Semi-analytic Radiative Shock Solutions with Grey S_n Transport

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Overview

What is a semi-analytic shock solution

- Relevant PDE's reduced to system of ODE's and solved using a standard solver with error control.
- Provides rad-hydro benchmark solutions assuming certain physics models.
- Improve our theoretical understanding.
  - Equilibrium Diffusion - Radiative shocks can be continuous for small and large values of $\mathcal{M}_0$.
  - Nonequilibrium Diffusion - A Zel'dovich spike may exist independently of the embedded hydrodynamic shock.
  - Radiative transfer - Anti-diffusive shocks exist for certain ranges of $\mathcal{M}_0$, which diffusion theory fails to model.

Previous approximate and semi-analytic solutions

- Sen & Guess (1957)
- Heaslet & Brown (1963)
- Drake (2007)
- Lowrie & Rauenzahn (2007)
- Lowrie and Edwards (2008)
- McClarren & Drake (2010)

Assumptions

- The local material is sufficiently hot for radiation to affect the hydrodynamics $> 10^6\ K$.
- Single material temperature.
- Radiation can be treated in the geometric optics limit.
- S_n radiation model.
- Grey opacities and an ideal-gas γ-law EOS
- An infinite medium (thick-thick shocks).
- Material is non-relativistic.

RH equations and the EOS

The 1-D nondimensional steady-state lab-frame RH equations, correct through $\mathcal{O}(\beta)$ with $\mathcal{O}(\beta^2)$ conservation corrections, are

$$\partial_x \left(\rho u\right) = 0$$

$$\partial_x \left(\rho u^2 + p_m\right) = -P_0 S_{rp}$$

$$\partial_x \left[\beta \left(\frac{1}{2}\rho u^2 + \rho e + p_m\right)\right] = -P_0 S_{re}$$

$$\mathcal{C}\mu\partial_x I = -\sigma_t I + \frac{\sigma_s}{4\pi} E_r + \frac{\sigma_a}{4\pi} T^4 - 2\frac{\sigma_s}{4\pi\mathcal{C}}\beta F_r +$$

$$\beta\mu\left(\sigma_t I + \frac{3\sigma_s}{4\pi} E_r + \frac{3\sigma_a}{4\pi} T^4\right) + \frac{1}{4\pi}\left(\beta^2\sigma_t\left(E_r + P_r\right) + 6\mu\beta^2\frac{\sigma_a}{\mathcal{C}} F_r\right),$$

with an ideal-gas γ-law EOS

$$p_m = \left(\gamma - 1\right)\rho e \quad \& \quad e = \frac{T}{\gamma\left(\gamma - 1\right)}.$$

The Radiation Moment Equations

- The radiation energy equation and momentum equations are obtained by taking the zero'th and first angular moments, respectively, of the grey transport equation:

$$\frac{\partial F_r}{\partial x} = S_{re} \quad ,$$

$$\mathcal{C}\frac{\partial P_r}{\partial x} = S_{rp} \quad .$$

- The radiation moment equation have three unknowns: E_r, F_r, and P_r and become closed under the assumption that $P_r = fE_r$, where f is called the variable Eddington factor.
- However, the transport equation must be solved for f.
- This suggests a straightforward global iterative solution procedure.

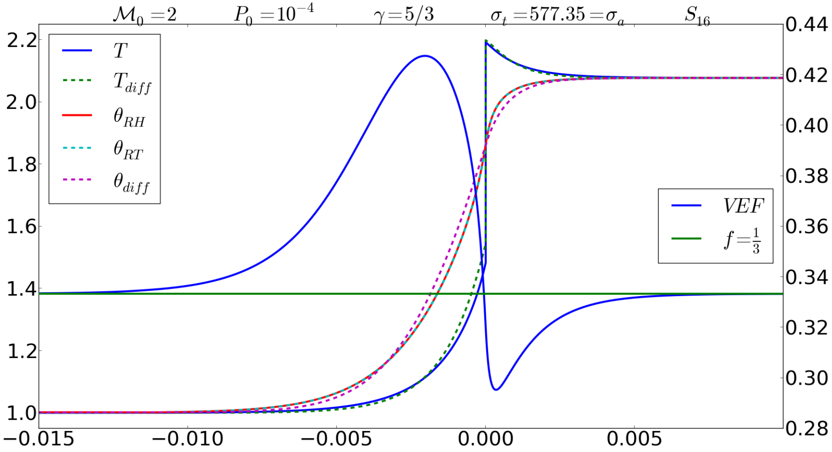
Global Solution Algorithm

- Begin with solution algorithm of Lowrie and Edwards.
- Their model assumes grey diffusion (EF=1/3).
- Replace their fixed EF with a VEF.
- Overall solution process is iterative:
  - Assume VEF = 1/3.
  - Solve "reduced" RH equations (Euler plus rad energy and momentum equations with assumed VEF dependence).
  - Use variables from rad-hydro solve to construct right side of transport equation, $\mathcal{C}\mu\partial_x I + \sigma_t I = q$.
  - Perform sweep (invert left-hand side S_n operator using ODE solver with error control).
  - Construct VEF function from intensity solution.
  - Repeat until converged: two versions of E_r, F_r, and P_r must agree.

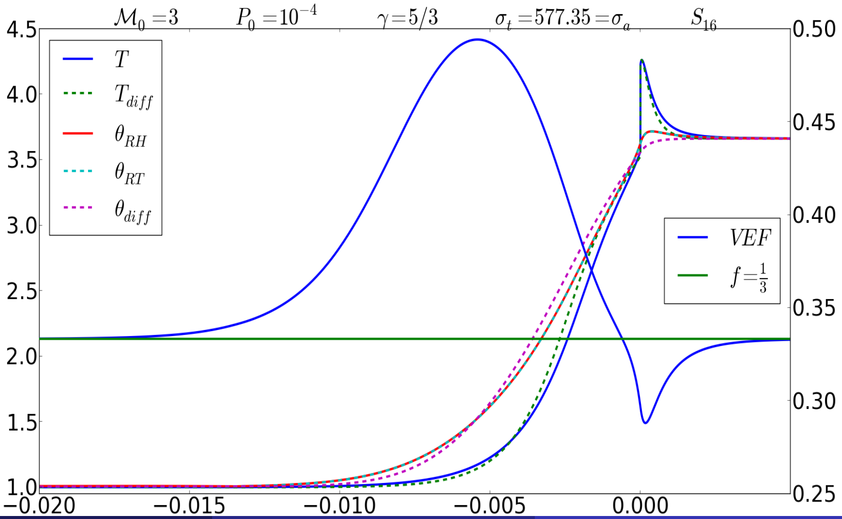
Reduced-System Solution Algorithm

- Define upstream conditions at $x = -\infty$.
- Derive downstream final conditions at $x = +\infty$ using continuity of flux (Rankine-Hugoniot conditions).
- Reduce system to two ODE's.
- Start with upstream boundary condition, integrate from upstream to downstream using starting trick.
- Start with downstream boundary, integrate from downstream to upstream using starting trick.
- Connect two solutions to obtain total solution where radiative flux and radiative intensity are continuous.

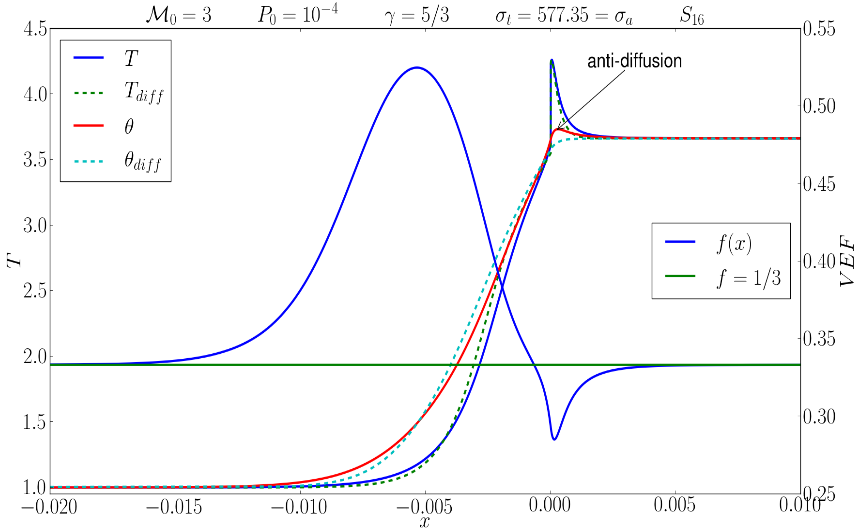
$\mathcal{M}_0 = 2$ Comparison to nonequilibrium diffusion



$\mathcal{M}_0 = 3$ Comparison to nonequilibrium diffusion



Anti-diffusion



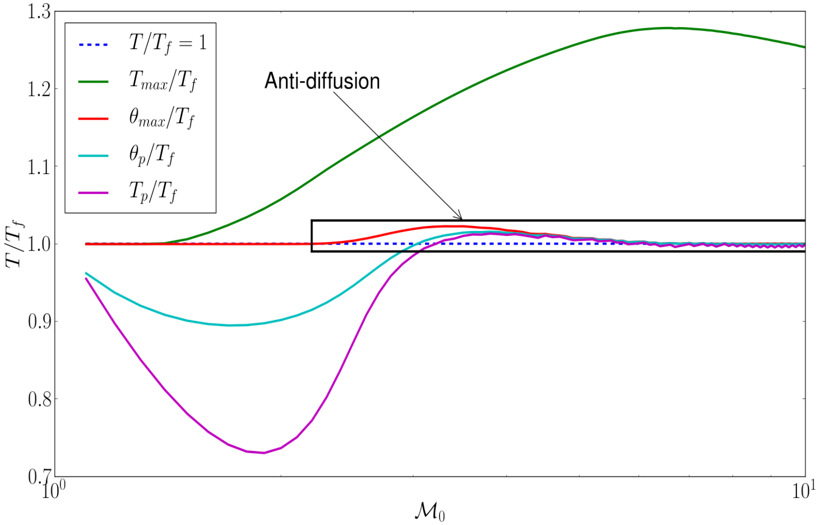
Origin of Anti-diffusion

- Anti-diffusion manifests itself in the shock structure as a non-monotonic dependence of the radiation temperature characterized by the attainment of values above the final downstream equilibrium material temperature.
- The radiative flux can be expressed as follows:

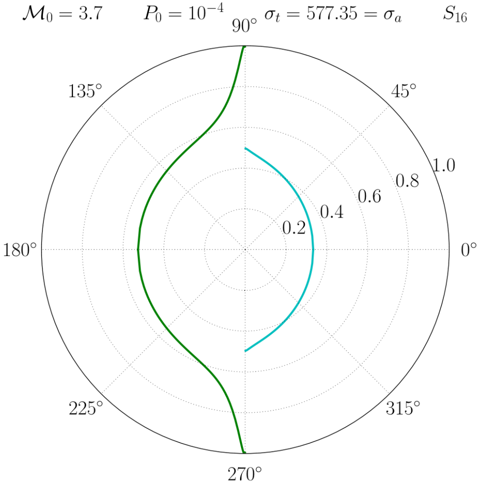
$$F_r = -\frac{1}{\sigma_t}\frac{\partial P_r}{\partial x} = -\frac{1}{\sigma_t}\frac{\partial f E_r}{\partial x} = -\frac{1}{\sigma_t}\left[f\frac{\partial E_r}{\partial x} + E_r\frac{\partial f}{\partial x}\right] \quad .$$

- Note that the flux can be directed along the gradient of E rather than opposite the gradient of E if $\frac{\partial f}{\partial x}$ is opposite in sign to $\frac{\partial E_r}{\partial x}$ and of sufficient magnitude. Thus radiation flows “uphill” rather than “downhill”. This can never happen with the diffusion approximation.

Anti-diffusion versus $\mathcal{M}_0$



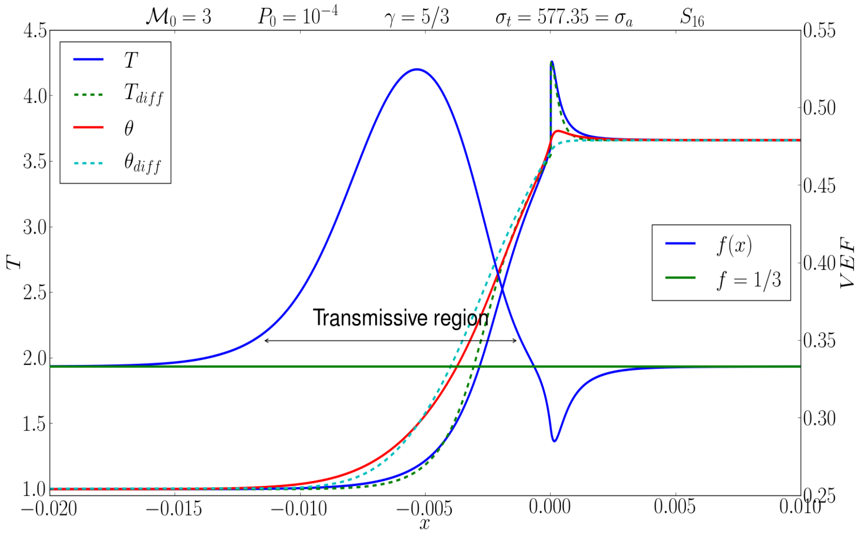
Angular Distribution at Shock



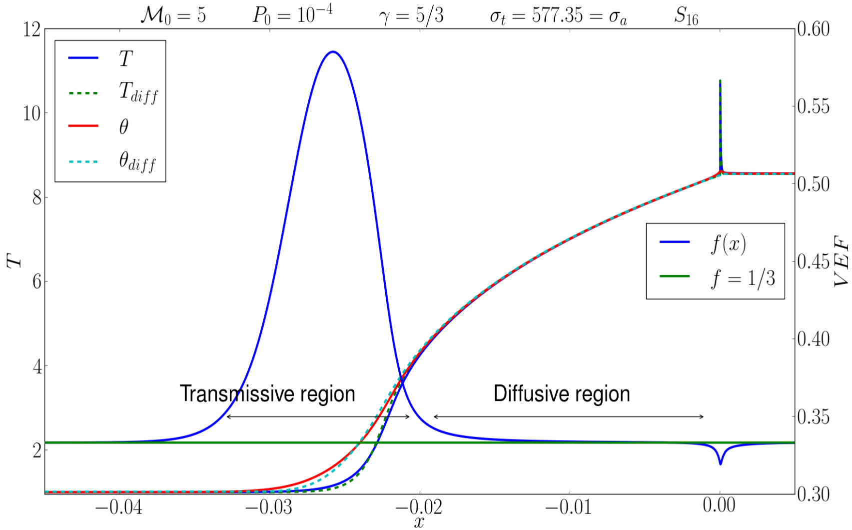
Transmissive and Diffusive Regions

- Transmissive:
  - $f > 1/3$, such that
  - the radiation temperature is greater than the material temperature, $\theta > T$,
  - and the temperatures decay exponentially when moving toward the equilibrium precursor state.
- Diffusive:
  - The Eddington approximation holds: $f \approx 1/3$.
  - This results in equal material and radiation temperatures (equilibrium), such that
  - the precursor temperatures decay linearly moving away from the embedded hydrodynamic shock.
  - Sits between the embedded hydrodynamic shock and the transmissive region.

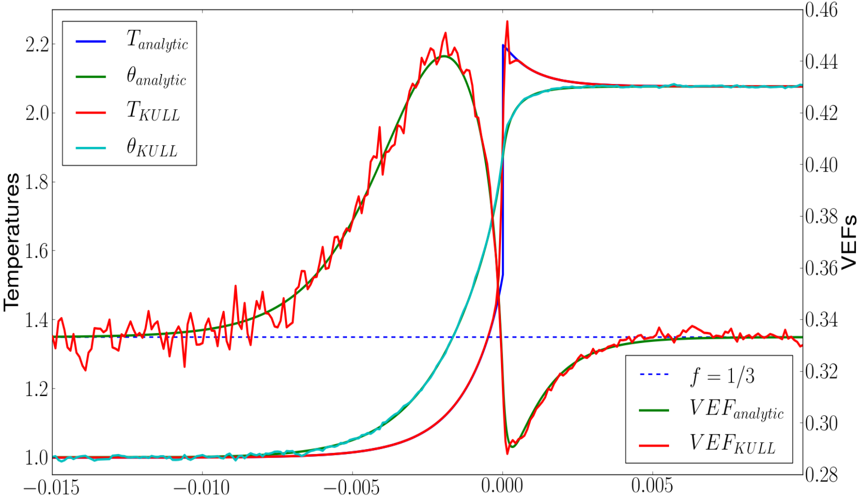
$\mathcal{M}_0 = 3$ Transmissive Region Only



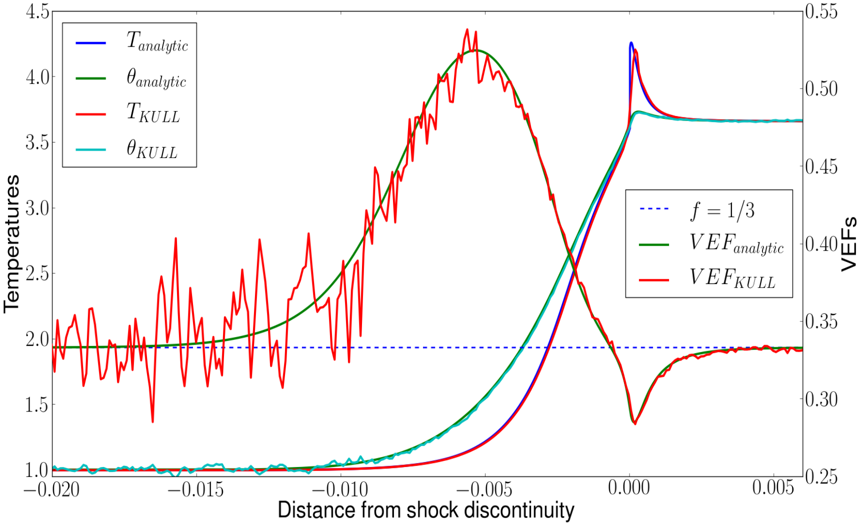
$\mathcal{M}_0 = 5$ Transmissive and Diffusive Regions



$\mathcal{M}_0 = 2$ Comparison with Fully Relativistic IMC



$\mathcal{M}_0 = 3$ Comparison with Fully Relativistic IMC



Conclusions

- Presented grey S_n semi-analytic radiative shock solutions. These solutions are a useful code-verification tool of RH codes that solve the radiation transport equation.
- Have confirmed anti-diffusion.
- Have confirmed angular distributions peaked about $\mu = 0$.
- Have confirmed transmissive/diffusive regions.
- Much more physical understanding remains to be extracted.

Future Work

- Incorporate frequency-dependent diffusion and frequency-dependent transport.
- Incorporate separate elecron and ion temperatures.
- Investigate validity of various material-motion models for radiation.