Smoothed Voronoi Particle Hydrodynamics: a new meshless method

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Where we left off...

- In the last Multimat meeting I discussed an extension to standard ASPH (Adaptive Smoothed Particle Hydrodynamics) which used the Voronoi diagram of the ASPH points to define a volume per ASPH node.
- At the time we used this knowledge solely to augment the mass density evolution:

$$\text{ASPH} : \rho_i = \sum_j m_j W_i$$
$$\text{Voronoi sum} : \rho_i = \frac{\sum_j m_j W_i}{\sum_j V_j W_i}$$

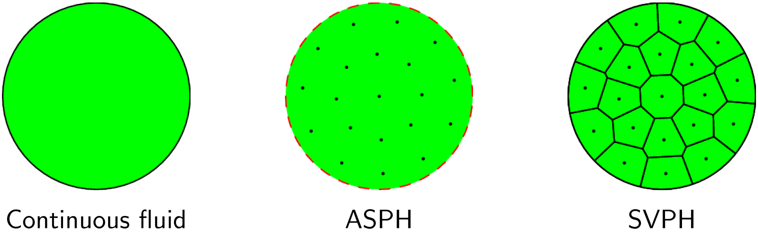
- ASPH using this Voronoi summed density was more stable and accurate.
- This suggested that we might be able to further improve things by exploiting the partition of unity volumes offered by the Voronoi in other parts of our algorithm.
- This talk is about the first stages of such an investigation...

The good and the bad about ASPH: what are we trying to accomplish?

- Good:
  - Robust and Lagrangian.
  - Conservative.
  - Adapts to strongly anisotropic flows well.
  - Resistant to mesh/discretization imprinting.
  - Simplest and quickest of the meshless methods.
- Bad:
  - Boundary conditions are difficult to handle.
  - Tensile instability/parasitic modes generally ignored.
  - Underlying interpolation inaccurate on distorted point distributions.
- Our goal is to use the extra geometric information provided by the Voronoi diagram to correct some of these shortcomings without destroying what we like in ASPH.
- We refer to this experimental new method as SVPH (Smoothed Voronoi Point Hydrodynamics).

Different methods of discretizing a disk of fluid.

- Consider a homogeneous 2D fluid within a bounding surface.



- ASPH simply places sampling points down in the fluid.
  - There is not a distinct boundary.
  - ASPH sampling falls off smoothly as we approach the surface of the fluid.
- SVPH also places the sampling points in the fluid, but in addition we compute the Voronoi diagram to define a unique volume for each sampling position.
  - This also allows us to account for any bounding surfaces through a constrained Voronoi diagram.

ASPH interpolation.

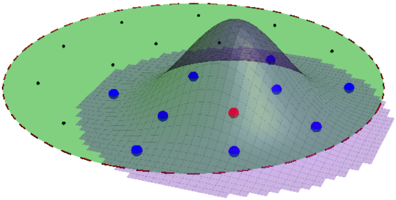
- ASPH interpolates according to the relation

$$\langle F(x^\alpha)\rangle = \int dV\ W(\eta^\alpha)\ F(x'^\alpha) \approx \sum_j \frac{m_j}{\rho_j} W_j F_j, \quad \eta^\alpha \equiv (x^\alpha - x'^\alpha)/h_i.$$

  - This assumes the interpolation kernel $W(\eta^\alpha)$ is normalized:

$$1 = \int_V dV\ W(\eta^\alpha) \approx \sum_j \frac{m_j}{\rho_j} W(\eta_i^\alpha), \Leftarrow \text{Only approximately true!}$$

- ASPH sampling from the red point interacts with the blue neighbors.
- The empty volume where the interpolation kernel overlaps the boundary results in error.



SPH sampling of the bounded disk in 2D

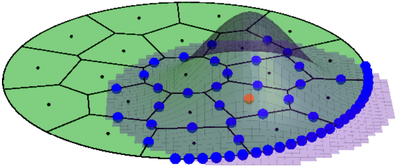
SVPH interpolation.

- SVPH interpolation uses a floating normalization based on the per node volumes,

$$\langle F(x^\alpha) \rangle = \frac{\int dV \; W(\eta^\alpha) \; F(x'^\alpha)}{\int dV \; W(\eta^\alpha)} \approx \frac{\sum_j V_j W_j F_j}{\sum_j V_j W_j}.$$

- In our current formalism, however, we do not interpolate to the neighbor node positions but rather to the faces.

  - ASPH sampling from the red point interacts with the blue faces.
  - In this case we know the volumes including the boundary, so overlapping the boundary should be accounted for.



SVPH sampling of the bounded disk in 2D

Why interpolate to the faces?

- In our original concept for SVPH, our intention was to interpolate solely to node positions similarly to ASPH.
- However, due to the floating normalization of the SVPH interpolation it much more difficult to enforce symmetries in point–point interactions.
  - By cleverly writing how the Lagrangian is differenced in ASPH, pair-wise point interactions are naturally symmetrized such that $F_{ij} = -F_{ji}$.
  - The floating volume normalization of SVPH breaks these symmetries however, making it less clear how to enforce momentum conservation to round-off.
- By interpolating the hydrodynamic state to faces, we can use a finite-volume integral over the faces of each SVPH point to form our differencing.
- This restores pair-wise symmetries in forces between points.
  - However, note these face forces only affect the two nodes on either side of the face, despite the fact we sample pressures on the face from a larger stencil!

Meshless gradient operators.

- The basic ASPH gradient is given by

$$\langle \partial^\alpha F(x_i^\alpha) \rangle = \sum_j \frac{m_j}{\rho_j} F_j \partial^\alpha W_j.$$

- The meshless SVPH gradient can be similarly derived as

$$\langle \partial^\alpha F(x_i^\alpha \rangle = \frac{\sum_j V_j \left(F_j - \langle F(x_i^\alpha) \rangle_i \right) \partial^\alpha W_j}{\sum_j V_j W_j} \approx \frac{\sum_j V_j \left(F_j - F_i \right) \partial^\alpha W_j}{\sum_j V_j W_j}$$

  - Vanishes exactly for a constant field.
  - However, difficult to symmetrize for particle $i \leftrightarrow j$ interactions.
- While we use the finite-volume integral in the succeeding hydrodynamic relations, we are not necessarily done with this form yet...

SVPH hydrodynamic equations.

Persistent Node properties:

$$\rho_i = \frac{\sum_{f(i)} m_f A_f \left(x_{if}^\alpha x_{if}^\alpha\right)^{1/2}}{\sum_{f(i)} V_f A_f \left(x_{if}^\alpha x_{if}^\alpha\right)^{1/2}}$$

$$\frac{Dv_i^\alpha}{Dt} = m_i^{-1} \sum_{f(i)} F_f$$

$$\Delta\varepsilon_i = m_i^{-1} \sum_{f(i)} w_{ij} F_f^\alpha (v_{ij}^\alpha(t_{1/2}))\Delta t$$

$$v_{ij}^\alpha(t_{1/2}) \equiv v_i^\alpha(t_{1/2}) - v_j^\alpha(t_{1/2})$$

Inferred Face properties:

$$m_f = \sum_j m_j W_j$$

$$V_f = \sum_j V_j W_j$$

$$P_f = \frac{\sum_j V_j P_j}{\sum_j V_j W_j}$$

$$F_f^\alpha = -\left(P_f A_f^\alpha + Q_f^{\alpha\beta} A_f^\beta\right)$$

$$x_{if}^\alpha \equiv x_i^\alpha - x_f^\alpha$$

- The thermal energy equation is compatibly differenced similarly to how I handle the ASPH energy.
- Mass, linear momentum, and energy are conserved to machine precision.

The artificial viscosity.

- We have written the face-centered artificial viscous term appropriately for a tensor form: $Q_f^{\alpha\beta}$.
  - We have adapted the ASPH forms [1, 2].
  - In most of this talk we use the unlimited scalar form.

$$Q_f = \frac{\sum_j V_j W_j \rho_f^2 \Pi_{fj}}{\sum V_j W_j}$$

$$\Pi_{fj} = -C_l c_{sf} \mu_{fj} + C_q \mu_{fj}^2$$

$$\mu_{fj} = \frac{v_{fj}^\alpha \eta_f^\alpha}{\eta_f^2 + \epsilon^2}$$

$$\eta_f = (x_f^\alpha - x_j^\alpha)/h_f$$

$$v_{fj}^\alpha = v_f^\alpha - v_j^\alpha$$

$$\rho_f = \frac{1}{2}(\rho_{f1} + \rho_{f2})$$

$$c_{sf} = \frac{1}{2}(c_{sf1} + c_{sf2})$$

$$h_f = \frac{1}{2}(h_{f1} + h_{f2})$$

- $f1$ and $f2$ refer to the SVPH points on either side of face f.

[1] J. J. Monaghan, Rep. Prog. Phys., 68, 1703–1759, 2005.

[2] J. M. Owen, J. Comp. Phys., 201, 601–629, 2004.

Extending to higher orders: reproducing kernel methods.

- The SVPH interpolation method is already zeroth order consistent.
- We can use reproducing kernel estimation to extend our ability to exactly reproduce fields to whatever order desired.
- For instance, to get linear fields exactly correct:

$$W^R_{fj} = \left(1.0 + B^\alpha_f (x^\alpha_f - x^\alpha_j)\right) W_j$$

$$B^\alpha_f = -\left(m_2^{-1}\right)^{\alpha\beta} m_1^\beta$$

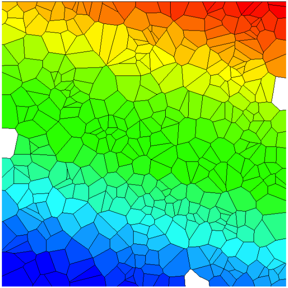
$$m_1^\alpha \equiv \sum_j V_j \, W_j \, (x^\alpha_f - x^\alpha_j)$$

$$m_2^{\alpha\beta} \equiv \sum_j V_j \, W_j \, (x^\alpha_f - x^\alpha_j)(x^\beta_f - x^\beta_j)$$

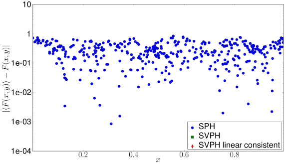
- Note these corrections are simply weighted moments of the local point geometry.
- For a given configuration of points, you compute these corrections once and they are applicable for any number of fields.
- Extensions to higher order consistency involves progressively higher moments of the point geometry.

Interpolation tests.

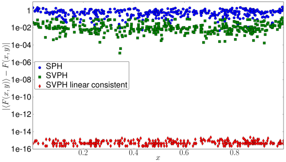
- Our first goal was to improve accuracy...
- For constant fields SVPH error vanishes as expected.
- SVPH without the reproducing kernel is always more accurate than SPH.
- SVPH with reproducing kernel is the most accurate.



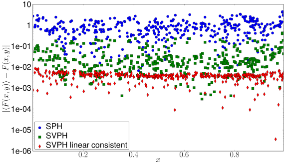
SVPH interpolated linear field on random points



Constant field errors



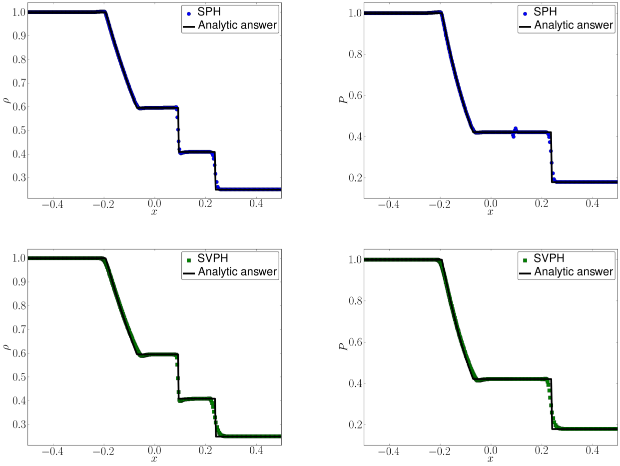
Linear field errors



Quadratic field error

1D hydro solutions: Sod

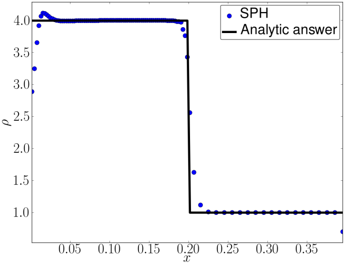
- The first place we start of course is 1D hydro...



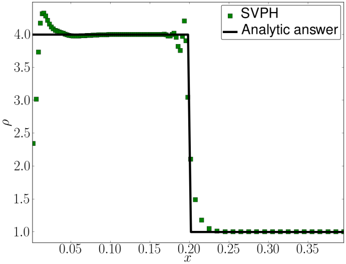
- SVPH avoids the blip in the pressure between the two materials.

1D hydro solutions: planar Noh

- The single material planar Noh problem looks fairly similar between SPH and SVPH.



SPH mass density

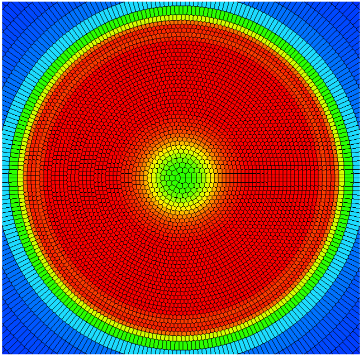


SVPH mass density

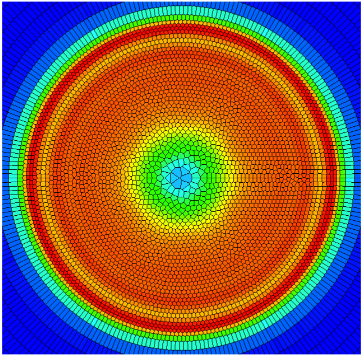
- SVPH sees more of a wall-heating effect and a bit more post-shock oscillation.
- Certainly more work to be done on the viscosity...

2D hydro solutions: cylindrical Noh (paved points)

- We have to have 1D working, but the real questions about SVPH are in higher dimensions.
  - The biggest question probably is have we maintained the nice "mesh" independence of ASPH?



SPH

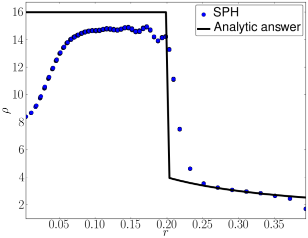


SVPH

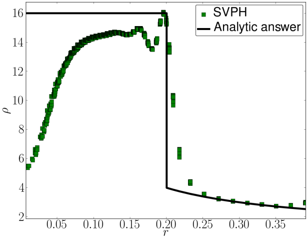
- When we initialize the SVPH points on rings ("paving"), symmetry is maintained well.

2D hydro solutions: cylindrical Noh (paved points)

- Looking at the scatter plot profiles of the density, we can see that while SVPH maintains the expected radial solution, there is evidence of symmetry breaking in the disordering of the points.
- Classic (A)SPH has the edge in symmetry here still.



SPH

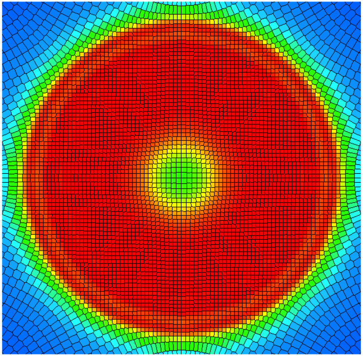


SVPH

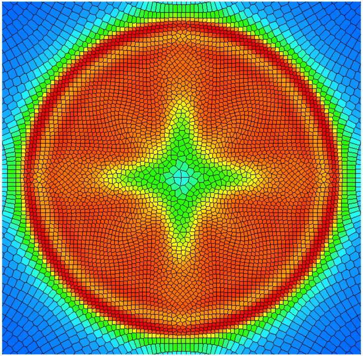
- Note we are using the unlimited scalar form for the artificial viscosity here.
  - This is why the post-shock density is low.

2D hydro solutions: cylindrical Noh (points on lattice)

- Seeding the points on a lattice is a more difficult symmetry test.
- The inner (post-shock) solution for SVPH shows more deviation from radial symmetry than SPH.



SPH

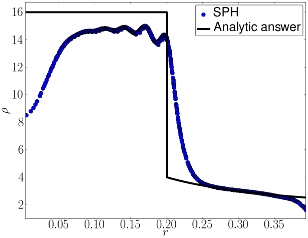


SVPH

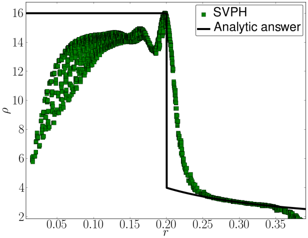
- The shock symmetry is not bad however.

2D hydro solutions: cylindrical Noh (points on lattice)

- Not surprisingly we find the lattice seeded points exacerbates the post-shock symmetry breaking in SVPH.



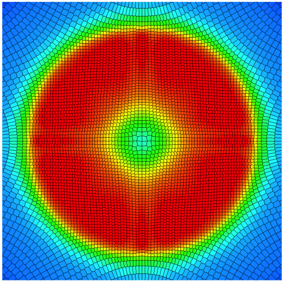
SPH



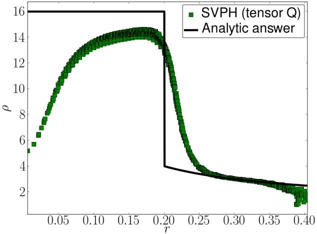
SVPH

- We clearly have more work to do.
- The inner profiles show our artificial viscous formalism is not quite right yet.
- Could this also improve the symmetry we're seeing?

Preliminary Tensor viscosity results.



Density image



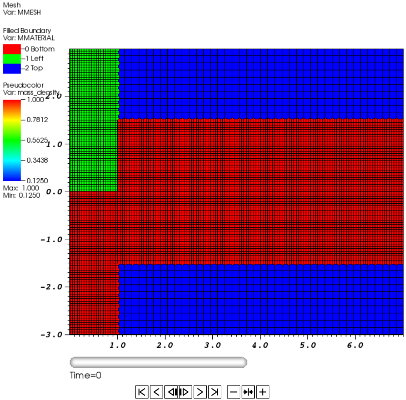
Radial profile of density

- This is an SVPH port of the tensor viscosity discussed in [3] to the faces.
  - Using the point-wise SVPH gradient for the velocity.
  - No limiter as yet.
- Symmetry is significantly improved.

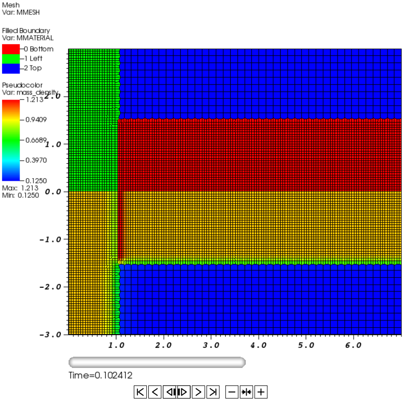
[3] J. M. Owen, J. Comp. Phys., 201, 601–629, 2004.

The triple point rollup test.

- I've never run the triple point with these methods before, but it wouldn't be a Multimat talk without some triple point results.



SPH

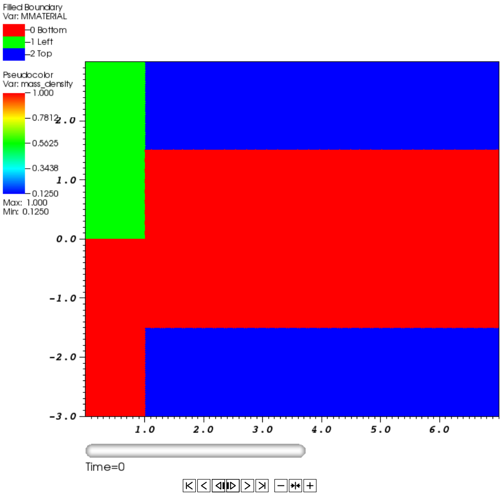


SVPH

Triple point – SPH high resolution.

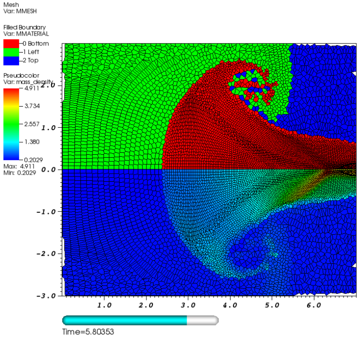
- The SPH calculation has difficulty rolling up the vortex.

- Some authors suggest implementing an artificial heat conduction to acount for entropy diffusion at small scales.
- What about just running at higher resolution?
- Here we repeat the SPH calculation with 4× resolution.



What about helping order the SVPH points?

- We have not yet discussed any methods to help keep the SVPH points ordered.
  - We want to remain in the Lagrangian frame, so options are limited.
- As an experiment, here we try allowing the points to relax toward the local cell centroid by 0.5% of the Lagrangian displacement.



Pure Lagrangian

With relaxation

Conclusions.

- What we have developed here is a bit of a strange hybrid method.
- From a users perspective this is still a meshless method.
  - The user chooses their geometry, lays down discretization points and initializes their properties, and off the problem goes.
  - The Voronoi diagram is entirely internal.
- In this early state we can demonstrate clear accuracy improvements.
- Vortical problems like the triple point show definite advantages for SVPH over (A)SPH.
- Areas we are still investigating:
  - Symmetry properties need to be better characterized and improved.
  - Artificial viscous formulation still under development.
    - Preliminary results with the port of ASPHs tensor viscosity promising.
  - Need to test reproducing kernel methods in hydro problems.
  - Determine how to enforce boundary conditions using faces.