - structured/unstructured grids
- Plots of manufactured solutions for pressure:



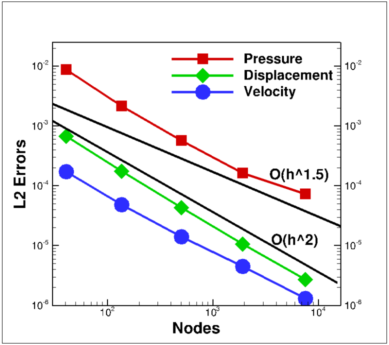
Compressible (left) and nearly incompressible (right)

Linear Elasticity: Verification

- Verification test: analytic pressure/velocity/displacement with valid solution in the incompressible limit (here $\nu = 0.4995$)
- Expected convergence rates are order 2 except for pressure which is order 1.5.



Static



Dynamic

Nonlinear Dynamics: Hyper-elasticity

- We concentrate on mixed formulations using a pressure:

$$\sigma = p\,I + \bar{\sigma}$$

- Pressure is assumed a function of the volumetric part of the deformation gradient:

$$p \equiv \kappa\,U'(J) \tag{2}$$

where $J = det(F)$, F is the deformation gradient, U is an energy function and κ is the bulk modulus (e.g. $U(J) = \frac{1}{2}(J-1)^2$)

- Deviatoric stress is defined in terms of J and $b = F\,F^T$ using another energy function, for example using a neo-Hookean law

$$\bar{\sigma} = \mu J^{-5/3}\,\bar{b}$$

Nonlinear Dynamics: J_2 Plasticity

- For plasticity, the pressure often remains a function of J
- The deviatoric stress is computed through an associative flow rule, with inclusion of constraints and plastic strain
- We have implemented a simple plasticity model [2] using linear hardening and a product factorization of the total deformation:

$$F = F^e F^p$$

- Extensions to other models (e.g. hypo-elasticity) should be possible provided that
  - we have separate models for $\bar{\sigma}$ and p, and
  - the pressure remains a function only of J

[2] JC Simo, CMAME (1992), pp. 61-102

Pressure Evolution Equation and VMS

- The nonlinear pressure equation in an evolution form:

$$\begin{aligned}\dot{p} &= \frac{\partial}{\partial t}\{\kappa\, U'(J)\} = \kappa\, U''(J)\,\dot{J} \\ &= \kappa\, U''(J)\,\{J\,\nabla\cdot v\} \\ &= \tilde{\kappa}(J)\,\nabla\cdot v.\end{aligned}$$

- We have defined an effective bulk modulus that varies as the material undergoes volume change.

$$\tilde{\kappa}(J) \equiv \kappa\, U''(J)\, J$$

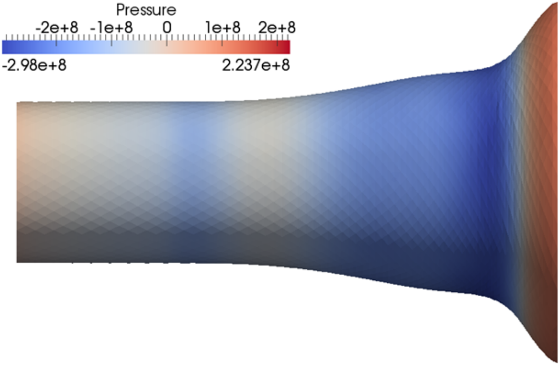
- VMS stabilization proceeds as in the linear case using a v' term

Nonlinear Examples Using Tet Meshes

- Dynamic
  - Taylor bar impact (elastic-plastic)
  - Bending beam (hyper-elastic)
- Quasistatic (run in dynamic mode)
  - Billet in compression (elastic-plastic)
  - Cylindrical bar in uniaxial tension (elastic-plastic)
- Impact test in complex geometry

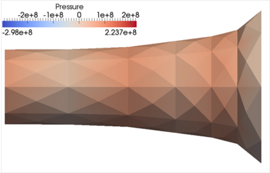
Taylor Bar

- Length/Radius: 3.24/0.32cm, density: 8930
- Elastic-plastic material (E=117.0e9, ν=0.35, σ_y=0.4e9, H=0.1e9)
- Zero normal velocity at wall, initial velocity 227m/s

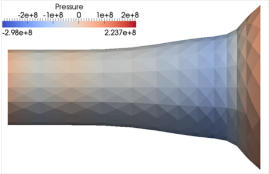


Pressure at final time

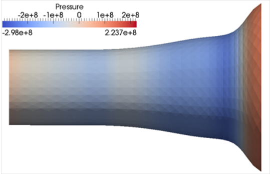
Taylor Bar: Pressure Convergence



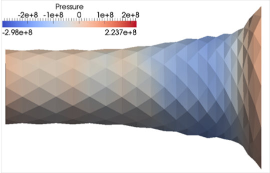
443 nodes



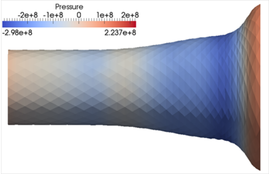
3189 nodes



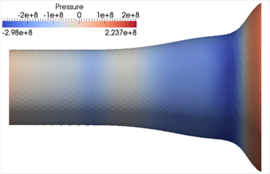
24233 nodes



466 nodes



2931 nodes

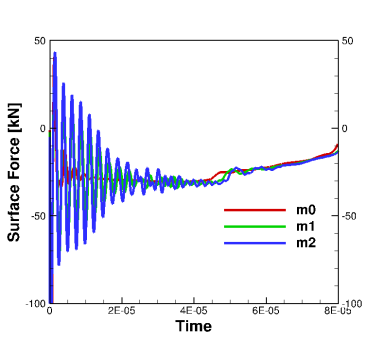


17239 nodes

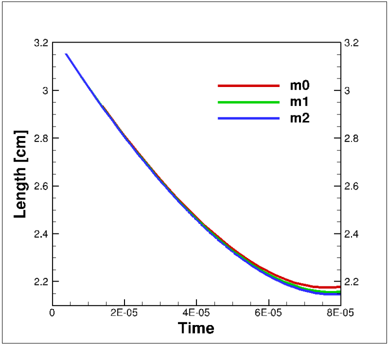
Note: unstructured grids fill space more evenly and resolve better than structured meshes derived from hex elements

Taylor Bar: Force and Length History

- Convergence of axial reaction force and final bar length:



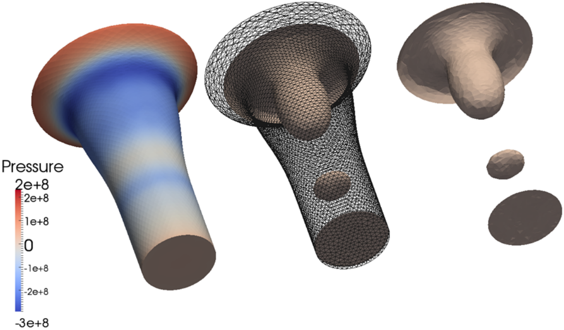
Wall Reaction Force



Final Length

Taylor Bar: Zero Pressure Isosurface

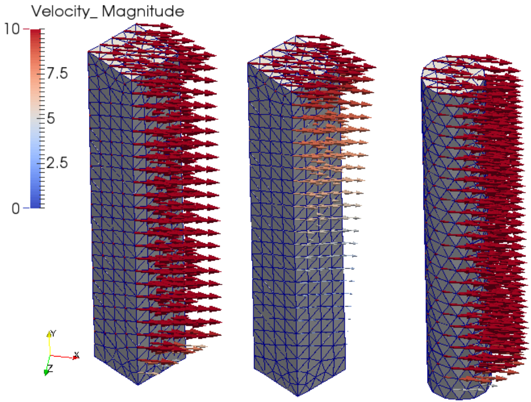
- Zero pressure isosurfaces: regions with no volume change
- We are able to resolve these surfaces very smoothly



Pressure and zero pressure iso-contours at final time

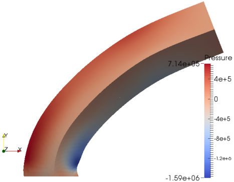
Bending Beam

- Length/Width: 6/1.4m, density: 1.1e3
- Elastic neo-Hookian material (E=1.7e7, ν=0.45)
- Fixed end, driven by initial x-velocity profile

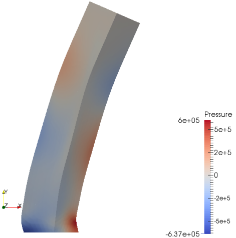


Versions of initial velocity and geometry

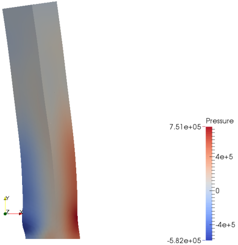
Bending Beam: Pressure



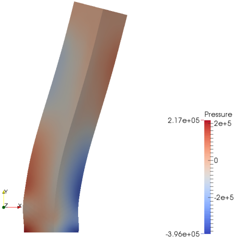
$t = 0.5$



$t = 1.0$



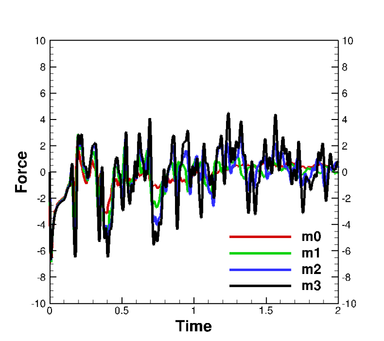
$t = 1.5$



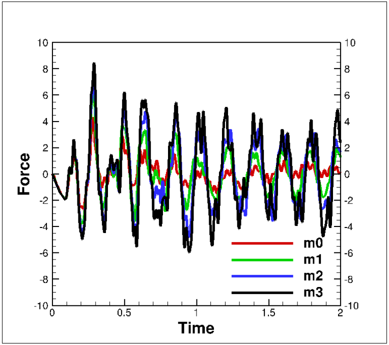
$t = 2.0$

Bending Beam: Force History

- We run on four uniform (unstructured) tet meshes (m_0-m_3)
- Convergence of reaction forces (x, y) at fixed surface:



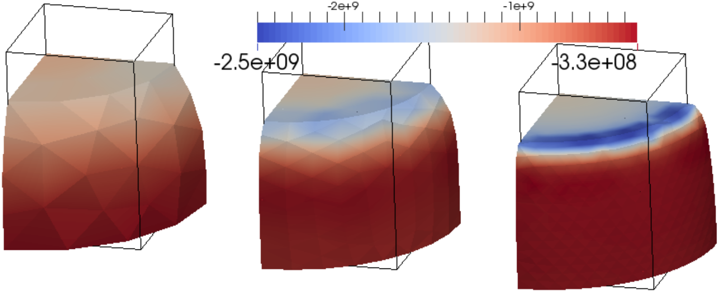
Wall Reaction Force (X)



Wall Reaction Force (Y)

Billet in Compression: Pressure

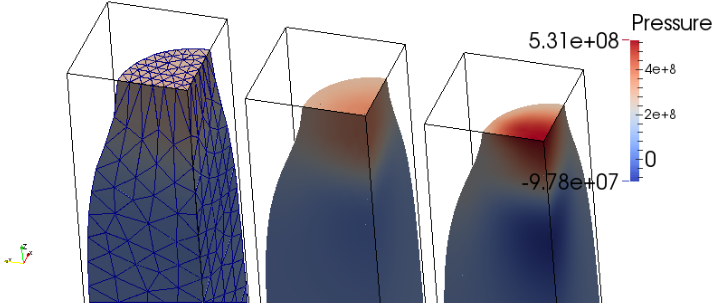
- Length/Radius: 1.5/1.0cm, density: 1e5
- Elastic-plastic material (E=384.62e9, ν=0.423, σ_y=1e9, H=3e9)
- Quasistatic approx. using dynamics (fictitious density, velocity)
- Top: dirichlet uniform velocity, bottom: zero normal displacement



Pressure contours for three meshes (25% compression)

Bar in Tension: Pressure

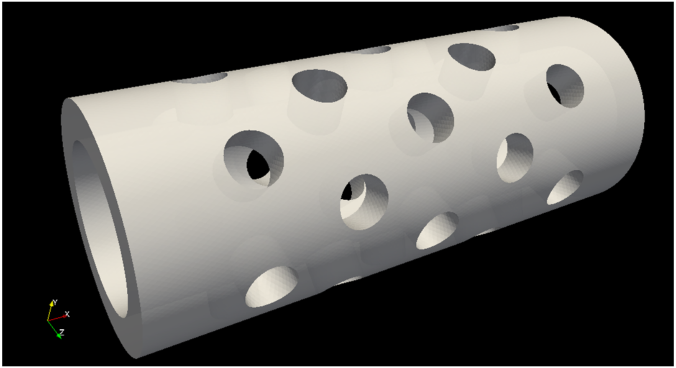
- Length/Radius: 5.33/0.641cm, density: 1e5
- Elastic-plastic material (E=80.2e9, ν=0.29, σ_y=0.45e9, H=0.13e9)
- Quasistatic approx. using dynamics (fictitious density, velocity)
- Top: zero normal displacement, bottom: dirichlet uniform velocity



Pressure contours for three meshes (0.4cm extension)

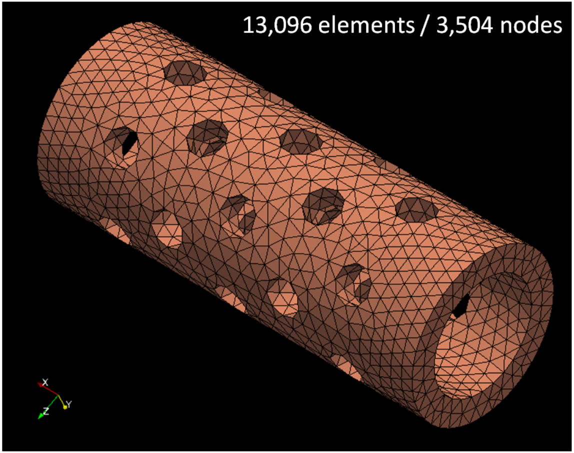
Impact Test in Complex Geometry

- Length/Radius: 32/3.24mm, density: 8930
- Elastic-plastic material (E=117.0e9, ν=0.35, σ_y=0.4e9, H=0.1e9)
- Zero normal velocity at wall, initial uniform x-velocity 100m/s

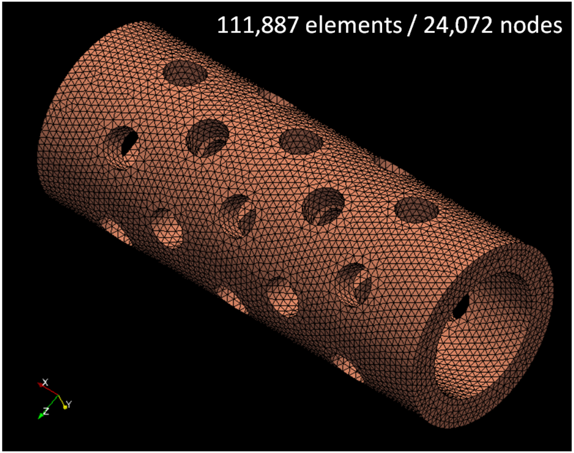


Initial Geometry

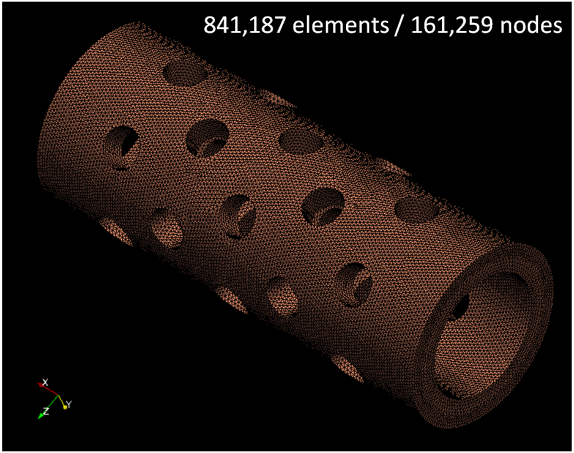
Impact Test: Meshes



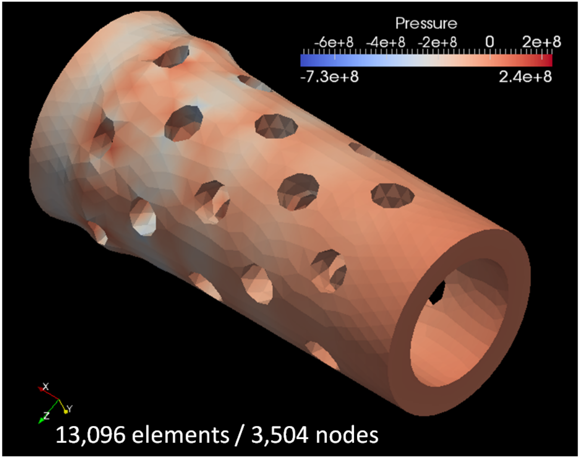
Impact Test: Meshes



Impact Test: Meshes

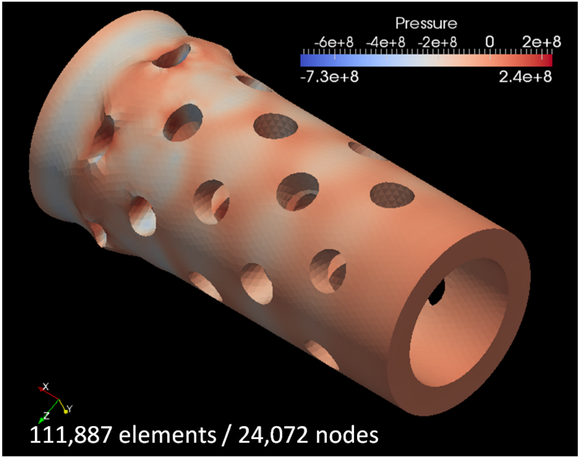


Impact Test: Deformation & Pressure



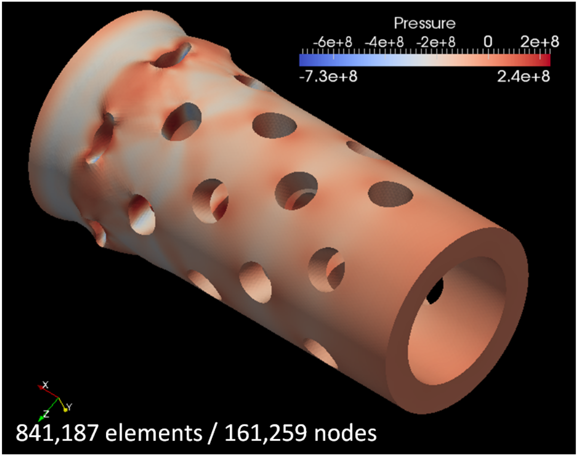
Pressure

Impact Test: Deformation & Pressure



Pressure

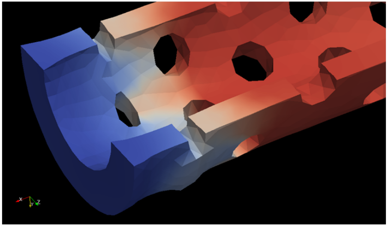
Impact Test: Deformation & Pressure



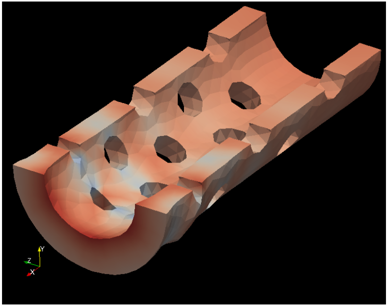
Pressure

Impact Test: Velocity & Pressure

Plot on half-domain:



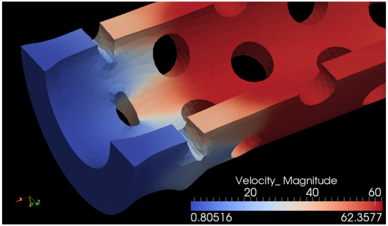
Velocity magnitude



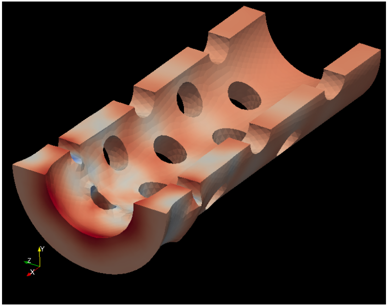
Pressure

Impact Test: Velocity & Pressure

Plot on half-domain:



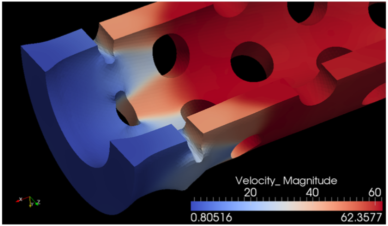
Velocity magnitude



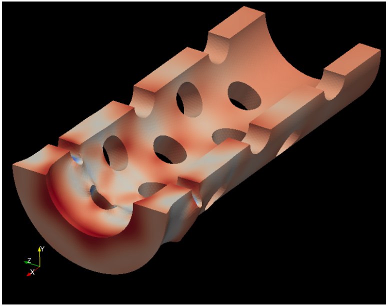
Pressure

Impact Test: Velocity & Pressure

Plot on half-domain:

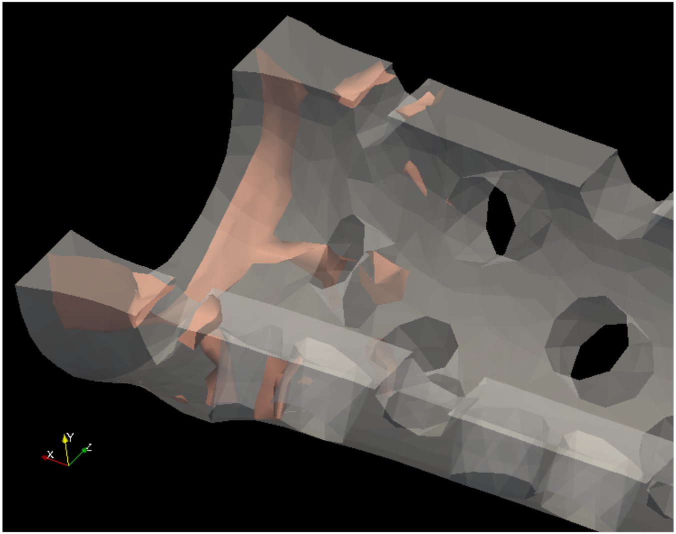


Velocity magnitude

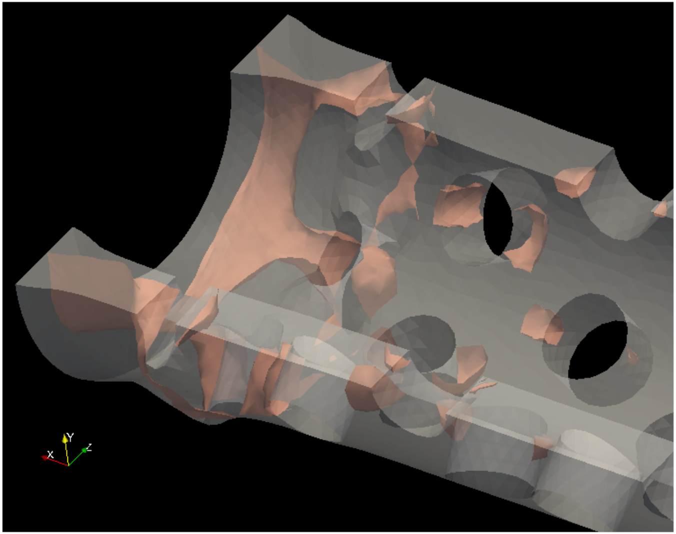


Pressure

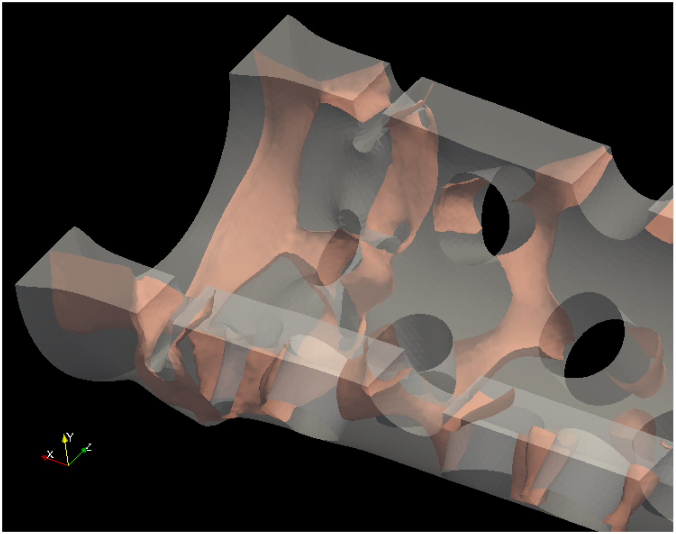
Impact Test: Zero Pressure Isosurfaces



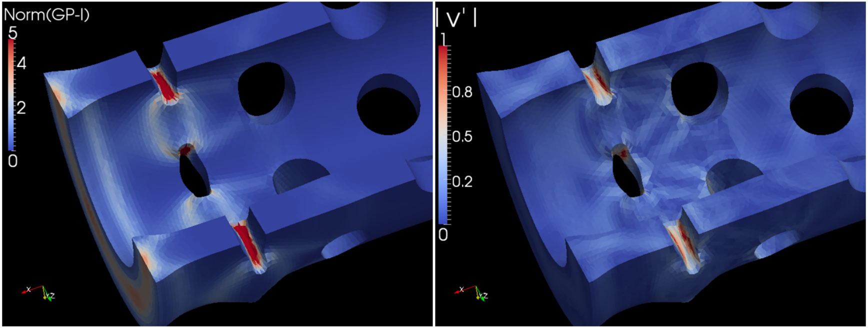
Impact Test: Zero Pressure Isosurfaces



Impact Test: Zero Pressure Isosurfaces



Impact Test: Plastic Strain



Plastic strain (left) and subgrid scale velocity (right)

Summary and Ongoing Work

- Current status
  - Finite deformation solid mechanics capability for tet meshes
  - Method is stable and accurate (based on VMS)
  - Compatible with VMS-based nodal hydrocode (we have a separate fluid/solid coupling module)
- Ongoing work
  - Additional formal code verification
  - Performance improvements
  - Comparisons with hex-based solid mechanics codes
  - Publications on solids and fluid/solid coupling