Simulation of Blast Wave Attenuation by Aqueous Foams

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Based on works done with:
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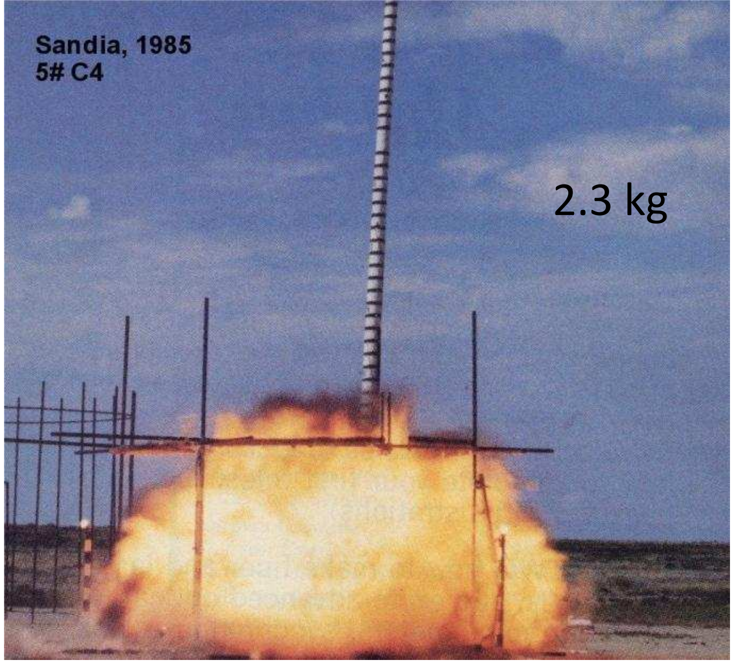
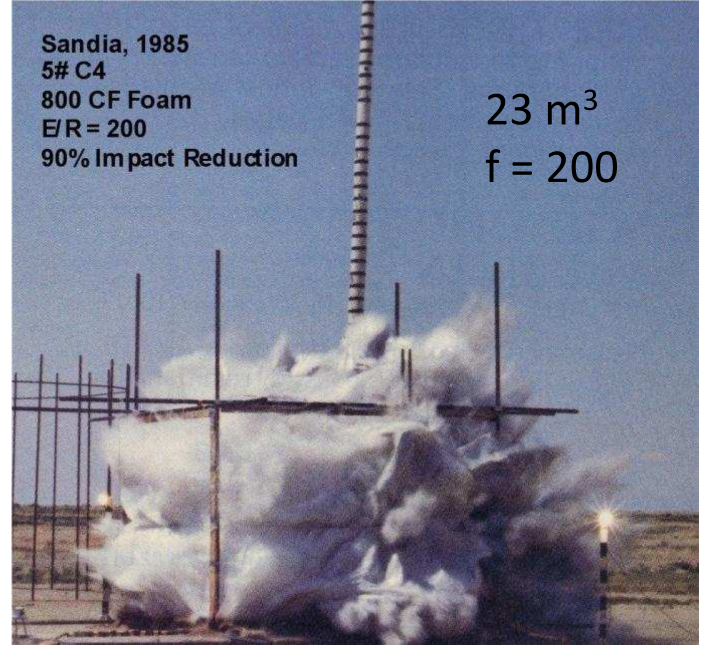




Effects on a van located at 9 meters from the charge

1 Introduction

Experimental evidences show that aqueous foam mitigate significantly the pressure loading produced by the detonation of high explosives

- Aim: Design of a numerical tool for the simulation of propagation of shock waves in aqueous foam
- Aqueous Foams: From the fluid dynamic point of view aqueous foams are a two phase medium containing water and air
- Physical model for simulation: Multi-Phase model with kinematic and thermal disequilibrium

2 Geometry

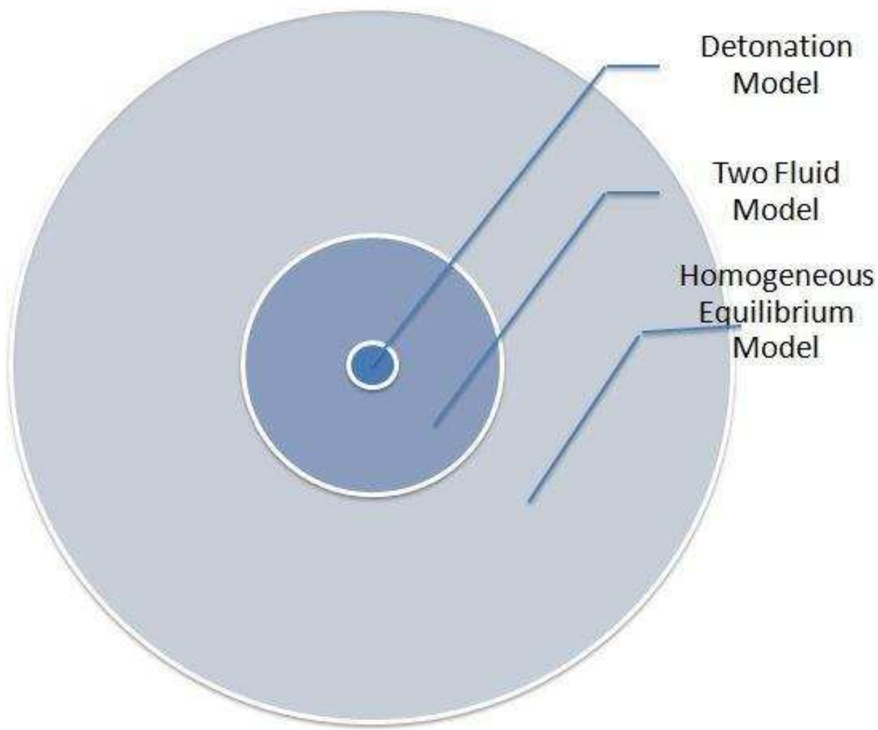


Figure 1: For each zone an appropriate model

3 Numerical Features

- Finite Volume Method
- Upwinding through Characteristic Fluxes
- $3D$ flows with spherical symmetry
- 2nd order (MUSCL), Time implicit

Some specific difficulties:

- Stiff source terms
- Non conservative terms for the two fluid models
- Very strong gradients
- Real Equation of State for Water and Steam in a very large range

4 Physical Models

4.1 The gaseous phase

The volume fractions α_s, α_a et α_w satisfy:

$$\alpha_s + \alpha_a + \alpha_w = 1$$

At one atmosphere and temperature of the room:

- $\alpha_s = 0$, $\alpha_w = 1/f$ and $\alpha_a = 1 - 1/f$
- $f > 1$ is called the expansion ratio of the foam

Denoting by α_g the gaseous volume fraction, by ρ_g the density of the gas and by e_g its specific internal energy, we have:

$$
\begin{aligned}
\alpha_g &= \alpha_s + \alpha_a\,, \\
\alpha_g \rho_g &= \alpha_s \rho_s + \alpha_a \rho_a\,, \\
\alpha_g \rho_g e_g &= \alpha_s \rho_s e_s + \alpha_a \rho_a e_a\,.
\end{aligned}
$$

Mass conservation of each gas and balance of momentum and total energy for the gaseous phase lead to:

$$
\begin{aligned}
(\alpha_s \rho_s)_t + \nabla \cdot (\alpha_s \rho_s u_g) &= Q_s, \\
(\alpha_a \rho_a)_t + \nabla \cdot (\alpha_a \rho_a u_g) &= 0, \\
(\alpha_g \rho_g u_g)_t + \nabla \cdot (\alpha_g \rho_g u_g \otimes u_g) + \alpha_g \nabla p &= Q_s\, u_i + C_{drag}(u_w - u_g)\,, \\
(\alpha_g \rho_g E_g)_t + \nabla \cdot (\alpha_g \rho_g H_g u_g) + p(\alpha_g)_t &= \\
= Q_s(h_{is} + \frac{|u_i|^2}{2}) &+ C_{drag}(u_w - u_g) \cdot u_i + Q_{is}\,.
\end{aligned}
$$

Air and steam have their own Equations of State (E.o.S.):

$$p = \mathcal{P}^k(\rho_k, e_k)\,, \quad T = \mathcal{T}^k(\rho_k, e_k)\,, \quad k \in \{a, s\}\,,$$

and one can show that the pressure p is in fact a function of 3 parameters: ρ_g, e_g and

$$\alpha \equiv \frac{\alpha_s - \alpha_a}{\alpha_g} = \frac{\alpha_s - \alpha_a}{\alpha_s + \alpha_a} \in [-1, +1]\,,$$

$$p = \mathcal{P}^g(\alpha, \rho_g, e_g)\,.$$

4.2 The liquid phase

Mass conservation of water and balance of momentum and total energy for the liquid phase lead to:

$$
\begin{aligned}
(\alpha_w \rho_w)_t + \nabla \cdot (\alpha_w \rho_w u_w) &= -Q_s, \\
(\alpha_w \rho_w u_w)_t + \nabla \cdot (\alpha_w \rho_w u_w \otimes u_w) + \alpha_w \nabla p &= \\
-Q_s\, u_i + C_{drag}(u_g - u_w)\,, & \\
(\alpha_w \rho_w E_w)_t + \nabla \cdot (\alpha_w \rho_w H_w u_w) + p(\alpha_w)_t &= Q_{iw} \\
-Q_s(h_{iw} + \frac{|u_i|^2}{2}) &+ C_{drag}(u_g - u_w) \cdot u_i\,.
\end{aligned}
$$

The liquid has also its own E.o.S.:

$$p = \mathcal{P}^w(\rho_w, e_w)\,, \quad T = \mathcal{T}^w(\rho_w, e_w)\,.$$

4.3 Closure relations

Mechanical closures The expression for the coefficient of the drag force, $\pm C_{drag}(u_w - u_g)$ is classical:

$$C_{drag} = \theta_\rho \frac{C^*}{r^*} \frac{\alpha_w \alpha_g \rho_w \rho_g}{\rho} |u_g - u_w| \, ,$$

where $\rho \equiv \alpha_g \rho_g + \alpha_w \rho_w$ and θ_ρ is a non dimensional number depending only on α_w, α_g, ρ_w, and ρ_g.

Concerning u_i, the interfacial velocity, several choices are possible, the more classical being:

$$u_i = \alpha_g u_g + \alpha_w u_w \, .$$

Thermodynamical closures: the case without phase change

In this case

$$Q_s = 0\,,$$

so we have to give Q_{is} et Q_{iw}. We have taken:

$$Q_{is} = Q(T_g - T_w)\,, \quad Q_{iw} = Q(T_w - T_g)\,,$$

with

$$Q \equiv \frac{3\lambda\alpha_w\rho_w}{\alpha_g\rho_g + \alpha_w\rho_w}\frac{1}{R_{mean}^2}\,, \quad R_{mean} = R_0\left(\frac{\rho_w}{\rho_w^0}\right)^{-\frac{1}{3}}\,,$$

$$\lambda = 0.033087\ W\,K\,m^{-1}\,, \quad \rho_w^0 = 1\ kg\,m^{-3}\,, \quad 10^{-3}\,m \leq R_0 \leq 10^{-6}\,m\,.$$

Thermodynamical closures: the case with phase change

In this case conservation of the total energy of the system (3 fluids) leads to:

$$Q_s(h_{is} - h_{iw}) + Q_{is} + Q_{iw} = 0\,.$$

Following classical modeling in Thermohydraulics, we take:

$$Q_{is} = \omega_{is}(h_{is} - h_s)\,, \quad Q_{iw} = \omega_{iw}(h_{iw} - h_w)\,,$$

where ω_{is} and ω_{iw} are interfacial liquid-vapor heat exchange coefficients. We choose to express these terms as functions of relaxation times. They take into account the fact that the liquid-vapor phase change is not instantaneous:

$$\omega_{is} = \frac{\alpha_s \alpha_w \rho_s}{\tau_{is}}\,, \quad \omega_{iw} = \frac{\alpha_s \alpha_w \rho_w}{\tau_{iw}}\,,$$

with the relaxation times $\tau_{is} = \tau_{iw} = 10^{-3}\, s$.
Finally, in case of phase change :

$$h_{is} = h_{sat,s}\,, \quad h_{iw} = h_{sat,w}\,,$$

where $h_{sat,s}$ and $h_{sat,w}$ are given in using the saturation curve of the equations of state.
Hence we find

$$Q_s = -\frac{Q_{is} + Q_{iw}}{h_{is} - h_{iw}}\,.$$

5 Equations of State

Air The usual perfect gas laws are used:

$$p = \mathcal{P}^a(\rho_a, e_a) = (\gamma_a - 1)\, \rho_a\, e_a\,, \quad T = \mathcal{T}^a(\rho_a, e_a) = \frac{e_a}{C_a^V}\,.$$

Steam and Water The International Association for the Properties of Water and Steam-IAPWS tables are used. They have been implemented as a Library (Freesteam) by John Pye and can be downloaded on sourceforge.

We have developed for our present purpose a new implementation, *Quicksteam*, designed for fluid flow simulation.

6 Numerical Method

We present the method for the discretization of the multi-phase system in the cartesian $1D$ case.

$$v_t + F(v)_x + \widetilde{C}(v)v_x + D(v)v_t = \widetilde{S}(v),$$

where $\delta \equiv \alpha_a\rho_a - \alpha_s\rho_s$

$$v = \begin{pmatrix} \delta \\ \alpha_g\rho_g \\ \alpha_w\rho_w \\ \alpha_g\rho_g u_g \\ \alpha_w\rho_w u_w \\ \alpha_g\rho_g E_g \\ \alpha_w\rho_w E_w \end{pmatrix}, \qquad F(v) = \begin{pmatrix} \delta u_g \\ \alpha_g\rho_g u_g \\ \alpha_w\rho_w u_w \\ \alpha_g(\rho_g u_g^2 + p - \pi) \\ \alpha_w(\rho_w u_w^2 + p - \pi) \\ \alpha_g\rho_g H_g u_g \\ \alpha_w\rho_w H_w u_w \end{pmatrix}.$$

We observe that the matrix $Id + D(v)$ is invertible and therefore we can rewrite our system as:

$$v_t + F(v)_x + C(v)\, v_x = S(v)\,,$$

and after spatial integration we get:

$$\frac{v_i^{n+1} - v_i^n}{\Delta t_n} + \frac{1}{\Delta x_i \Delta t_n} \int_{t_n}^{t_{n+1}} \int_{K_i} [Id + E(v)]\, F(v)_x \, dx\, dt =$$
$$= \frac{1}{\Delta x_i \Delta t_n} \int_{K_i} S(v(x,t))\, dx\, dt\,,$$

where the matrix $E(v)$ satisfies

$$E(v) J(v) = C(v)\,, \quad J(v) \equiv \nabla_v F(v)\,.$$

$$v_t + F(v)_x + C(v)\, v_x = S(v)$$

The scheme reads:

$$\frac{v_i^{n+1} - v_i^n}{\Delta t_n} + \frac{1}{\Delta x_i} \left(Id + E(v_i^n)\right) \left(\mathcal{F}(\mu^n_{i+\frac{1}{2}}; v_i^{n+1}, v_{i+1}^{n+1}) - \right.$$
$$\left. \mathcal{F}(\mu^n_{i-\frac{1}{2}}; v_{i-1}^{n+1}, v_i^{n+1})\right) - S(v_i^n, v_i^{n+1}) = 0\,,$$

where

$$\mu^n_{i+\frac{1}{2}} = \frac{\Delta x_i v_i^n + \Delta x_{i+1} v_{i+1}^n}{\Delta x_i + \Delta x_{i+1}}\,,$$

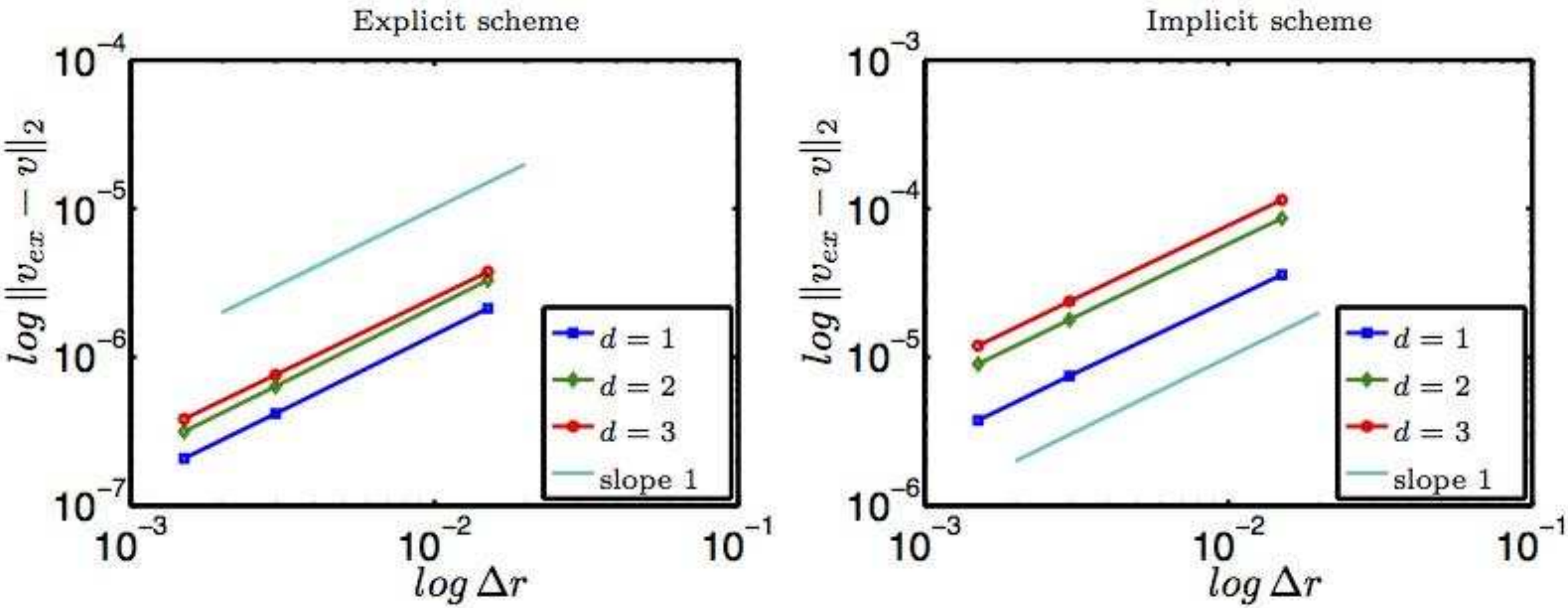
and

$$\mathcal{F}(\mu; v, w) = \frac{F(v) + F(w)}{2} - Sign(\widetilde{A}(\mu))\frac{F(w) - F(v)}{2}\,,$$

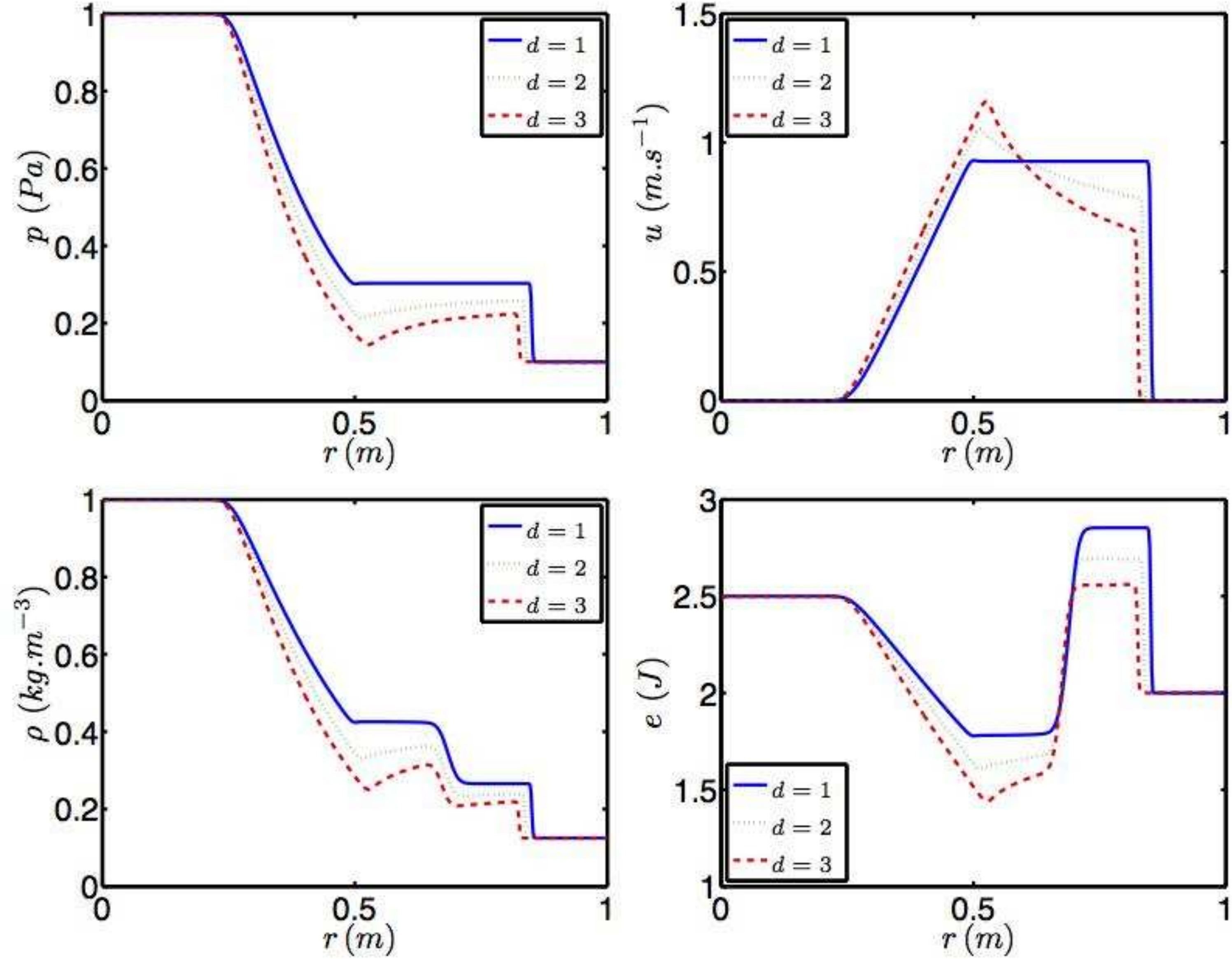
$\widetilde{A}(v) = J(v)[J(v) + C(v)]J(v)^{-1}.$

7 Numerical Results

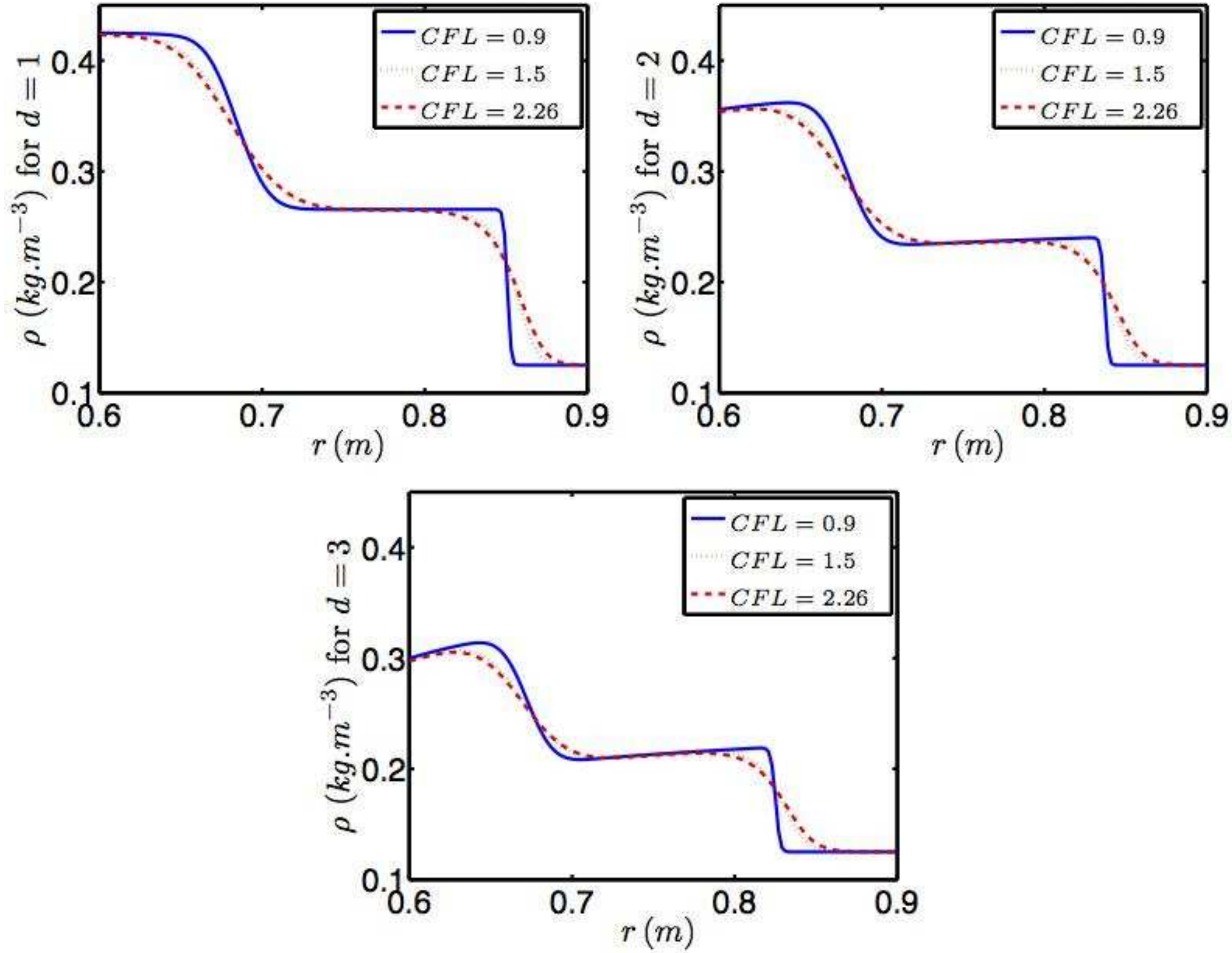
Test on Euler Equation



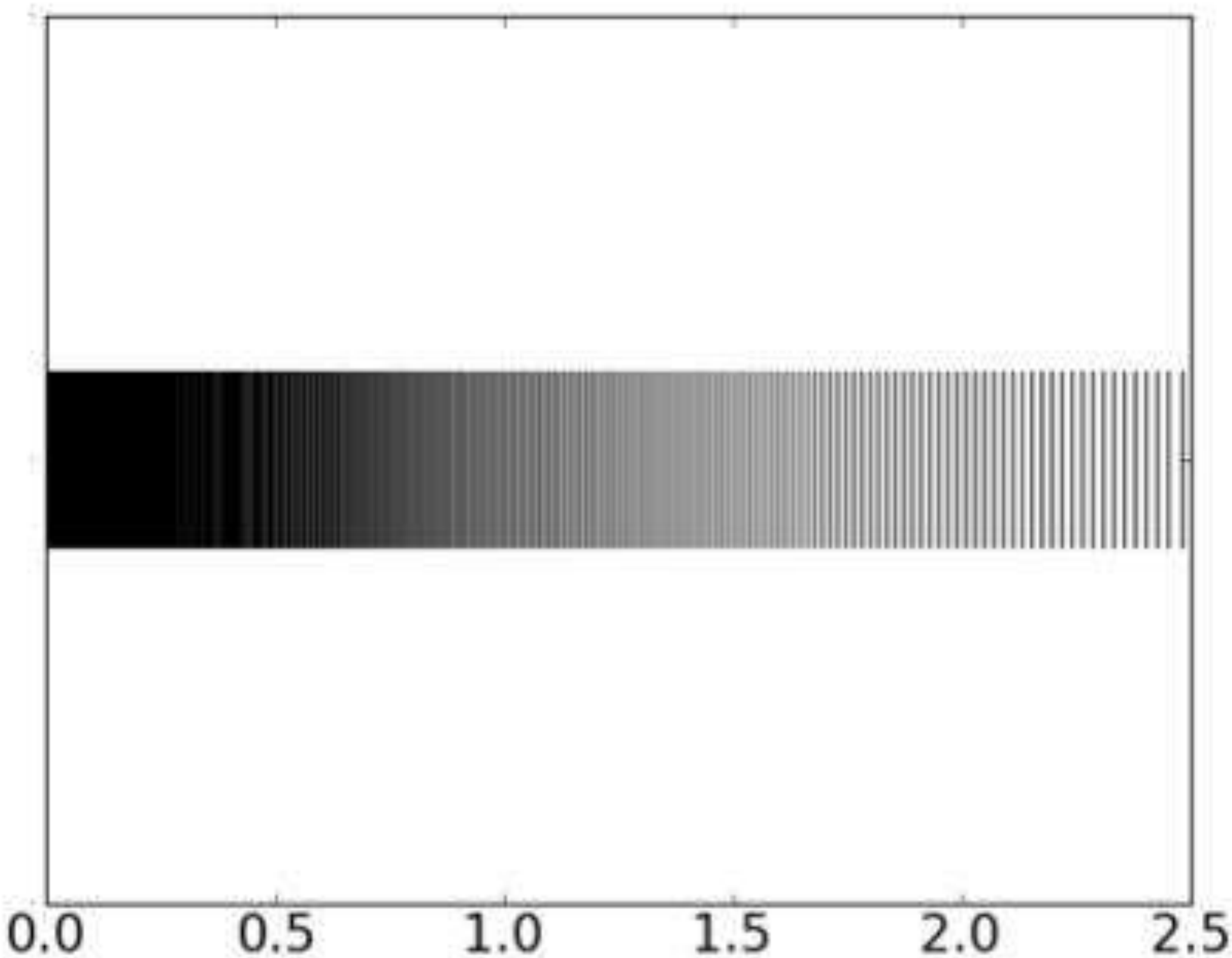
Euler's equations : validation with an analytical solution - Space discretization error on $v=(\rho,\rho u,\rho E)$ computed with the discrete L^2-norm at the final time $t=0.2\,s$ and for $d=1$, 2, 3.



Euler's equations : a shock tube problem - Numerical results obtained with the explicit FVCF scheme ($N=500$ and $CFL=0.9$) for $d=1$, 2, 3 at $t=0.2\,s$.



Euler's equations : a shock tube problem - Density profiles for several values of the Courant number CFL and for a dimension parameter d successively equal to 1 (top left), 2 (top right), and 3 (bottom).

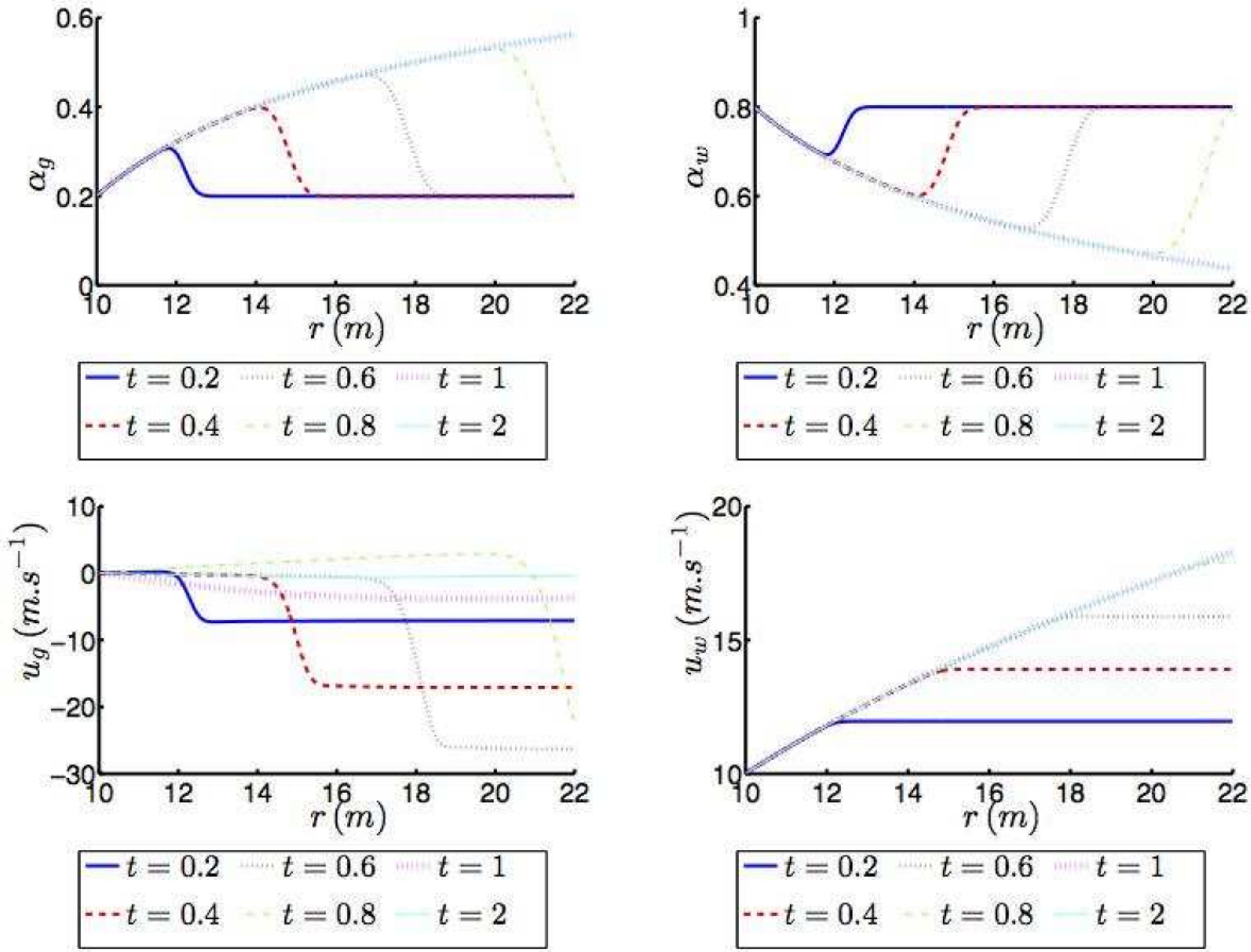


Euler's equations: spherically symmetric blast wave in air - The vertical lines represent the boundaries of the control volumes of the $1D$ mesh.

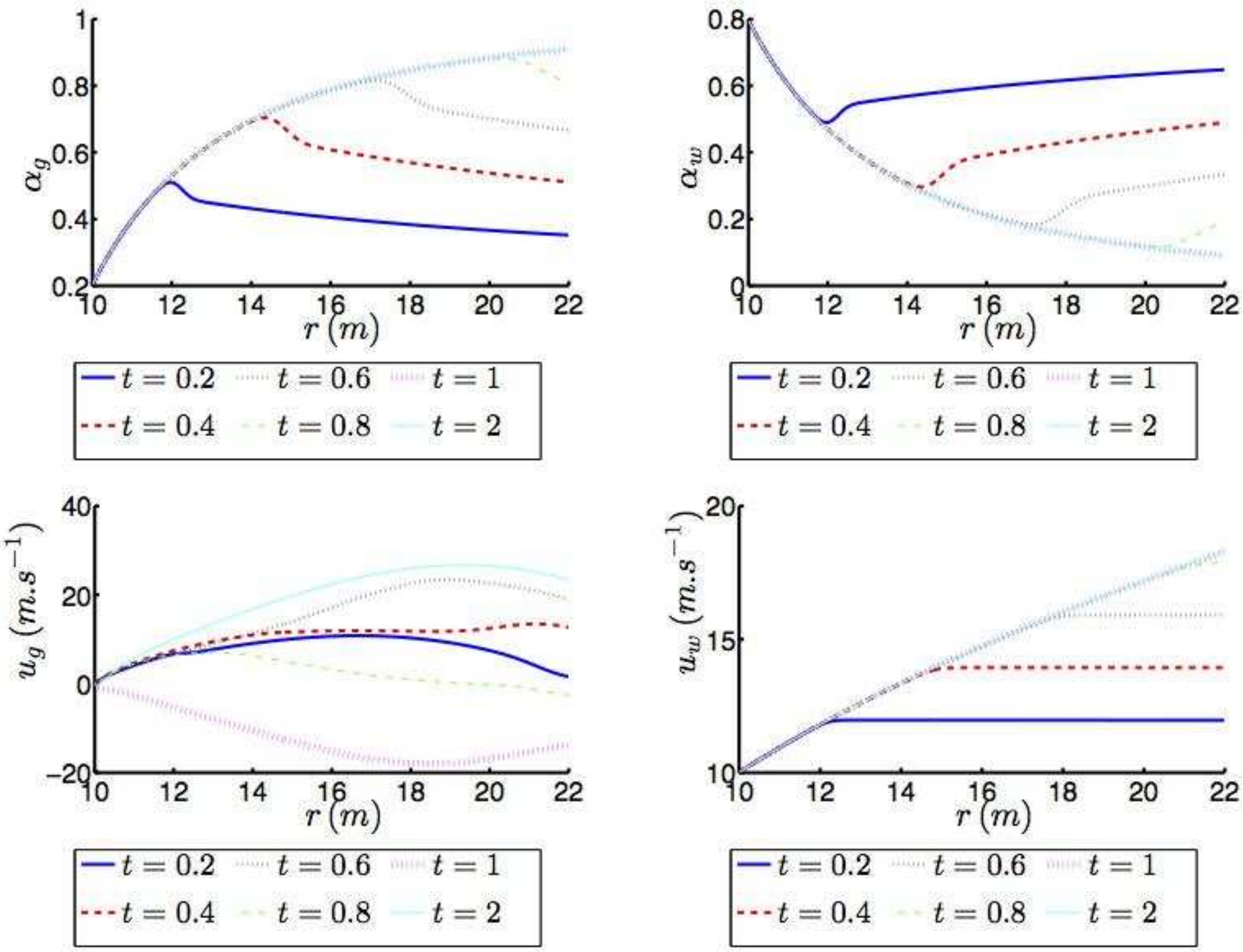
Time [ms]	0.05	0.15	0.35	0.5	1.	2.	3.
Distance (real) [m]	0.2	0.4	0.6	0.75	1.05	1.55	1.95
Distance (simulation) [m]	0.195	0.394	0.628	0.751	1.074	1.567	2.009

Table 1: Euler's equations: spherically symmetric blast wave in air - Shock front distance from the charge center at different times.

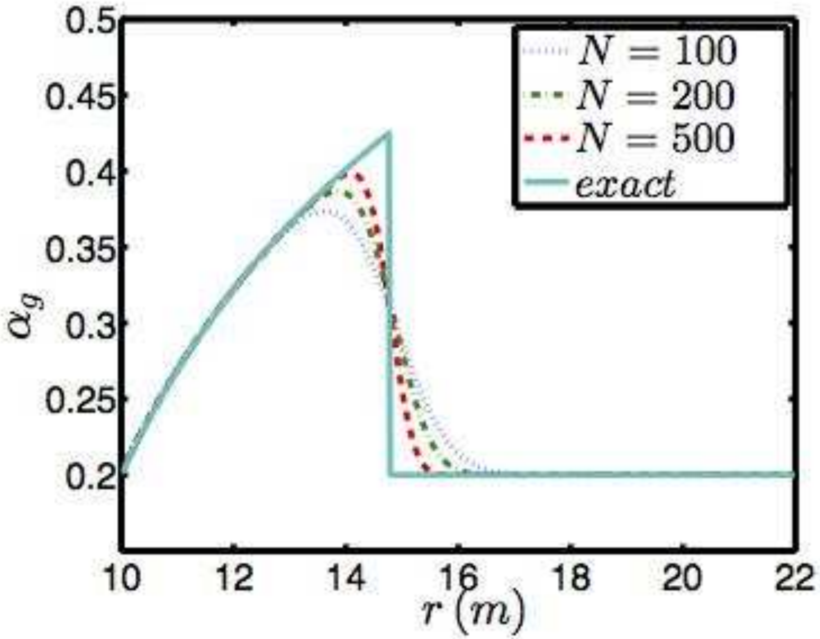
Test on Ransom’s faucet flow

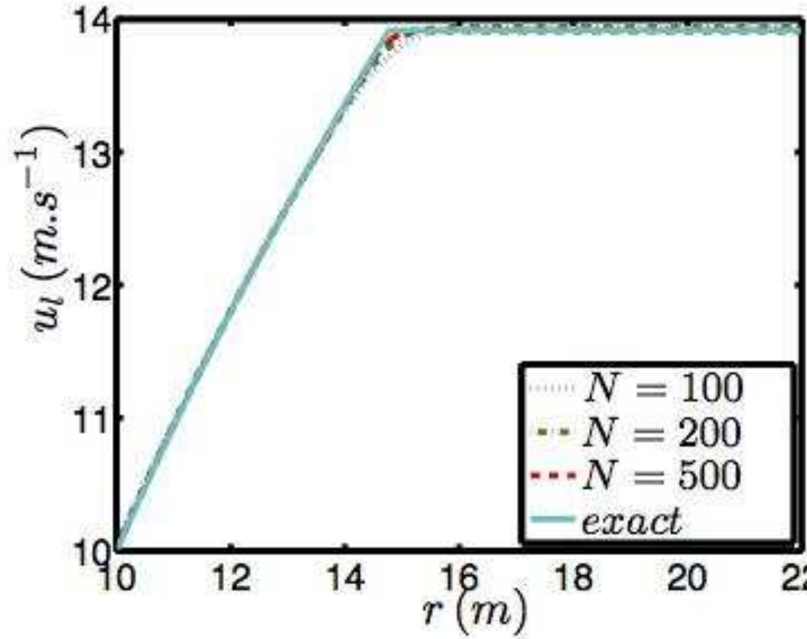


Ransom's faucet test case - Numerical solutions for $d=1$ at different times (s).

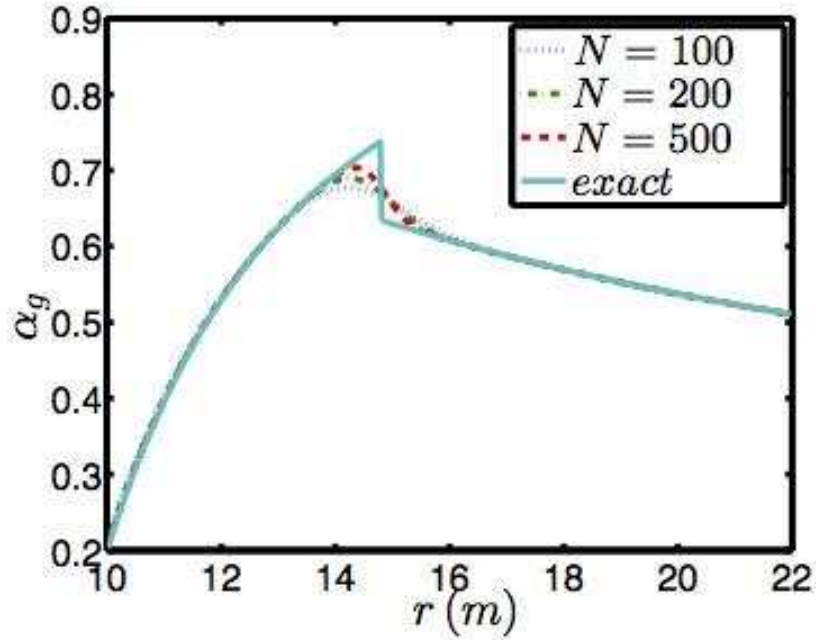


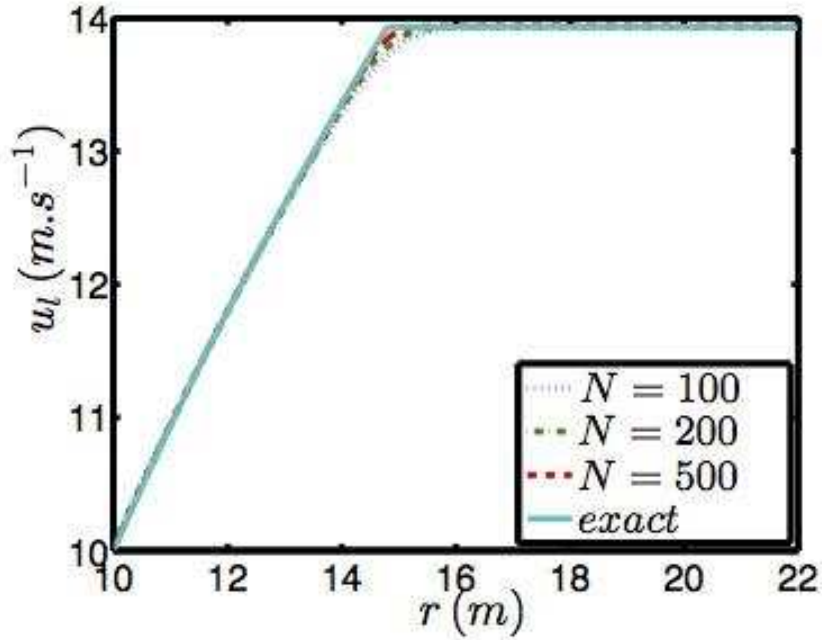
Ransom's faucet test case - Numerical solutions for $d=3$ at different times (s).





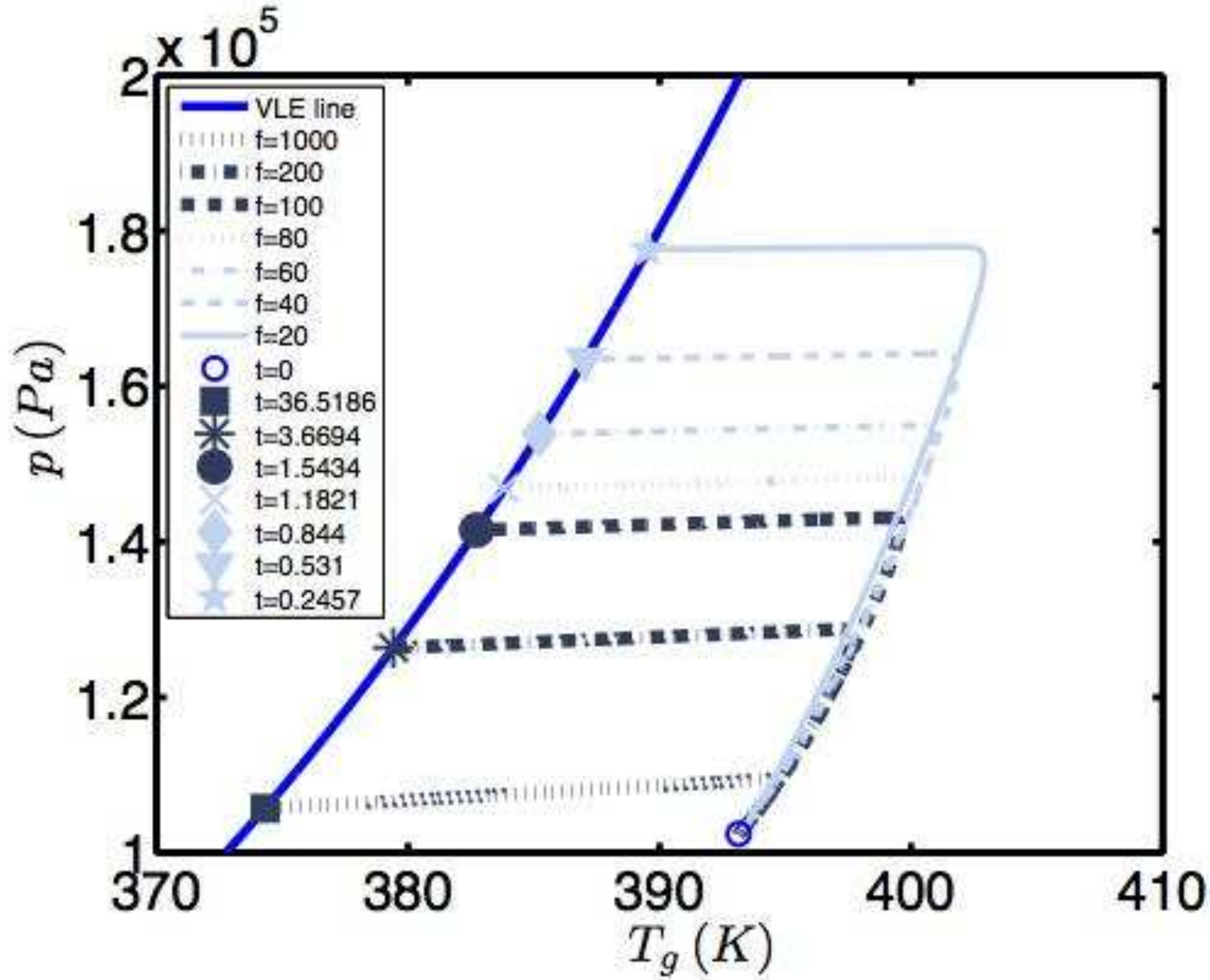
Ransom's faucet test case - Grid dependency for $d=1$



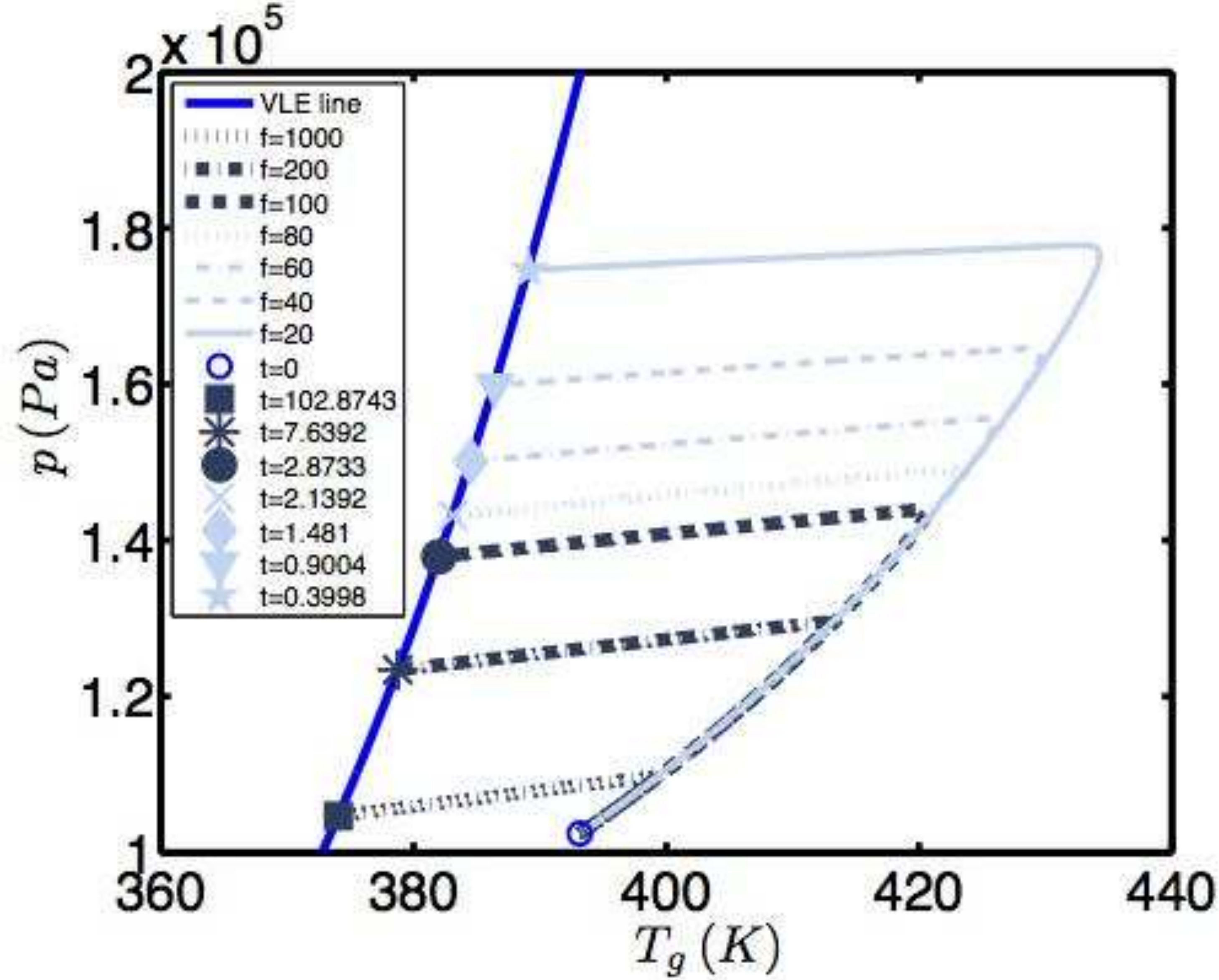


Ransom's faucet test case - Grid dependency for $d=3$

Test on change of phase

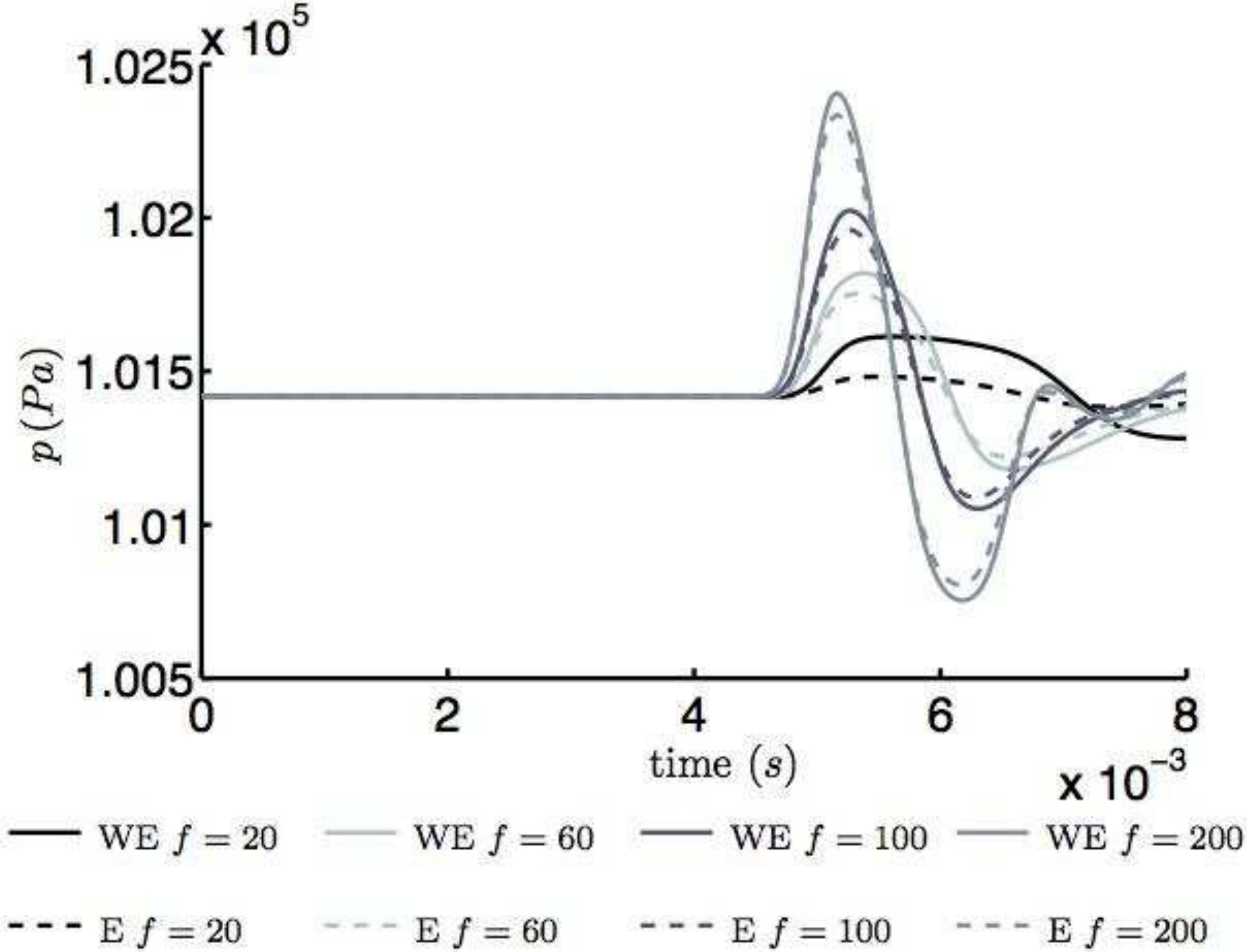


Time evolution of a water-air state (T_g,p) towards the liquid-vapor equilibrium when using the analytical thermodynamic ($d=1$).

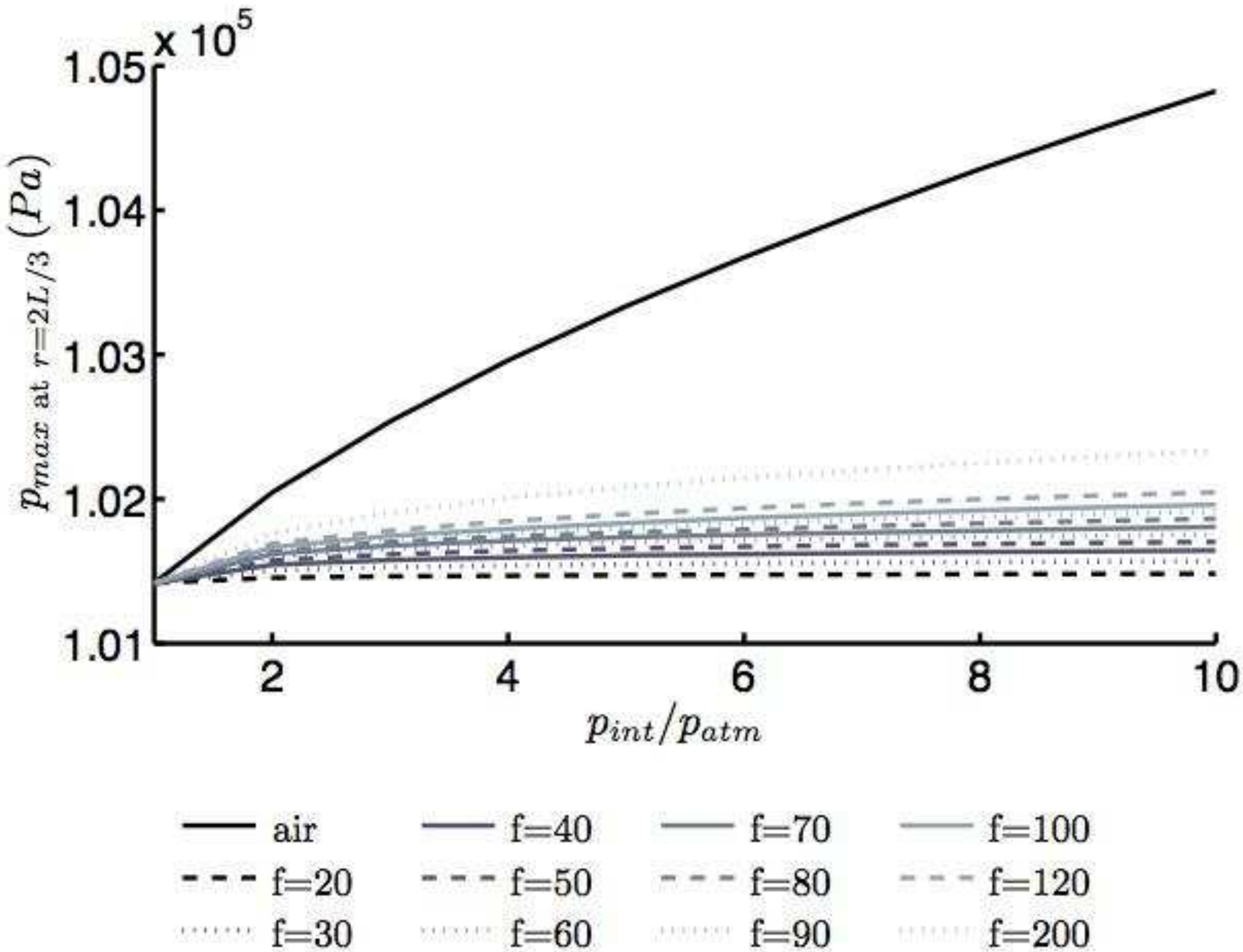


Time evolution of a water-air state (T_g,p) towards the liquid-vapor equilibrium when using the $Quicksteam$ software ($d=1$).

Pressure attenuation due to the presence of foam

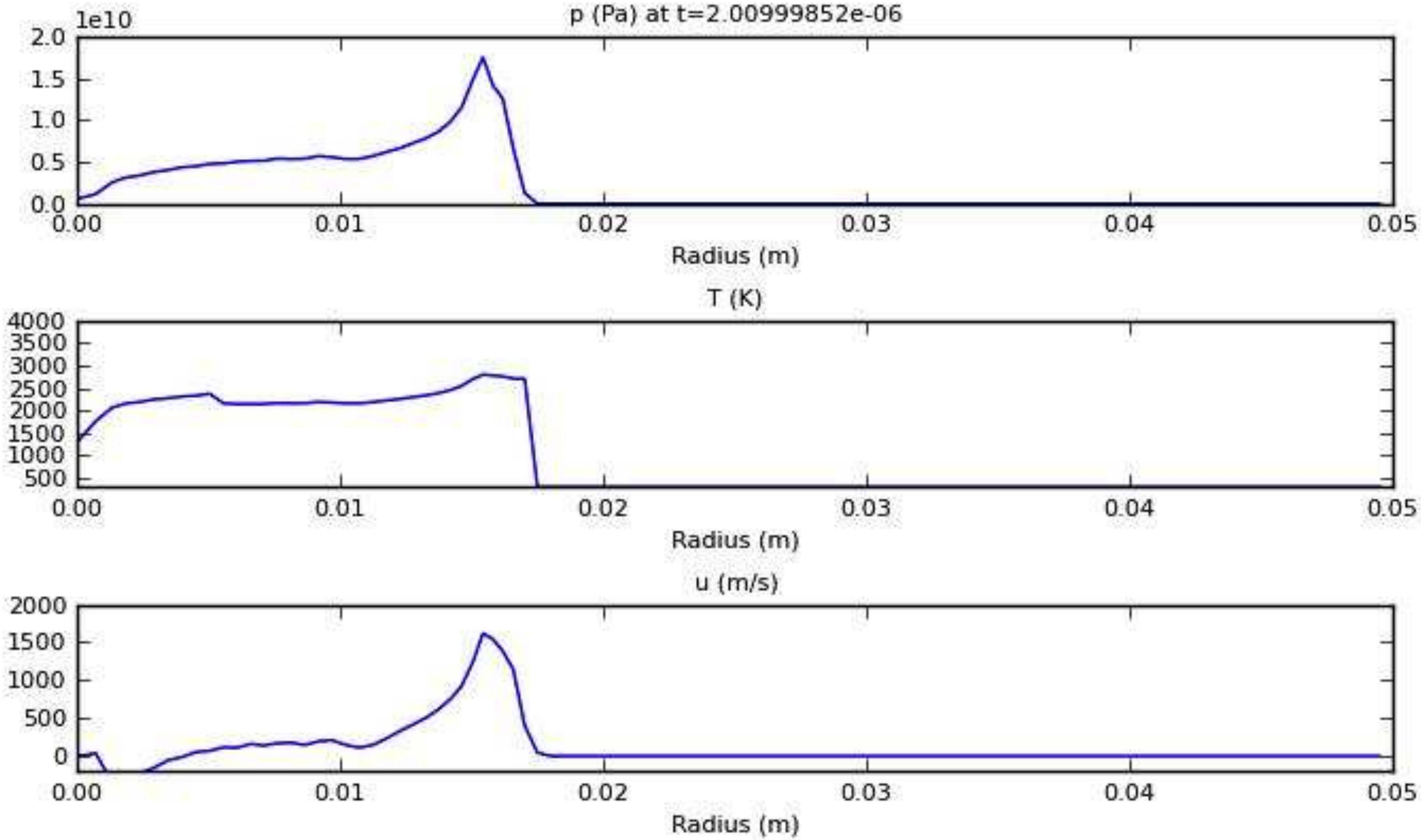


Foam blast surpression test: time evolution of the pressure p at $r=2L/3$, without evaporation (WE) and with evaporation (E).



Foam blast surpression test: overpressure reduction in aqueous foam of different expansion ratio.

Detonation model



Blast wave from a spherical charge of TNT (radius equal to \$5cm\$) computed with C.L. Mader's SIN code.