Second-order Accurate Interface- and Discontinuity-aware Diffusion Solvers in Two- and Three-dimensions

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Joint Work with

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Acknowledgement: Chong Chang, Ted Frederick

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LA-UR-13-26728

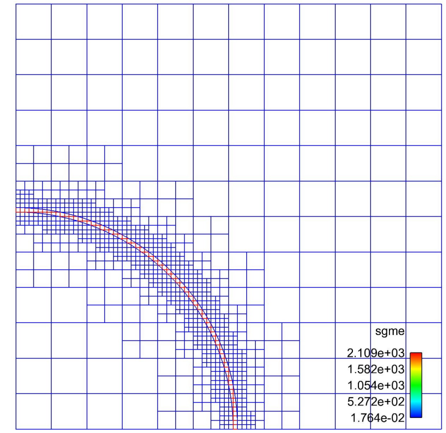
Outlines

- Motivations
- Solver for simple 1D equations for illustration
- Interface reconstruction
- Solvers on 2D/3D general polyhedral meshes
- Numerical examples
- Conclusions and future work

Motivations

- Previous research & codes, including Roxane
  - Interface reconstruction
  - Solvers on general polyhedral meshes

Motivations



- Previous research & codes, including Roxane
  - o Interface reconstruction
  - o Solvers on general polyhedral meshes
- Desired features of solvers*
  - o Accurate treatment for material discontinuity
  - o Second order accurate in space and time
  - o Correct steady states $\Delta t \rightarrow \infty$

*Dai & Woodward, *numerical simulations for nonlinear heat transfer in systems of multi-materials*, JCP, 1998.
*Dai & Woodward, *a second-order iterative implicit-explicit hybrid scheme for hyperbolic systems of conservation laws*, JCP, 1996.

1D Illustration

$$\frac{\partial T}{\partial t} + \frac{\partial F}{\partial x} = S \qquad F \equiv -\kappa \frac{\partial T}{\partial x}$$

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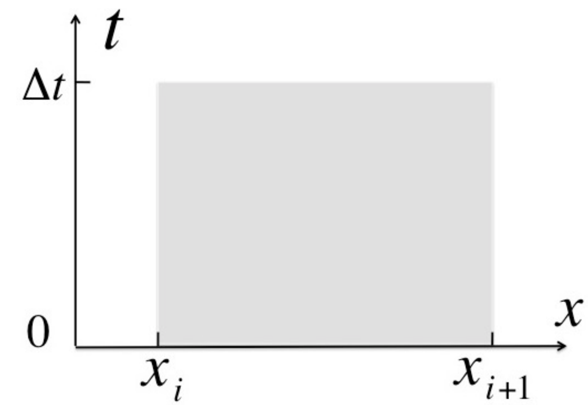
"Simultaneous discretization in space & time"



$$T_i^n = T_i + \frac{\Delta t}{\Delta x}(\overline{F}_i - \overline{F}_{i+1}) + \overline{S}_i \Delta t \quad \text{(exact)}$$

$$T_i^n \equiv \frac{1}{\Delta x}\int_{x_i}^{x_{i+1}} T(\Delta t, x)\,dx$$

Notations:

^{n} : at "new" time, $t = \Delta t$

$^-$: time integral / average

$$\overline{F}_i \equiv \frac{1}{\Delta t}\int_0^{\Delta t} F(t, x_i)\,dt$$

Typical Methods

$$T_i^n = T_i + \frac{\Delta t}{\Delta x}(\overline{F}_i - \overline{F}_{i+1}) + \overline{S}_i \Delta t$$

- Euler forward method

$$\overline{F}_i \approx F_i = -\frac{\kappa}{\Delta x}(T_i - T_{i-1})$$

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- Euler backward method

$$\overline{F}_i \approx F_i^n = -\frac{\kappa}{\Delta x}(T_i^n - T_{i-1}^n)$$

- Crank-Nicolson method

$$\overline{F}_i \approx \frac{1}{2}(F_i + F_i^n)$$

Goals

- 2^{nd} order accurate in time
- Stable for large time step
- Correct steady states for large time step
- Correct treatment of discontinuity of materials

Second-order Accuracy in Time

$$T_i^n = T_i + \frac{\Delta t}{\Delta x}(F_i^h - F_{i+1}^h) + S_i^h \Delta t$$

$$F^h = F(T^h)$$

Second-order Accuracy in Time

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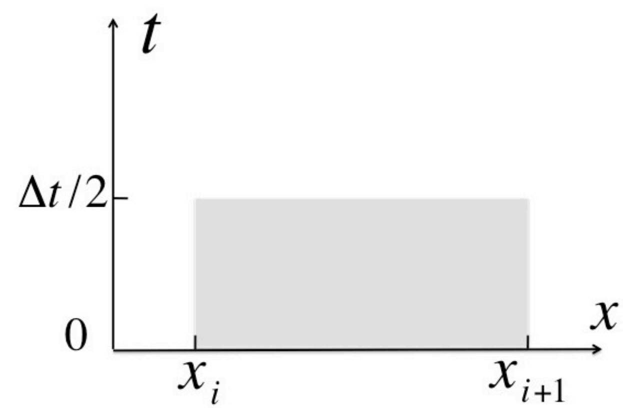
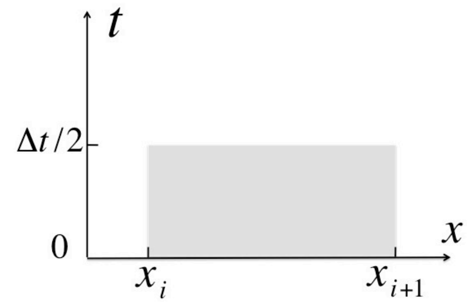


$$T_i^h = T_i + \frac{\Delta t}{2\Delta x_i}(\overline{F}_i^h - \overline{F}_{i+1}^h) + \frac{1}{2}\overline{S}_i^h \Delta t$$

$$T_i^h \equiv \frac{1}{\Delta x_i}\int_{\Delta x_i} T(\frac{1}{2}\Delta t, x)\,dx$$

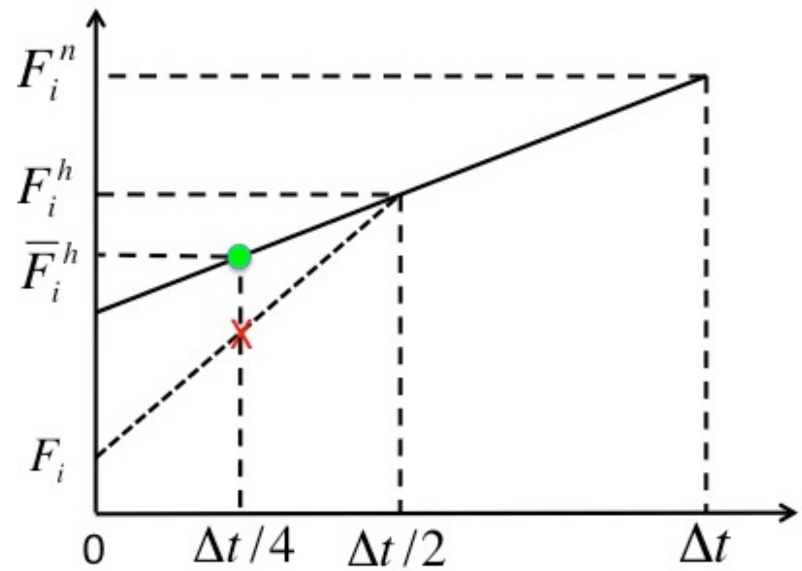
$$\overline{F}_i^h \equiv \frac{2}{\Delta t}\int_0^{\Delta t/2} F(t, x_i)\,dt$$

Correct Steady States



$$T_i^n = T_i + \frac{\Delta t}{\Delta x}(F_i^h - F_{i+1}^h) + S_i^h \Delta t$$

$$T_i^h = T_i + \frac{\Delta t}{2\Delta x_i}(\overline{F}_i^h - \overline{F}_{i+1}^h) + \frac{1}{2}\overline{S}_i^h \Delta t$$



$$\overline{F}_i^h \equiv \frac{2}{\Delta t}\int_0^{\Delta t/2} F(t, x_i)\, dt$$

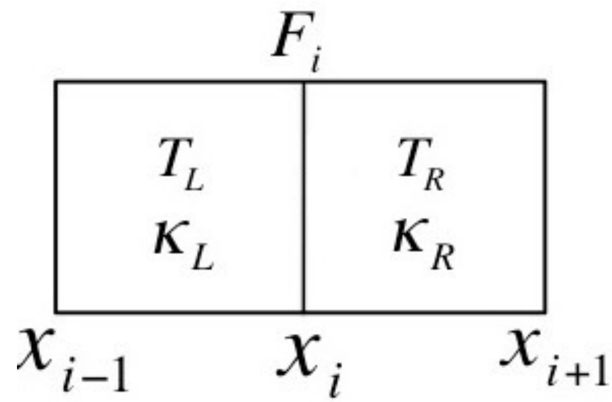
$$\overline{F}_i^h \approx \frac{3}{2}F_i^h - \frac{1}{2}F_i^n \qquad \overline{S}_i^h \approx \frac{3}{2}S_i^h - \frac{1}{2}S_i^n$$

- Stable for large time step, correct steady state

Treatment for discontinuity:

Effective Diffusion Coefficient

$$F = -\kappa \frac{\partial T}{\partial x}.$$



If single material $F_i \approx -\frac{\kappa}{\Delta x}(T_R - T_L).$

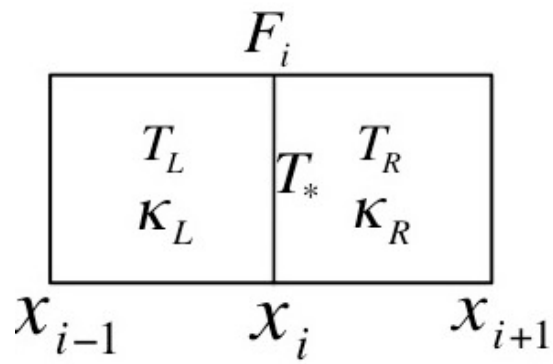
If different materials $F_i \approx -\frac{\bar{\kappa}}{\Delta x}(T_R - T_L).$

Note: algebraic average cannot be correct.

Material Discontinuity: effective diffusion coefficient

$$F_i = -\frac{\overline{\kappa}}{\Delta x}(T_R - T_L)$$

$$\overline{\kappa} \equiv \frac{2\kappa_L \kappa_R}{\kappa_L + \kappa_R}.^*$$



$$T_* = \frac{\kappa_L T_L + \kappa_R T_R}{\kappa_L + \kappa_R}.$$

*Dai & Woodward, numerical simulations for nonlinear heat transfer in systems of multi-materials, JCP, 1998.

Material Discontinuity: effective diffusion coefficient

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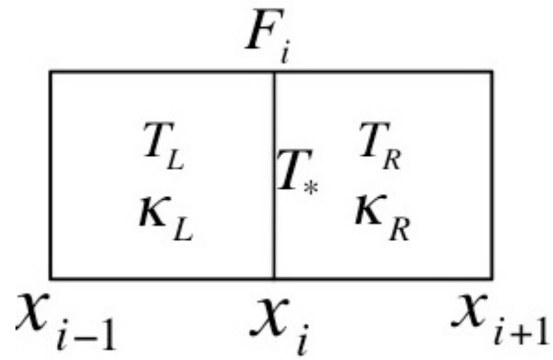

$$\overline{\kappa} \equiv \frac{2\kappa_L \kappa_R}{\kappa_L + \kappa_R}.^*$$

$$T_* = \frac{\kappa_L T_L + \kappa_R T_R}{\kappa_L + \kappa_R}.$$

$\overline{\kappa} = ?$ for general polyhedral cells

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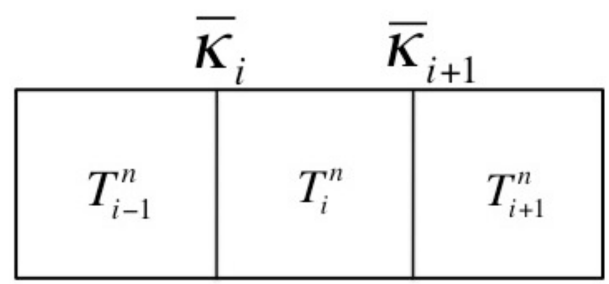
Difference Equations

$$T_i^n + \alpha_i T_i^h = T_i \; + N_i^h + S_i^h \Delta t.$$

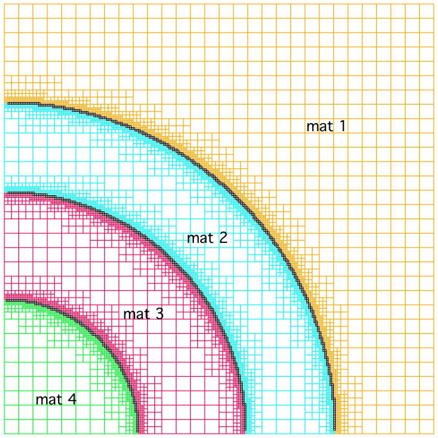
$$-\frac{1}{4}\alpha_i T_i^n + (1+\frac{3}{4}\alpha_i)T_i^h = T_i \; + \frac{3}{4}N_i^h - \frac{1}{4}N_i^n + \frac{1}{2}\overline{S}_i^h \Delta t.$$

$$\alpha_i \equiv \frac{\Delta t}{(\Delta x_i)^2}(\overline{\kappa}_i + \overline{\kappa}_{i+1})$$



$$N_i^h \equiv \frac{\Delta t}{(\Delta x_i)^2}(\overline{\kappa}_i T_{i-1}^h + \overline{\kappa}_{i+1} T_{i+1}^h)$$

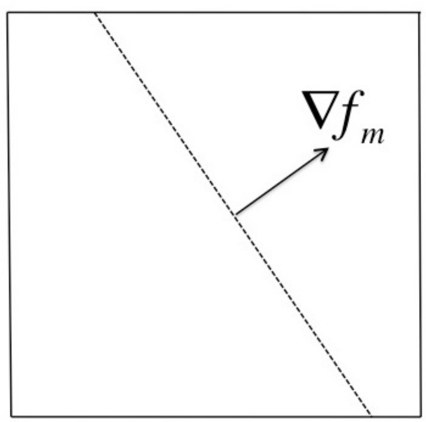
Interface Reconstruction

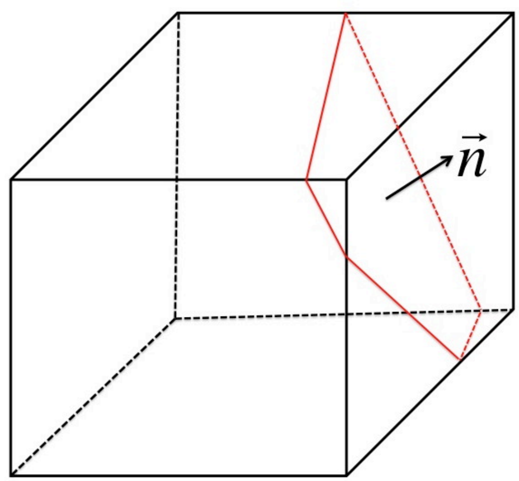


- Focus on mixing cells on structured AMR mesh
- Three-dimensional
- More than three materials in one cell

Procedure in Interface Reconstruction

- Find gradient of each material in each cell
- Determine gradient of the cell
- Determine the order of materials: p(m) and local order
- Find the interface

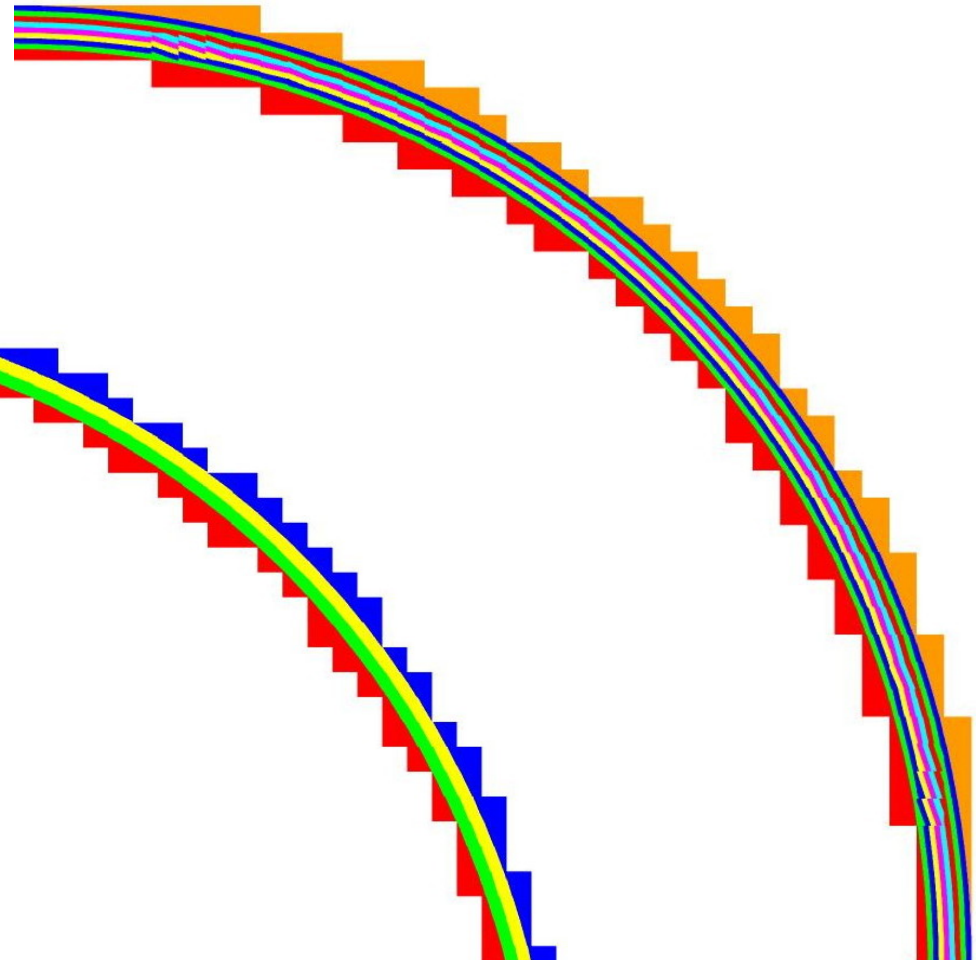


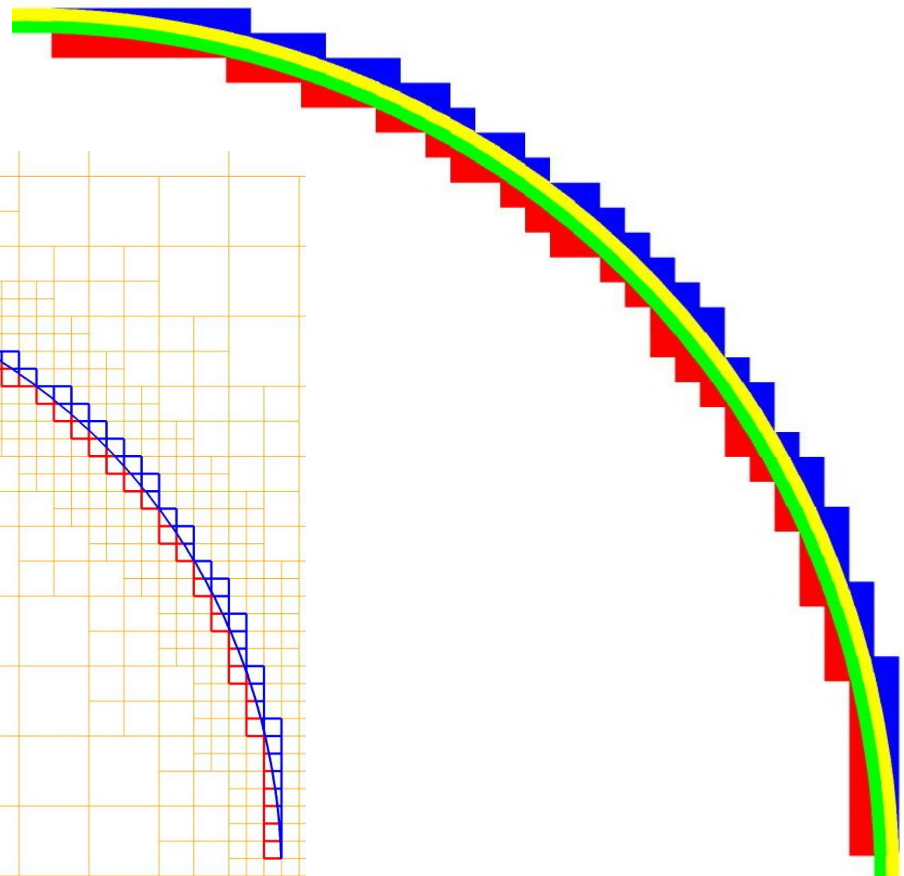


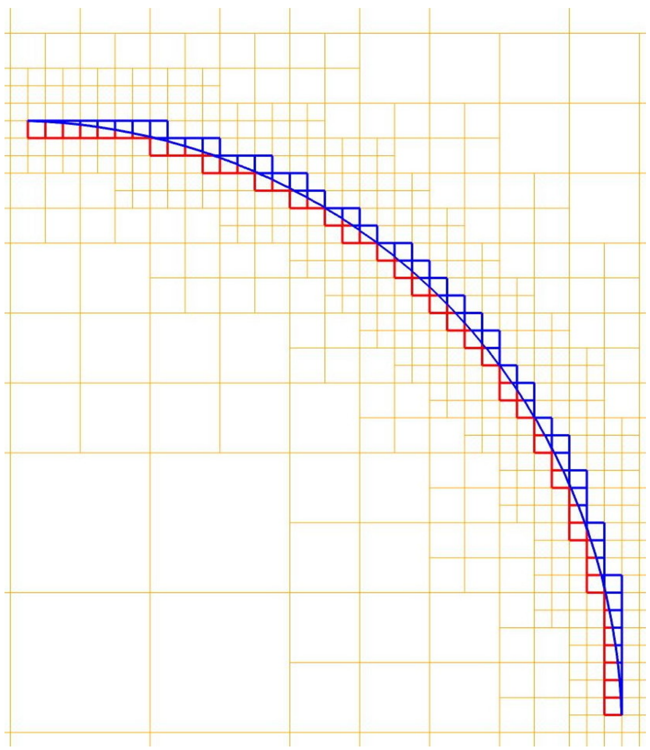
$$q(m) \equiv \| \nabla f_m \|^2 \sqrt{f_m}.$$

$$p(m) \equiv \vec{n}(m) \cdot \vec{n}(m_0).$$

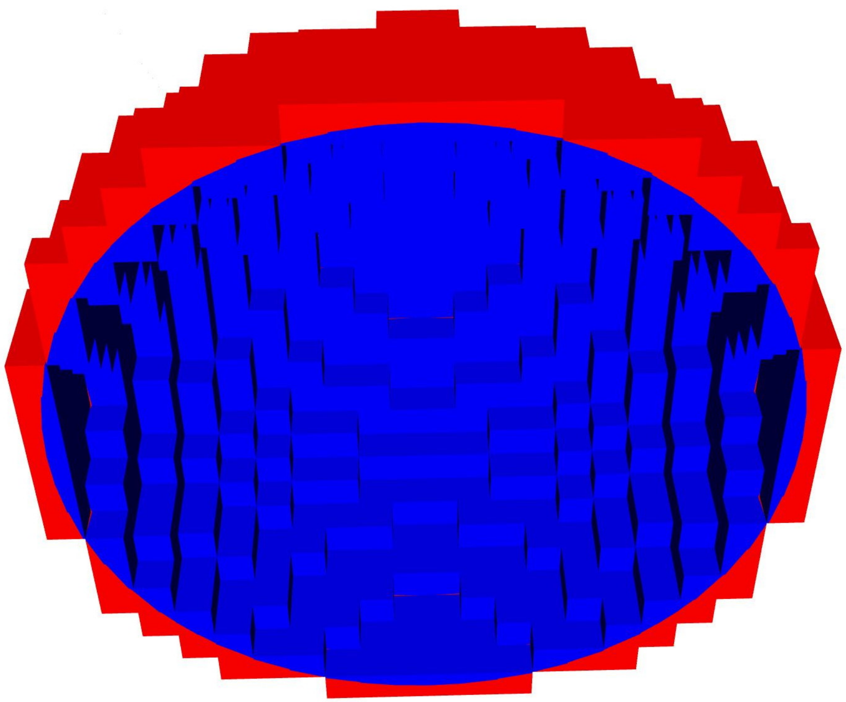
Reconstruction Examples

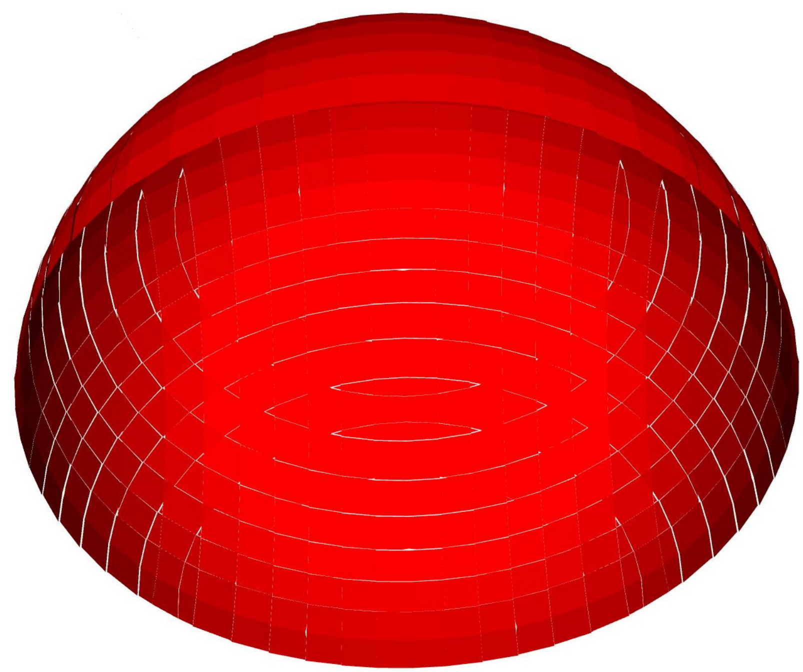




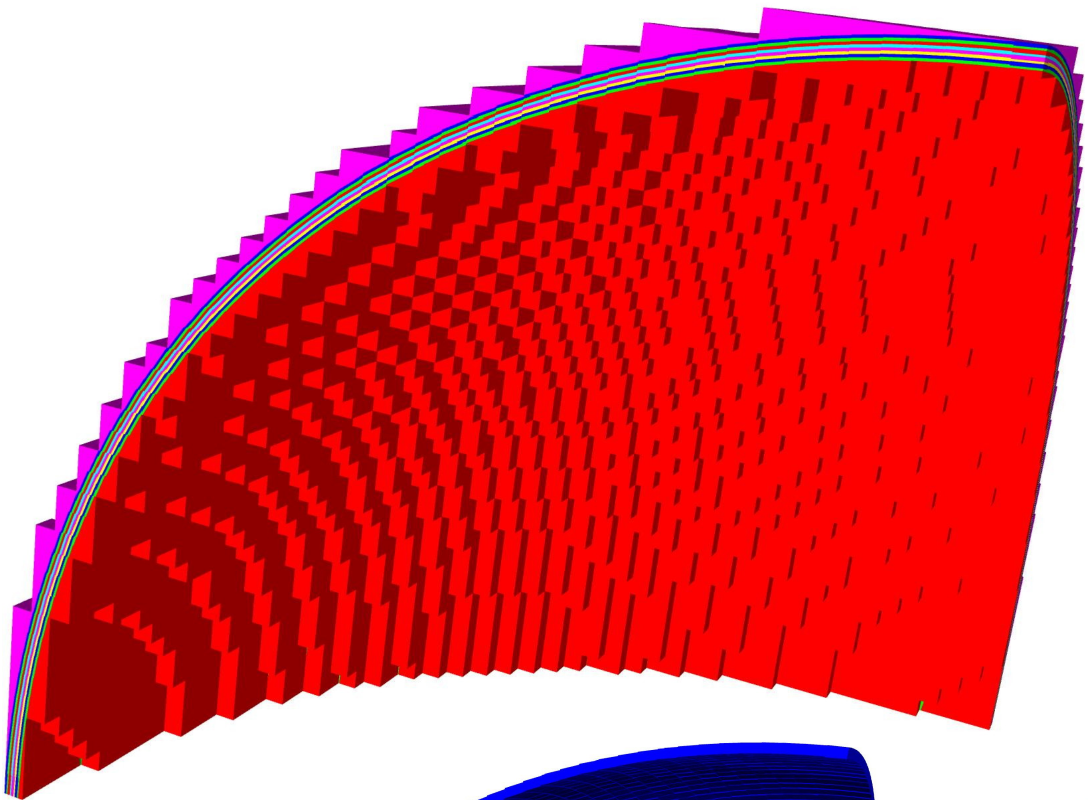


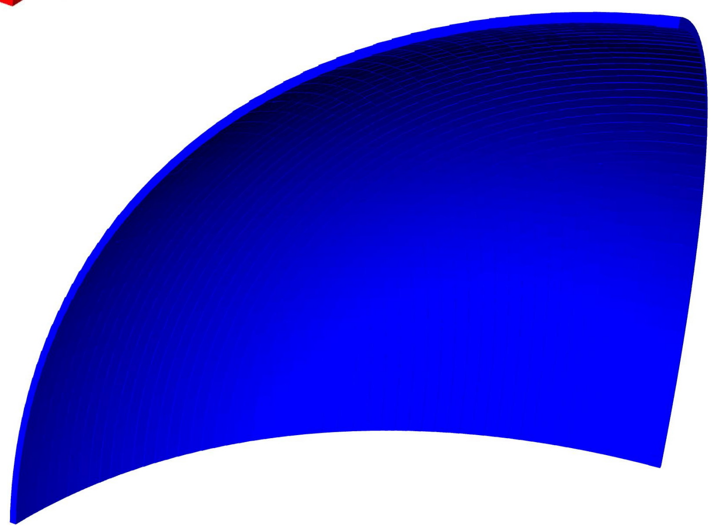
Reconstruction Examples

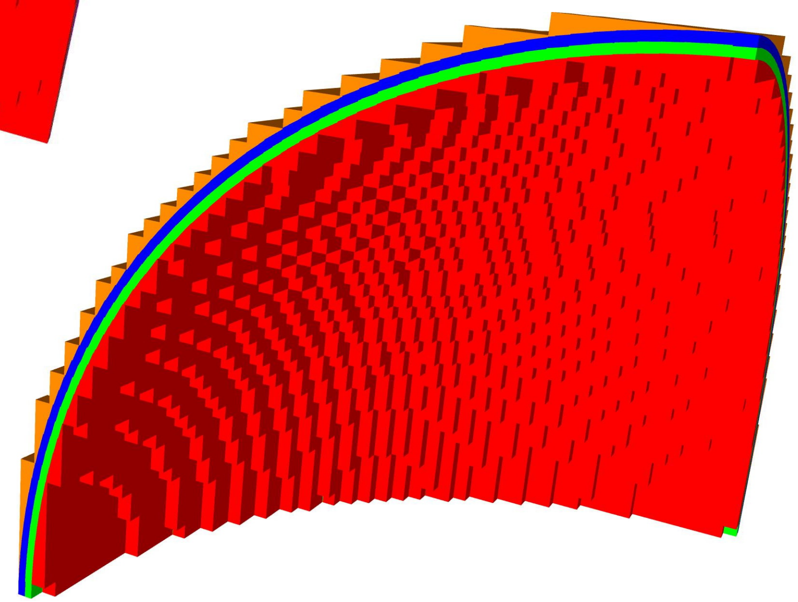




Reconstruction Examples







Diffusion Solvers on General Polyhedral Mesh

$$\frac{\partial u}{\partial t} = -\nabla \cdot \vec{F} + S.$$

$$u \equiv \rho c_v T + aT^4.$$

$$\vec{F} \equiv -[\kappa_0(T)\nabla T + \sigma(T)\nabla T^4].$$

- Discontinuity in material property
- Second order accurate in space and time
- Correct steady state for large time step

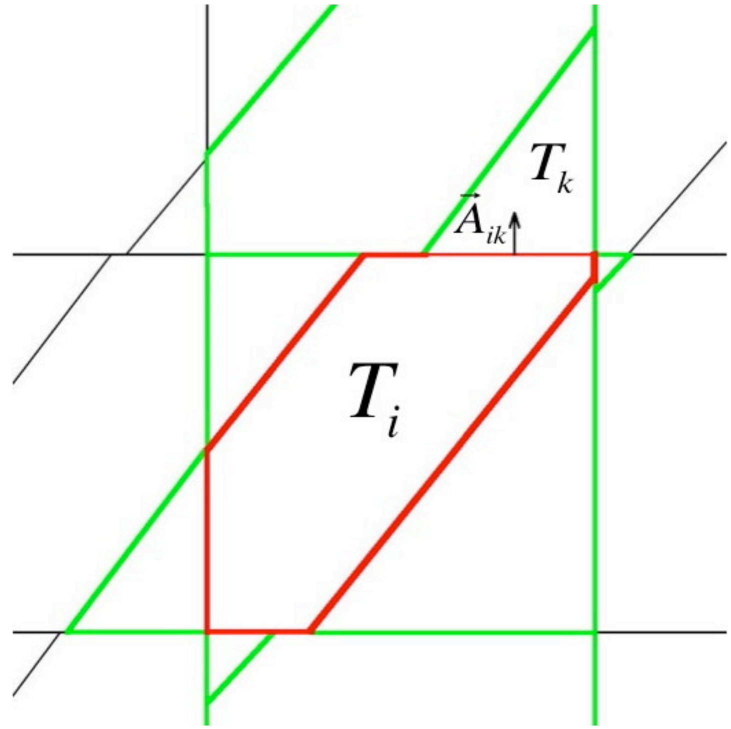
$$\frac{\partial u}{\partial t} = -\nabla \cdot \vec{F} + S.$$

Integrate Eq. over ΔV and Δt,

$$u_i^n = u_i - \frac{\Delta t}{\Delta V_i} \sum_k \overline{F}_{ik} A_{ik} + \overline{S}_i \Delta t.$$

$$u_i^n \equiv \frac{1}{\Delta V_i} \int_{\Delta V_i} u(\Delta t, \vec{r}) dV.$$

$$\overline{S}_i \equiv \frac{1}{\Delta t \Delta V_i} \int_0^{\Delta t} \int_{\Delta V_i} S(t, \vec{r}) dV dt.$$



k : all neighbors of cell i.

$$\overline{F}_{ik} \equiv \frac{1}{\Delta t} \int_0^{\Delta t} \left[\frac{1}{\Delta A_{ik}} \int_{A_{ik}} \vec{F} \cdot d\vec{a} \right] dt.$$

Second-order Accurate in Time

$$u_i^n = u_i - \frac{\Delta t}{\Delta V_i} \sum_k \overline{F}_{ik} A_{ik} + \overline{S}_i \Delta t.$$

Introduce another set of unknowns: $\mathrm{u}_\mathrm{i}^\mathrm{h}$ or $T_i^h (t = \frac{1}{2} \Delta t)$.

$$\overline{F}_{ik} \approx F_{ik}(\frac{1}{2} \Delta t) = F_{ik}^h$$

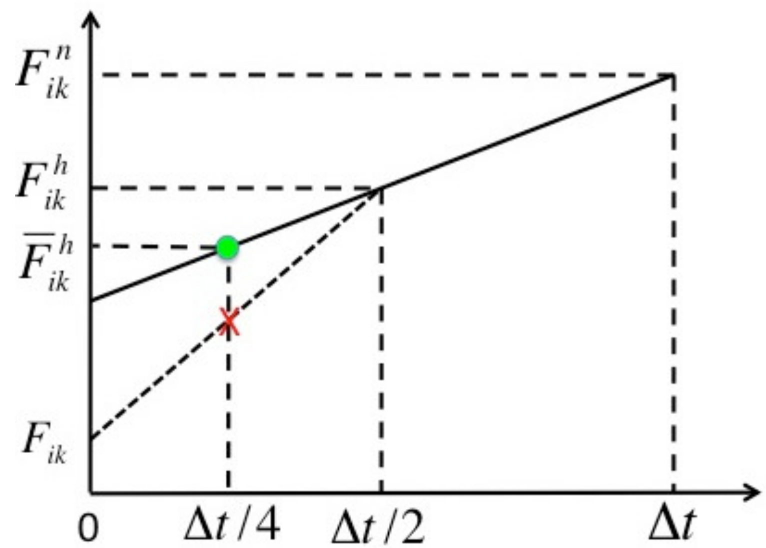
$$\overline{S}_i = S_i^h$$

Correct Steady States

$$u_i^h = u_i - \frac{\Delta t}{2\Delta V_i}\sum_k \overline{F}_{ik}^h A_{ik} + \frac{1}{2}\overline{S}_i^h \Delta t.$$

$$\overline{F}_{ik}^h \approx \frac{3}{2}F_{ik}^h - \frac{1}{2}F_{ik}^n.$$

$$\overline{S}_i^h \approx \frac{3}{2}S_i^h - \frac{1}{2}S_i^n.$$



Difference Form

$$u_i^n = u_i - \frac{\Delta t}{\Delta V_i} \sum_k F_{ik}^h A_{ik} + S_i^h \Delta t.$$

$$u_i^h = u_i - \frac{\Delta t}{2\Delta V_i} \sum_k \overline{F}_{ik}^h A_{ik} + \frac{1}{2} \overline{S}_i^h \Delta t.$$

$$\overline{F}_{ik}^h \approx \frac{3}{2} F_{ik}^h - \frac{1}{2} F_{ik}^n$$

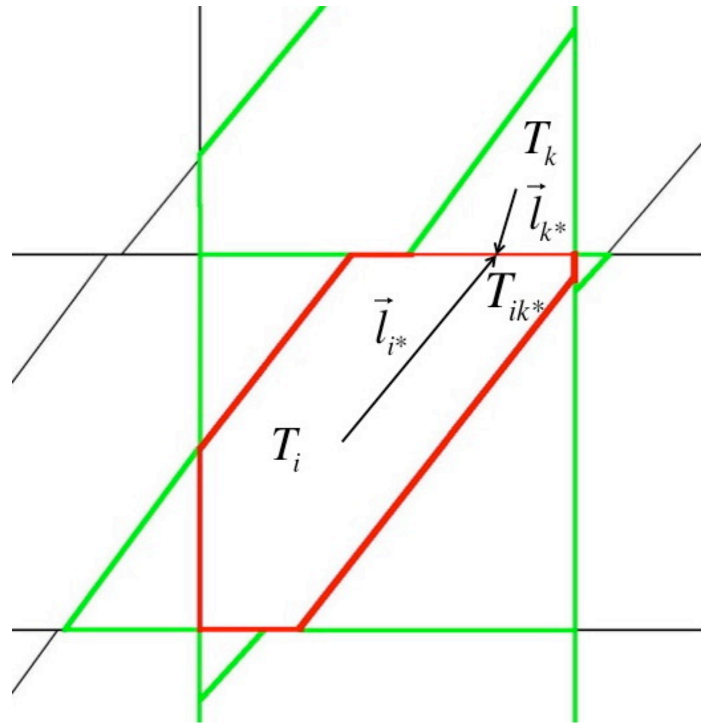
Treatment for material discontinuity

Calculate $F_{ik}(T_i, T_k)$.

$$\kappa(T) \equiv \kappa_0(T) + 4T^3\sigma(T).$$

$$\vec{F} \equiv -[\kappa_0(T)\nabla T + \sigma(T)\nabla T^4].$$

$$F_{ik} \approx -\tilde{\kappa}_{ik}(T_k - T_i)$$



$$\tilde{\kappa}_{ik} = \frac{\kappa_i \kappa_k}{l_k \alpha_i \kappa_i + l_l \alpha_k \kappa_k} \alpha_i \alpha_k \sim \frac{\kappa}{l}$$

$l_i, l_k, \alpha_i, and\ \alpha_k$ are geometry factors.

$\alpha_i \equiv \vec{n}_{i*} \cdot \vec{n}_{ik} > 0, \quad \alpha_k \equiv -\vec{n}_{k*} \cdot \vec{n}_{ik} > 0$

$$T_{ik*} = \frac{l_{k*}\alpha_i \kappa_i T_i + l_{l*}\alpha_k \kappa_k T_k}{l_{k*}\alpha_i \kappa_i + l_{l*}\alpha_k \kappa_k}$$

Difference Equations (nonlinear)

$$u_i^n + \frac{\Delta t}{\Delta V_i}\Big(\sum_{k\in N_i}\tilde{\kappa}_{ik}^h A_{ik}\Big)T_i^h = u_i + \frac{\Delta t}{\Delta V_i}\sum_{k\in N_i}(\tilde{\kappa}_{ik}^h A_{ik} T_k^h) + S_i^h \Delta t.$$

$$-\frac{\Delta t}{4\Delta V_i}\Big(\sum_{k\in N_i}\tilde{\kappa}_{ik}^n A_{ik}\Big)T_i^n + u_i^h + \frac{3\Delta t}{4\Delta V_i^u}\Big(\sum_{k\in N_i}\tilde{\kappa}_{ik}^h A_{ik}\Big)T_i^h$$

$$= u_i + \frac{3\Delta t}{4\Delta V_i}\sum_{k\in N_i}(\tilde{\kappa}_{ik}^h A_{ik} T_k^h) - \frac{\Delta t}{4\Delta V_i}\sum_{k\in N_i}(\tilde{\kappa}_{ik}^n A_{ik} T_k^n) + \frac{1}{2}\overline{S}_i^h \Delta t.$$

Difference Equations (nonlinear)

$$\hat{u}_i T_i^n + \frac{\Delta t}{\Delta V_i}\Big(\sum_{k\in N_i}\tilde{\kappa}_{ik}A_{ik}\Big)T_i^h = \hat{u}_i T_i + \frac{\Delta t}{\Delta V_i}\sum_{k\in N_i}(\tilde{\kappa}_{ik}A_{ik}T_k^h) + S_i^h\Delta t.$$

$$-\frac{\Delta t}{4\Delta V_i}\Big(\sum_{k\in N_i}\tilde{\kappa}_{ik}A_{ik}\Big)T_i^n + \hat{u}_i T_i^h + \frac{3\Delta t}{4\Delta V_i}\Big(\sum_{k\in N_i}\tilde{\kappa}_{ik}A_{ik}\Big)T_i^h$$

$$= \hat{u}_i T_i + \frac{3\Delta t}{4\Delta V_i}\sum_{k\in N_i}(\tilde{\kappa}_{ik}A_{ik}T_k^h) - \frac{\Delta t}{4\Delta V_i}\sum_{k\in N_i}(\tilde{\kappa}_{ik}A_{ik}T_k^n) + \frac{1}{2}\bar{S}_i^h\Delta t.$$

$$\hat{u}_i \equiv C_{vi} + 4aT_i^3.$$

Linearized $S_i^h = S(T_i^h)$ and $\bar{S}_i^h = \bar{S}_i^h(T_i^h, T_i^n)$.

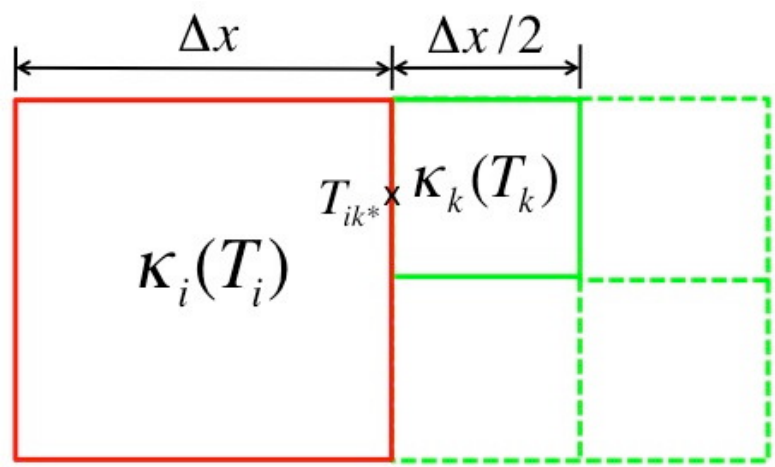
Implication for Structure AMR

$$T_{ik*} = \frac{\kappa_i T_i + 3\kappa_k T_k}{\kappa_i + 3\kappa_k} \quad \text{for 3D.} \qquad (4.16)$$

$$T_{ik*} = \frac{2\kappa_i T_i + 5\kappa_k T_k}{2\kappa_i + 5\kappa_k} \quad \text{for 2D.} \qquad (4.17)$$

$$\kappa_{ik}^{(c)} \equiv \frac{4\kappa_i \kappa_k}{\kappa_i + 3\kappa_k} \quad \text{for 3D.} \qquad (4.19)$$

$$\kappa_{ik}^{(c)} \equiv \frac{\kappa_i \kappa_k}{\kappa_i/4 + 5\kappa_k/8} \quad \text{for 2D.} \qquad (4.20)$$



Steady States

$$\left(\sum_{k\in N_i}\kappa_{ik}^n A_{ik}\right)T_i^n = \left(\sum_{k\in N_i}\kappa_{ik}^n A_{ik} T_k^n\right) + S(T_i^n)\Delta V_i\,.$$

3-T Equations

$$\frac{\partial a T_r^4}{\partial t} = -\nabla \cdot \vec{F}_r + S_r,$$

$$\rho c_{ve} \frac{\partial T_e}{\partial t} = -\nabla \cdot \vec{F}_e - S_r + S_e,$$

$$\rho c_{vi} \frac{\partial T_i}{\partial t} = -\nabla \cdot \vec{F}_i - S_e.$$

$$\vec{F}_r \equiv -\sigma_r \nabla T_r^4,$$

$$\vec{F}_e \equiv -\sigma_e \nabla T_e$$

$$\vec{F}_i \equiv -\sigma_i \nabla T_i$$

$$S_r \equiv ac\rho\kappa_\rho (T_e^4 - T_r^4)$$

$$S_e \equiv \rho c_{ve} \kappa_{ie} (T_i - T_e)$$

a: radiation constant.

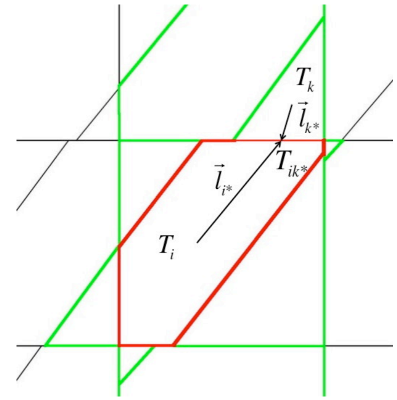
c_{ve}, c_{vi}: specific heat capacities.

$\sigma_r, \sigma_e, \sigma_i$: heat conductivities.

κ_ρ: material absorption coefficient.

κ_{ie}: coeffiecient for interaction.

Nonlinear Difference Equations



$$a(T_r^4)_i^n + \frac{\Delta t}{\Delta V_i}\Big(\sum_k \tilde{\kappa}_{ik}^h A_{ik}\Big)T_{ri}^h = a(T_r^4)_i + \frac{\Delta t}{\Delta V_i}\Big(\sum_k \tilde{\kappa}_{ik}^h A_{ik}\Big)T_{rk}^h + \overline{S}_{ri}^n \Delta t$$

$$a(T_r^4)_i^h + \frac{3\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\kappa}_{ik}^h A_{ik}\Big)T_{ri}^h - \frac{\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\kappa}_{ik}^n A_{ik}\Big)T_{ri}^n = a(T_r^4)_i + \frac{3\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\kappa}_{ik}^h A_{ik}\Big)T_{rk}^h - \frac{\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\kappa}_{ik}^n A_{ik}\Big)T_{rk}^n + \frac{1}{2}\Delta t \overline{S}_{ri}^h$$

$$(\rho c_e)_i T_{ei}^n + \frac{\Delta t}{\Delta V_i}\Big(\sum_k \tilde{\sigma}_{eik}^h A_{ik}\Big)T_{ei}^h = (\rho c_e)_i T_{ei} + \frac{\Delta t}{\Delta V_i}\Big(\sum_k \tilde{\sigma}_{eik}^h A_{ik}\Big)T_{ek}^h - \Delta t(\overline{S}_{ri}^n - \overline{S}_{ei}^n)$$

$$(\rho c_e)_i T_{ei}^h + \frac{3\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\sigma}_{eik}^h A_{ik}\Big)T_{ei}^h - \frac{\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\sigma}_{eik}^n A_{ik}\Big)T_{ei}^n = (\rho c_e)_i T_{ei} + \frac{3\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\sigma}_{eik}^h A_{ik}\Big)T_{ek}^h - \frac{\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\sigma}_{eik}^n A_{ik}\Big)T_{ek}^n - \frac{1}{2}\Delta t(\overline{S}_{ri}^h - \overline{S}_{ei}^h)$$

$$(\rho c_p)_i T_{pi}^n + \frac{\Delta t}{\Delta V_i}\Big(\sum_k \tilde{\sigma}_{pik}^h A_{ik}\Big)T_{pi}^h = (\rho c_p)_i T_{pi} + \frac{\Delta t}{\Delta V_i}\Big(\sum_k \tilde{\sigma}_{pik}^h A_{ik}\Big)T_{pk}^h - \overline{S}_{pi}^n \Delta t$$

$$(\rho c_p)_i T_{pi}^h + \frac{3\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\sigma}_{pik}^h A_{ik}\Big)T_{pi}^h - \frac{\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\sigma}_{pik}^n A_{ik}\Big)T_{pi}^n = (\rho c_p)_i T_{pi} + \frac{3\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\sigma}_{pik}^h A_{ik}\Big)T_{pk}^h - \frac{\Delta t}{4\Delta V_i}\Big(\sum_k \tilde{\sigma}_{pik}^n A_{ik}\Big)T_{ek}^n - \frac{1}{2}\Delta t \overline{S}_{pi}^h$$

Linear Difference Equations

$$4a(T_r^3)_i T_{ri}^n + \frac{\Delta t}{\Delta V_i}(\sum_k \tilde{\kappa}_{ik}^h A_{ik}) T_{ri}^h = 4a(T_r^4)_i + \frac{\Delta t}{\Delta V_i}(\sum_k \tilde{\kappa}_{ik}^h A_{ik}) T_{rk}^h + \overline{S}_{ri}^n \Delta t$$

$$4a(T_r^3)_i T_{ri}^h + \frac{3\Delta t}{4\Delta V_i}(\sum_k \tilde{\kappa}_{ik}^h A_{ik}) T_{ri}^h - \frac{\Delta t}{4\Delta V_i}(\sum_k \tilde{\kappa}_{ik}^n A_{ik}) T_{ri}^n = 4a(T_r^4)_i + \frac{3\Delta t}{4\Delta V_i}(\sum_k \tilde{\kappa}_{ik}^h A_{ik}) T_{rk}^h - \frac{\Delta t}{4\Delta V_i}(\sum_k \tilde{\kappa}_{ik}^n A_{ik}) T_{rk}^n + \frac{\Delta t}{2}\overline{S}_{ri}^h$$

$$(\rho c_{ve})_i T_{ei}^n + \frac{\Delta t}{\Delta V_i}(\sum_k \tilde{\sigma}_{eik} A_{ik}) T_{ei}^h = (\rho c_{ve})_i T_{ei} + \frac{\Delta t}{\Delta V_i}(\sum_k \tilde{\sigma}_{eik} A_{ik}) T_{ek}^h - \overline{S}_{ri}^n \Delta t + \overline{S}_{ei}^n \Delta t$$

$$(\rho c_{ve})_i T_{ei}^h + \frac{3\Delta t}{4\Delta V_i}(\sum_k \tilde{\sigma}_{eik} A_{ik}) T_{ei}^h - \frac{\Delta t}{4\Delta V_i}(\sum_k \tilde{\sigma}_{eik} A_{ik}) T_{ei}^n = (\rho c_{ve})_i T_{ei} + \frac{3\Delta t}{4\Delta V_i}(\sum_k \tilde{\sigma}_{eik} A_{ik}) T_{ek}^h - \frac{\Delta t}{4\Delta V_i}(\sum_k \tilde{\sigma}_{eik} A_{ik}) T_{ek}^n - \frac{\Delta t}{2}(\overline{S}_{ri}^h - \overline{S}_{ei}^h)$$

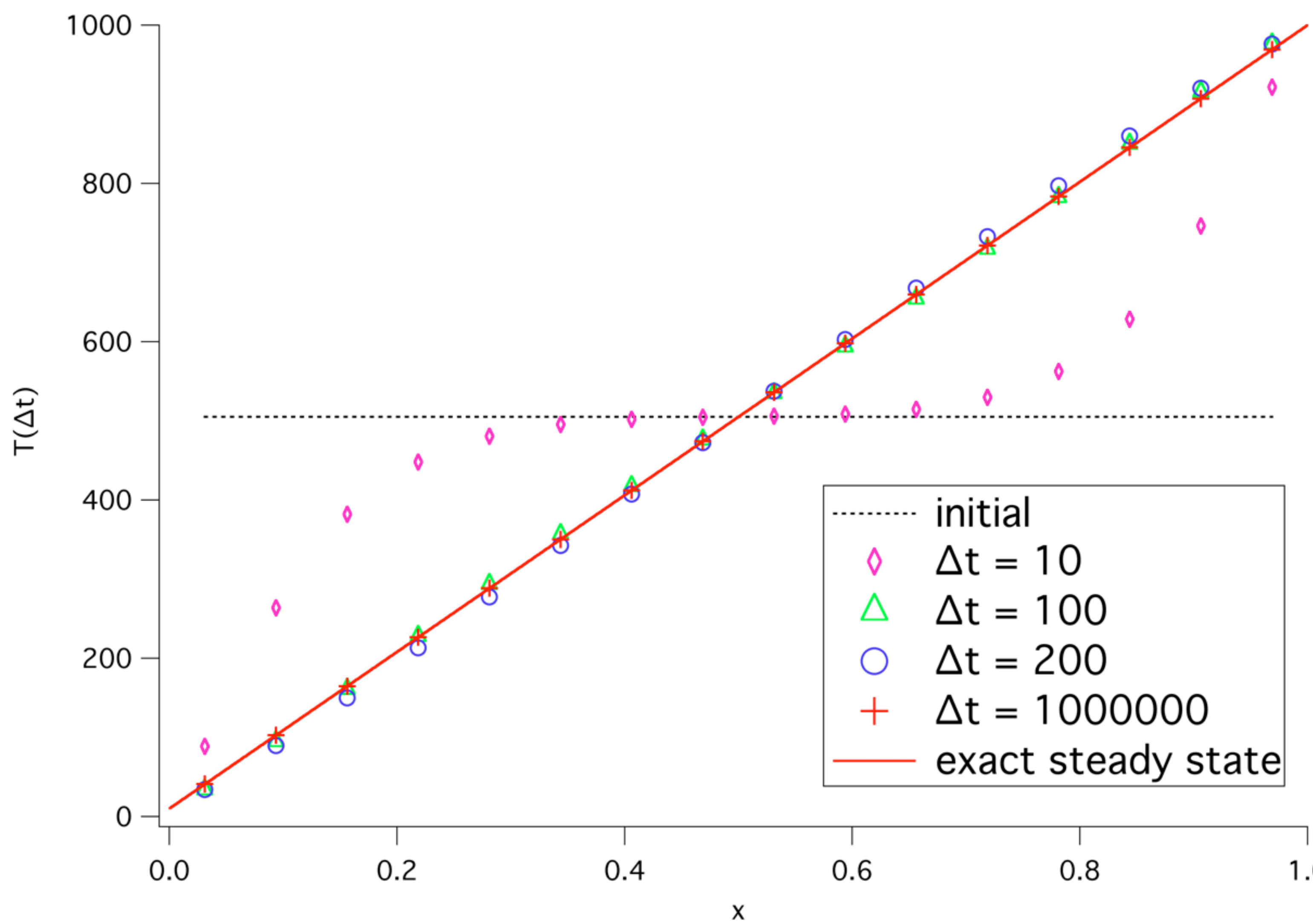
$$(\rho c_{vp})_i T_{pi}^n + \frac{\Delta t}{\Delta V_i}(\sum_k \tilde{\sigma}_{pik} A_{ik}) T_{pi}^h = (\rho c_{vp})_i T_{pi} + \frac{\Delta t}{\Delta V_i}(\sum_k \tilde{\sigma}_{pik} A_{ik}) T_{pk}^h - \overline{S}_{pi}^n \Delta t$$

$$(\rho c_{vp})_i T_{pi}^h + \frac{3\Delta t}{4\Delta V_i}(\sum_k \tilde{\sigma}_{pik} A_{ik}) T_{pi}^h - \frac{\Delta t}{4\Delta V_i}(\sum_k \tilde{\sigma}_{pik} A_{ik}) T_{pi}^n = (\rho c_{vp})_i T_{pi} + \frac{3\Delta t}{4\Delta V_i}(\sum_k \tilde{\sigma}_{pik} A_{ik}) T_{pk}^h - \frac{\Delta t}{4\Delta V_i}(\sum_k \tilde{\sigma}_{pik} A_{ik}) T_{pk}^n - \frac{\Delta t}{2}\overline{S}_{ei}^h$$

Numerical examples

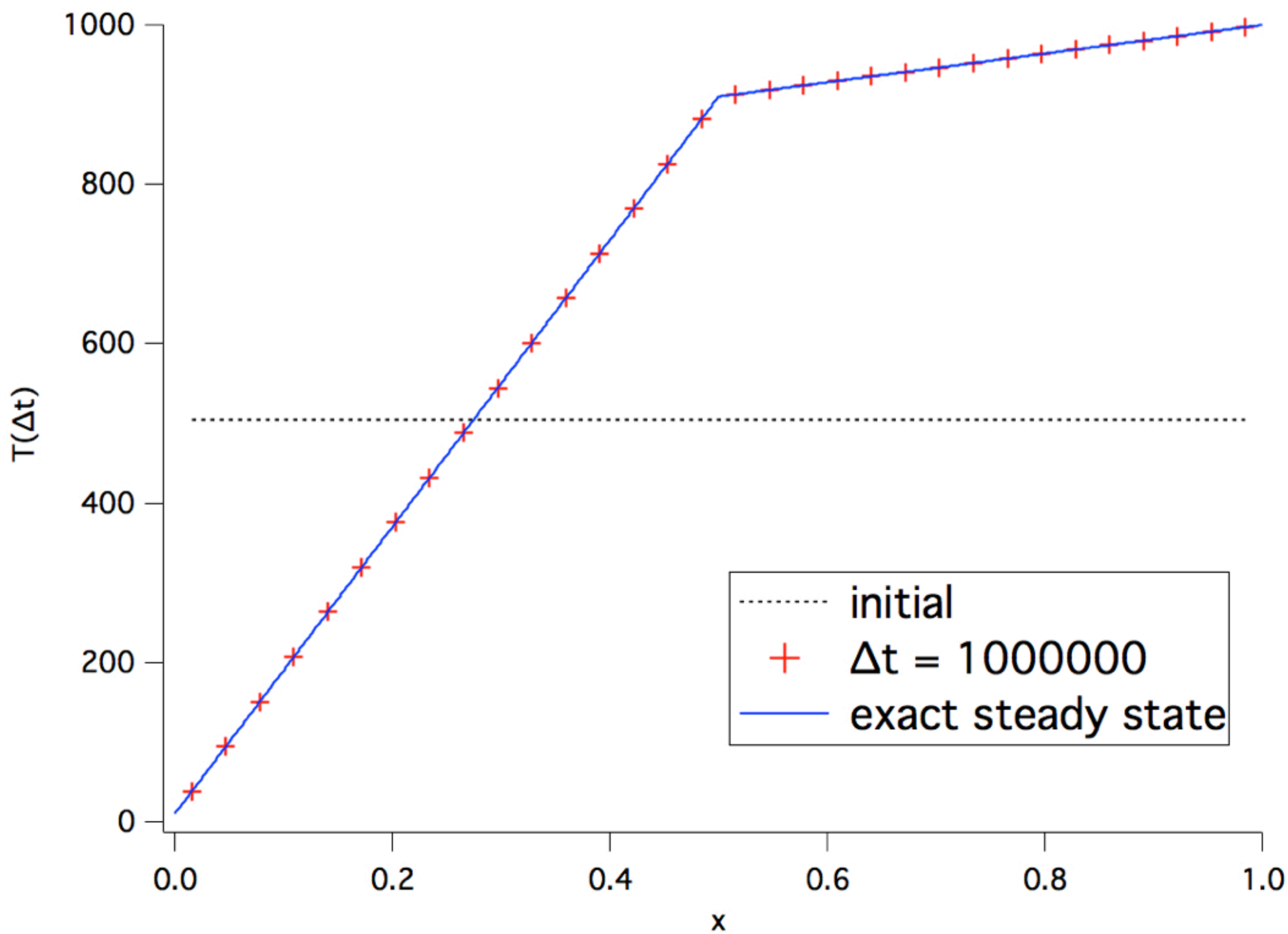
Numerical Example

1D temperature after one time step for $N = 16$

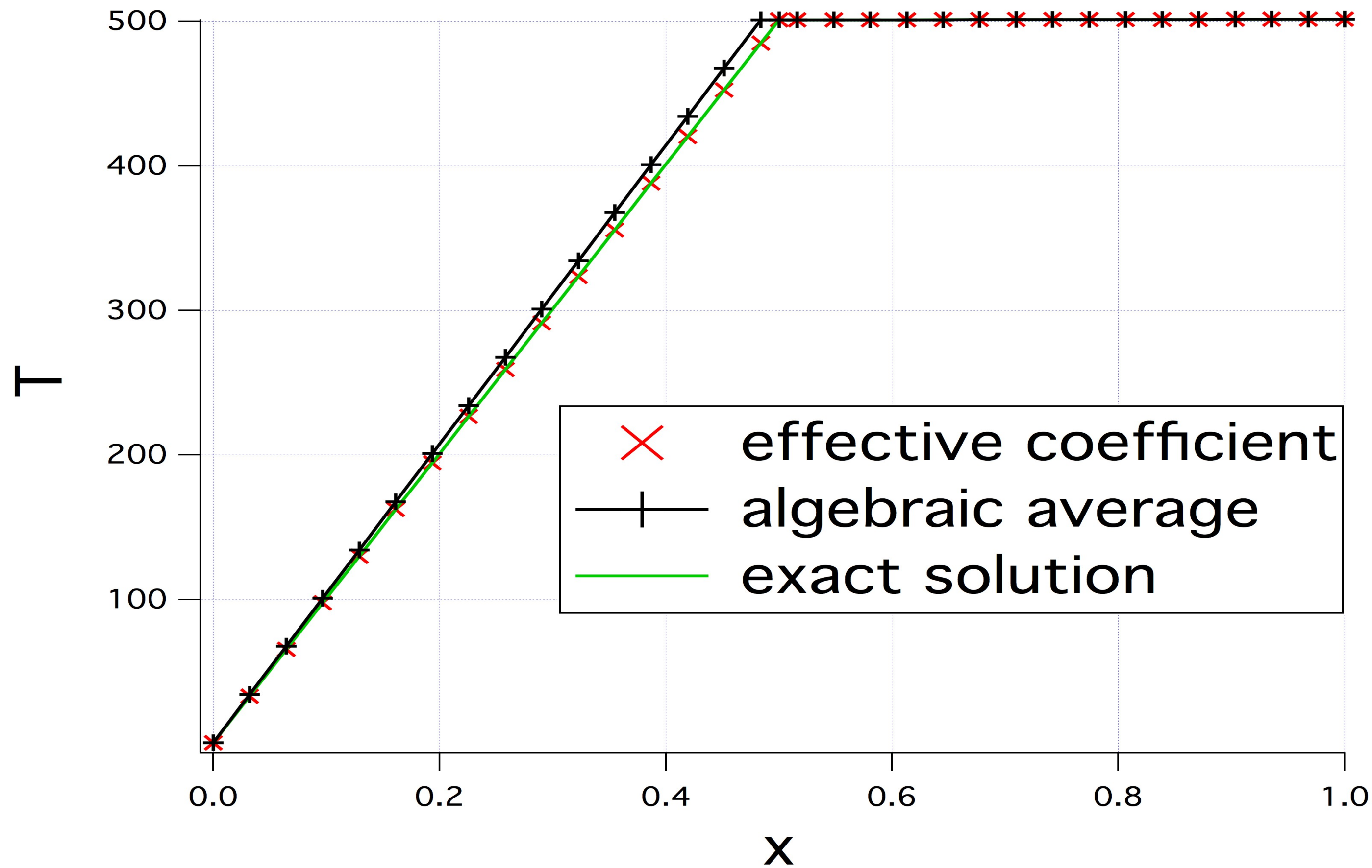


Numerical Examples

1D temperature after one time step for N = 32, 2 mats



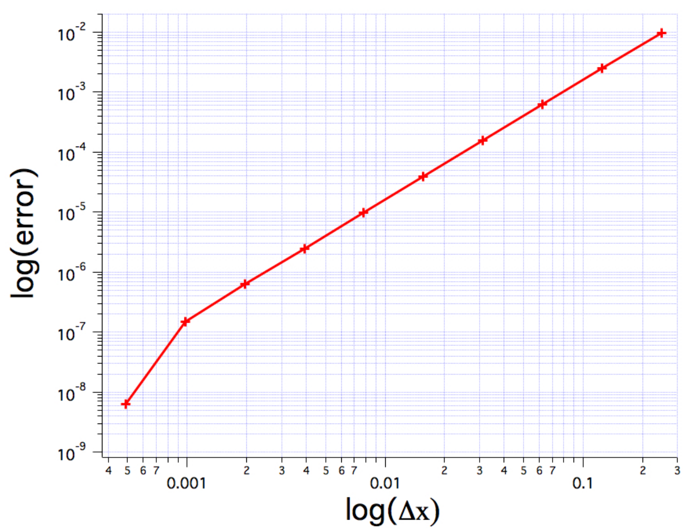
Numerical Examples

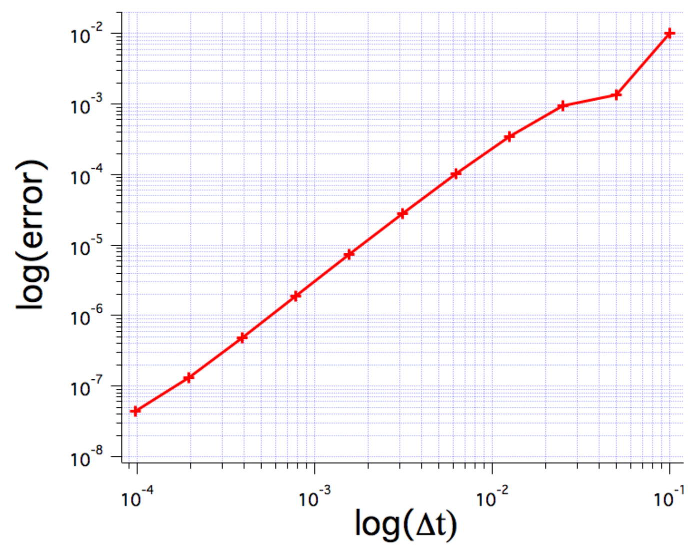


Example for Accuracy

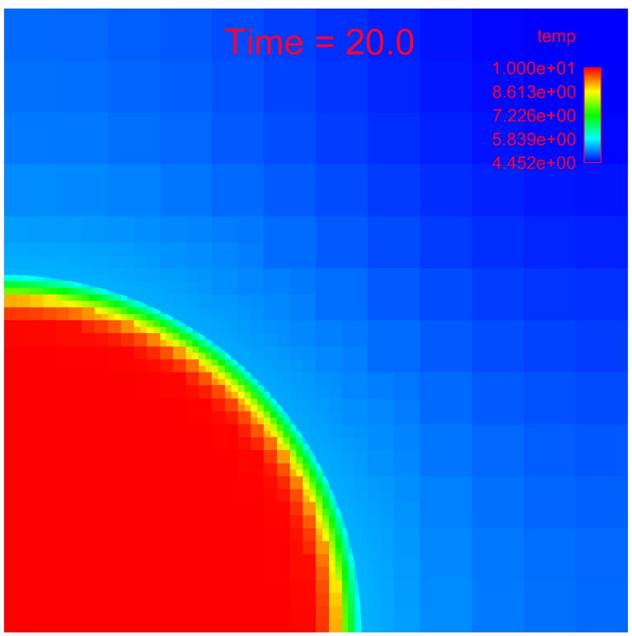
$$T(t,x) = 1 + e^{-(2\pi)^2 \kappa t} \sin(2\pi x). \qquad (5.1)$$

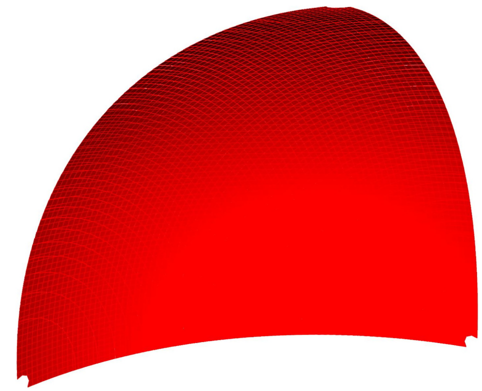
$$\text{error} \equiv \sum_{i=0}^{n} \left\| T_i(t) - T_i^{exact}(t) \right\| \Delta x_i$$

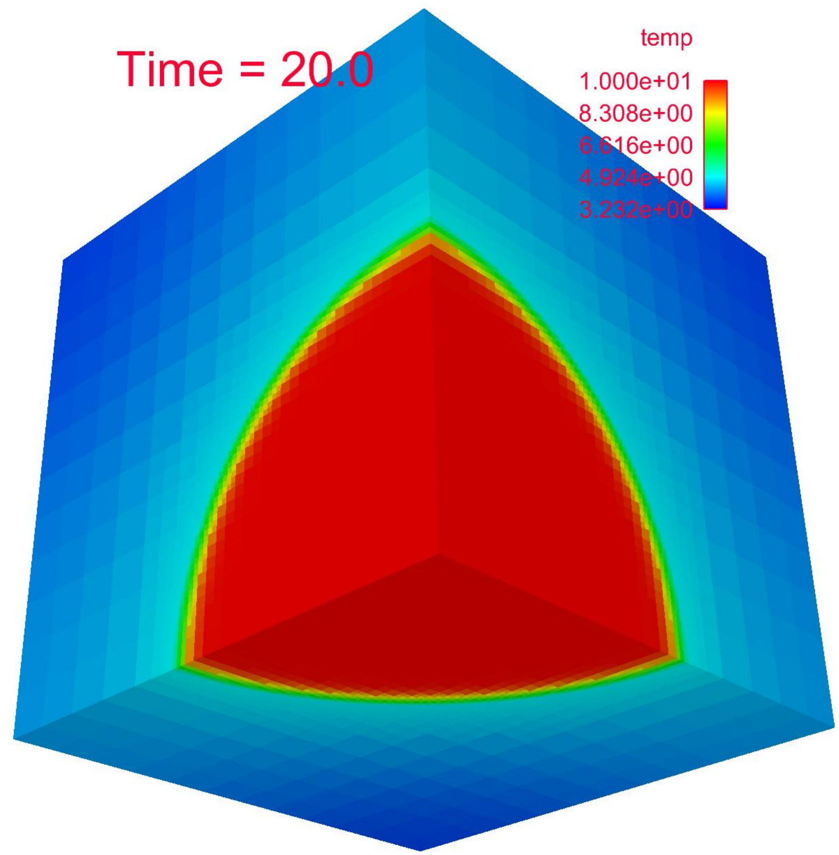




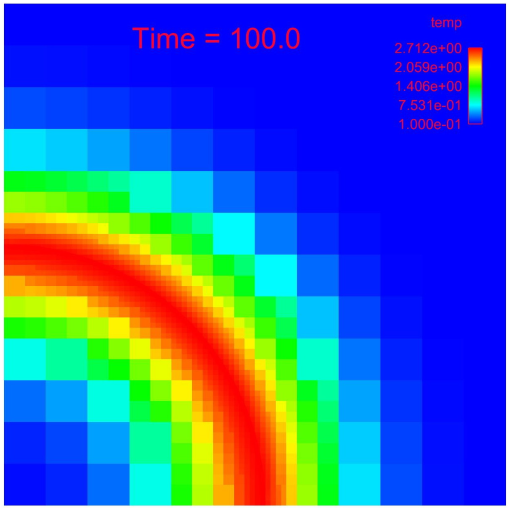
Example on polyhedral meshe

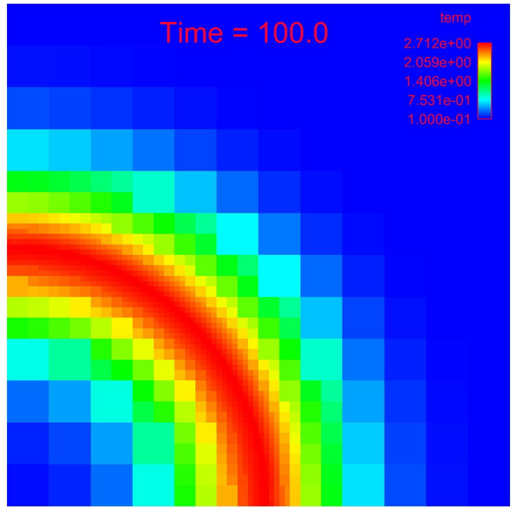


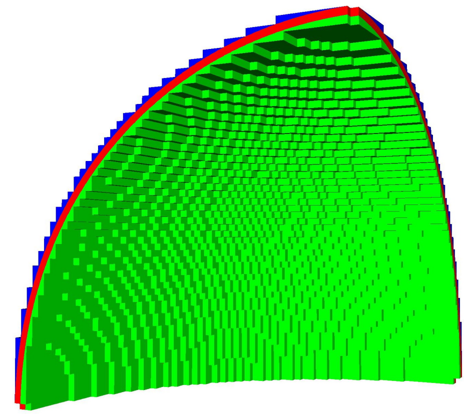


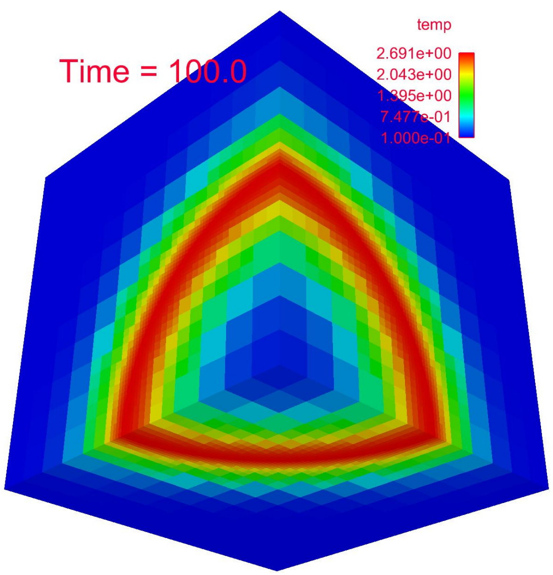


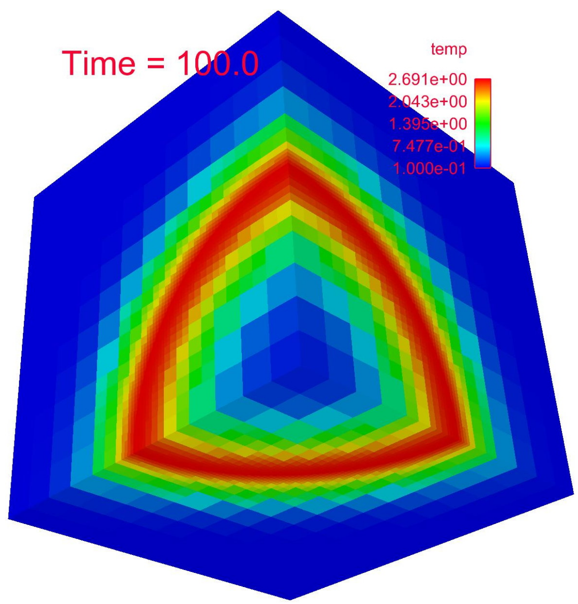
Example for different time steps

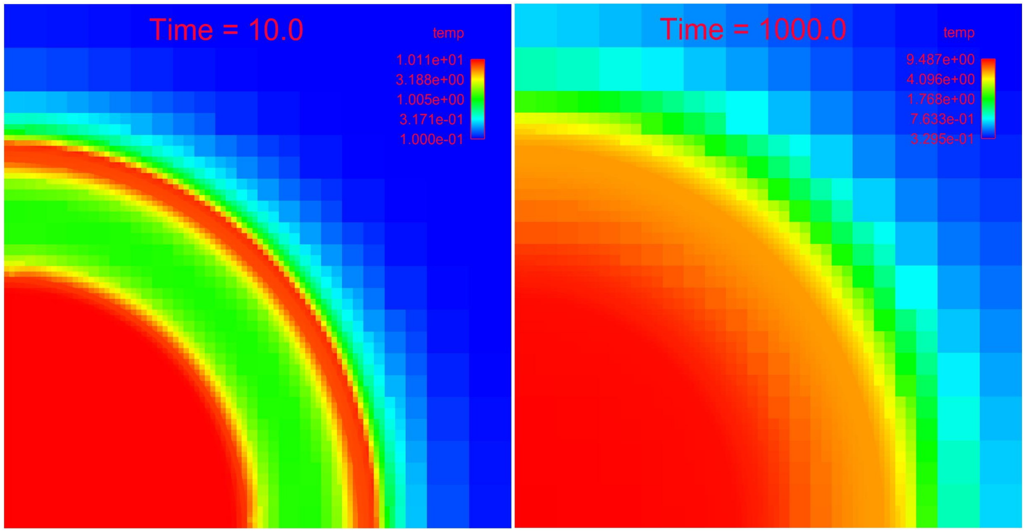


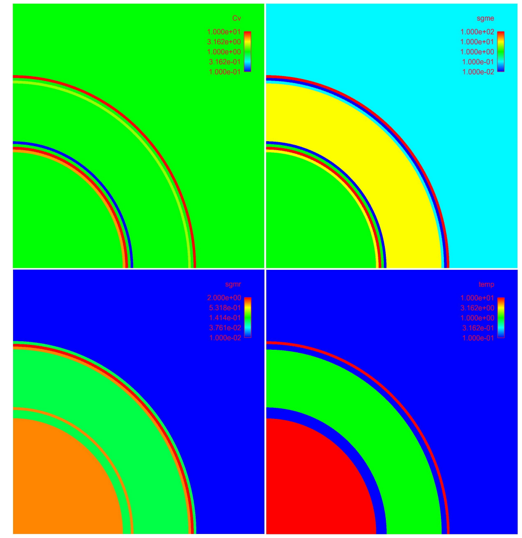




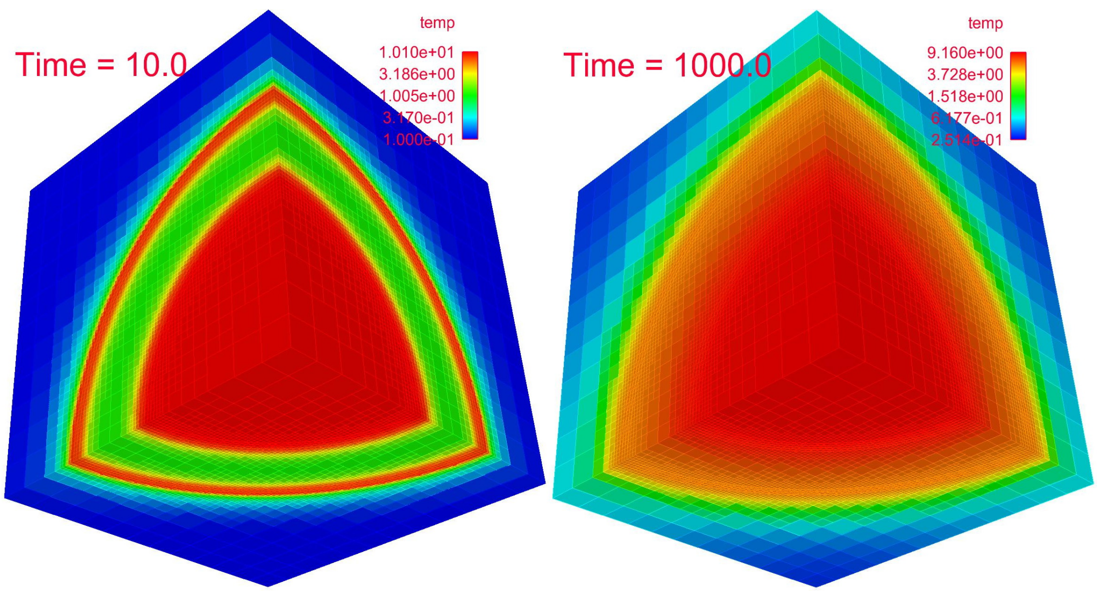


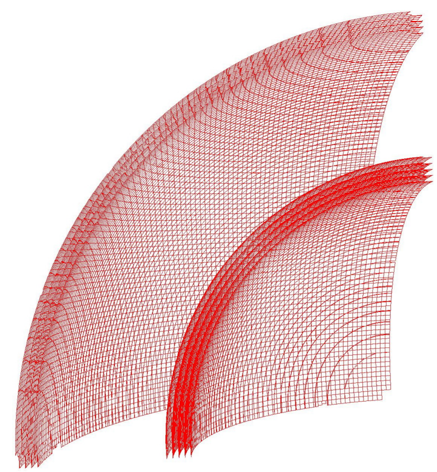




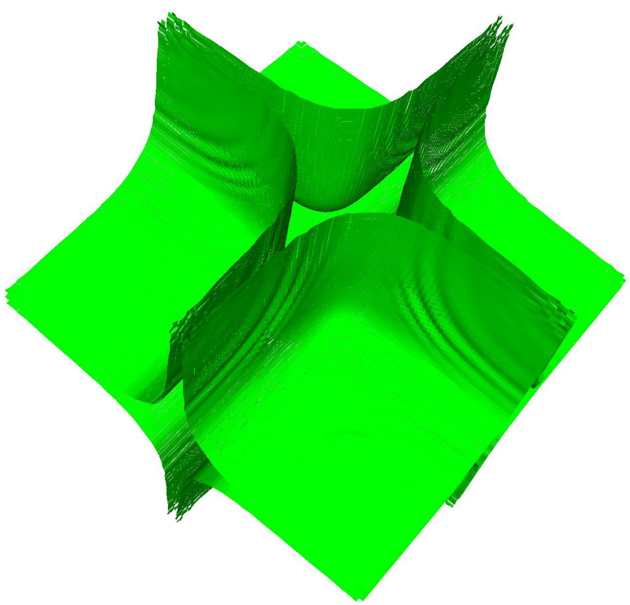


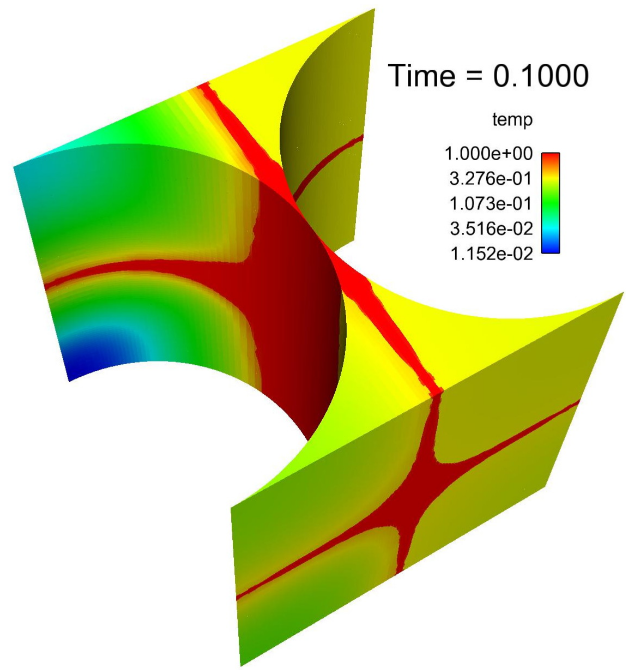
Cv: 0.1 to 10
mat dif coef: 0.01 to 100
rad dif coef: 0.01 to 2





More Example





Conclusions

- Implemented interface reconstruction on 3D AMR meshes.
- Developed diffusion solver on 2D & 3D general polyhedral meshes.
  - Second order accuracy in space and time.
  - Correct steady states for large time step.
  - Correct treatment of discontinuity on general polyhedral meshes.
- Features demonstrated through numerical examples.

Future Work

- Further complete this work in Roxane.
- Develop new linear solver with much less communication (but with much more computing).