Contact with friction for the eXtended Eulerian Method

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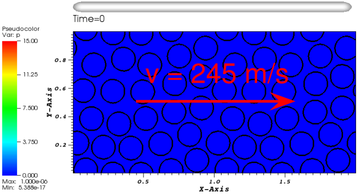
Outline

- **Motivation**
- **eXtended Eulerian Method (background)**
- **Friction Algorithm (current work)**
- **Numerical Results**

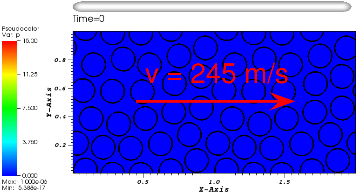
Motivation

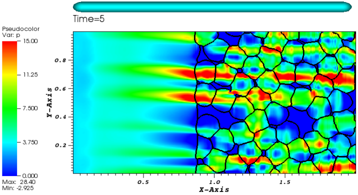
Compaction of Granular Material

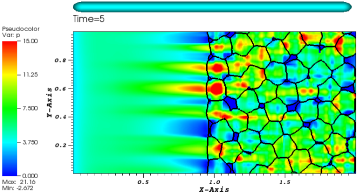


Compaction of Granular Material

- Motivation
  - **Granular Material**
  - Taylor Anvil
- XEM
- Friction Algorithm
- Numerical Results
- The End

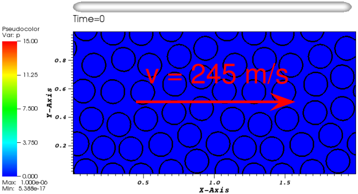


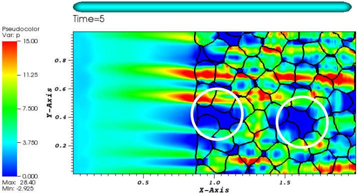


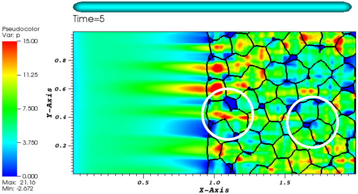


Fully Bonded Solution **Frictionless Slip Solution**

Compaction of Granular Material

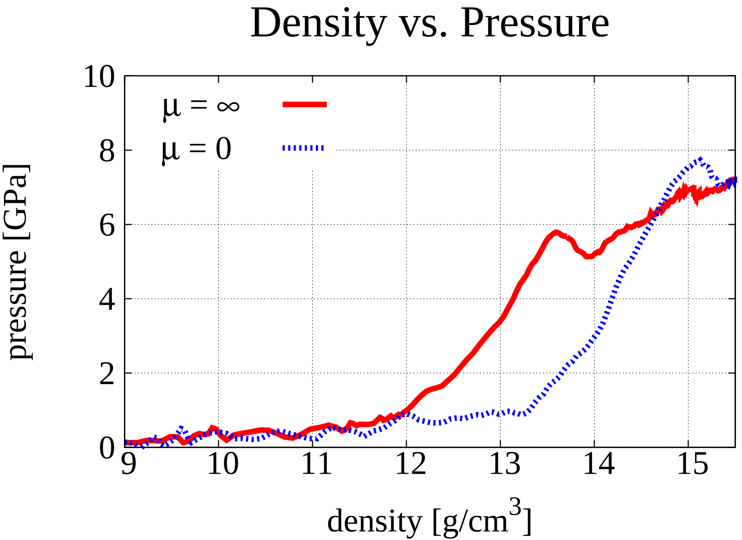




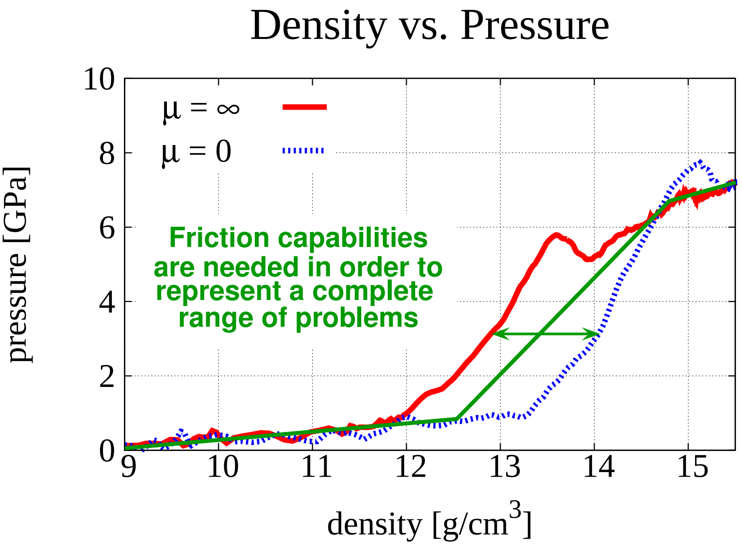


Fully Bonded Solution **Frictionless Slip Solution**

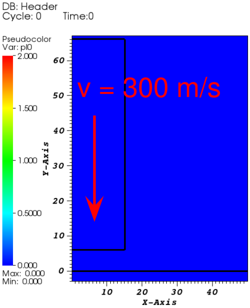
Compaction Curves



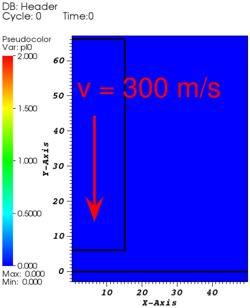
Compaction Curves

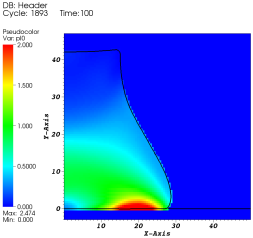


Taylor Anvil Test



Taylor Anvil Test





Fully Bonded Solution

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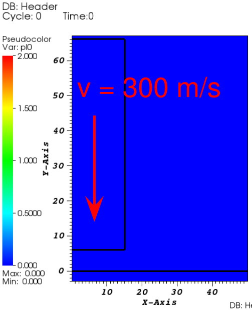
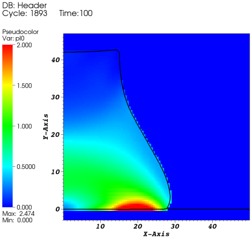
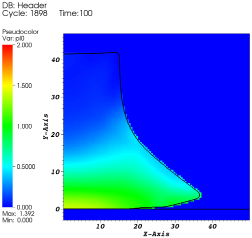




Fully Bonded Solution



Frictionless Slip Solution

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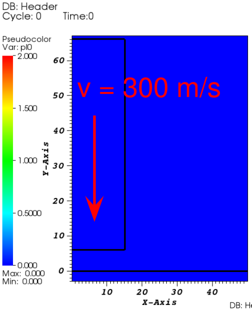
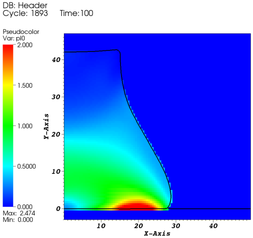
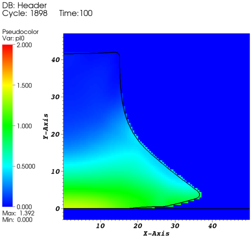
Taylor Anvil Test



Friction capabilities are needed in order to represent a complete range of problems



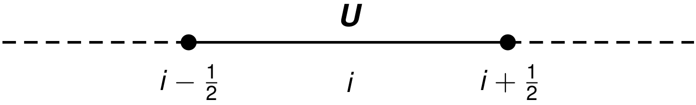
Fully Bonded Solution



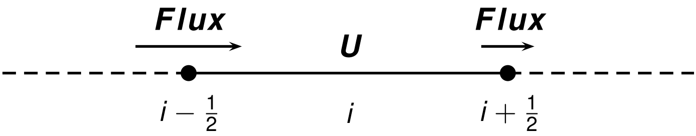
Frictionless Slip Solution

eXtended Eulerian Method (background)

Overview



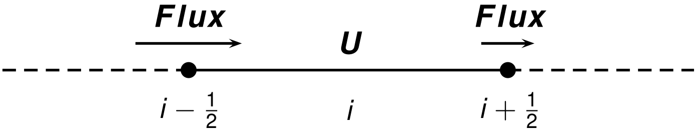
Overview



Time Evolution

$$\boldsymbol{U}_i^{n+1} = \boldsymbol{U}_i^n - \frac{\Delta t}{\Delta x}\left[\boldsymbol{F}_{i+\frac{1}{2}}(\boldsymbol{U}^n) - \boldsymbol{F}_{i-\frac{1}{2}}(\boldsymbol{U}^n)\right]$$

Overview



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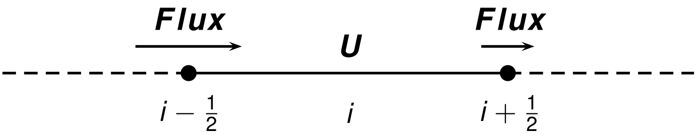
$$\boldsymbol{F}(\boldsymbol{U}) = f(\boldsymbol{q}_S) \qquad \boldsymbol{q}_S = Riemann(\boldsymbol{q}_L, \boldsymbol{q}_R)$$

Overview



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$$\boldsymbol{F}(\boldsymbol{U}) = f(\boldsymbol{q}_S) \qquad \boldsymbol{q}_S = Riemann(\boldsymbol{q}_L, \boldsymbol{q}_R)$$

Constitutive Equation

$$\boldsymbol{\sigma}^{n+1} = f(\boldsymbol{\sigma}^n, \boldsymbol{L}, \Delta t, \dots)$$

Conservation Equations

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}(\boldsymbol{U})}{\partial x} + \frac{\partial \boldsymbol{G}(\boldsymbol{U})}{\partial y} + \frac{\partial \boldsymbol{H}(\boldsymbol{U})}{\partial z} = \boldsymbol{0}$$

$$\boldsymbol{U} = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{Bmatrix} \qquad \boldsymbol{F}(\boldsymbol{U}) = \begin{Bmatrix} \rho u \\ \rho u^2 - \sigma_{xx} \\ \rho u v - \sigma_{yx} \\ \rho u w - \sigma_{zx} \\ uE - u\sigma_{xx} - v\sigma_{yx} - w\sigma_{zx} \end{Bmatrix}$$

$$\boldsymbol{G}(\boldsymbol{U}) = \begin{Bmatrix} \rho v \\ \rho u v - \sigma_{xy} \\ \rho v^2 - \sigma_{yy} \\ \rho v w - \sigma_{zy} \\ vE - u\sigma_{xy} - v\sigma_{yy} - w\sigma_{zy} \end{Bmatrix} \boldsymbol{H}(\boldsymbol{U}) = \begin{Bmatrix} \rho w \\ \rho u w - \sigma_{xz} \\ \rho v w - \sigma_{yz} \\ \rho w^2 - \sigma_{zz} \\ wE - u\sigma_{xz} - v\sigma_{yz} - w\sigma_{zz} \end{Bmatrix}$$

$$E = \rho \left[\tfrac{1}{2}\left(u^2 + v^2 + w^2\right) + e\right]$$

Discretization and Spacial Splitting

$$\frac{\partial \boldsymbol{U}}{\partial t} \approx \frac{\boldsymbol{U}^{n+1} - \boldsymbol{U}^{n}}{\Delta t} = \frac{\boldsymbol{U}^{*} - \boldsymbol{U}^{n}}{\Delta t} + \frac{\boldsymbol{U}^{**} - \boldsymbol{U}^{*}}{\Delta t} + \frac{\boldsymbol{U}^{n+1} - \boldsymbol{U}^{**}}{\Delta t}$$

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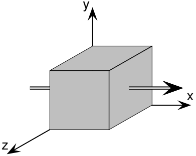
$$\frac{\boldsymbol{U}^{*} - \boldsymbol{U}^{n}}{\Delta t} + \frac{\Delta \boldsymbol{F}(\boldsymbol{U}^{n})}{\Delta x} = \boldsymbol{0}$$



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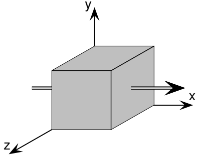
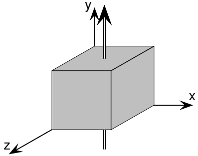


$$\frac{\boldsymbol{U}^{**} - \boldsymbol{U}^{*}}{\Delta t} + \frac{\Delta \boldsymbol{G}(\boldsymbol{U}^{*})}{\Delta y} = \boldsymbol{0}$$



Discretization and Spacial Splitting

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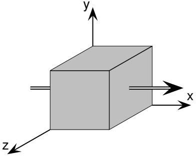
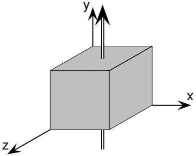
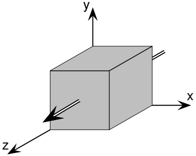


$$\frac{\boldsymbol{U}^{**} - \boldsymbol{U}^{*}}{\Delta t} + \frac{\Delta \boldsymbol{G}(\boldsymbol{U}^{*})}{\Delta y} = \mathbf{0}$$



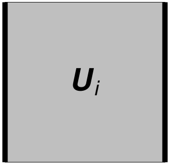
$$\frac{\boldsymbol{U}^{\mathbf{n+1}} - \boldsymbol{U}^{**}}{\Delta t} + \frac{\Delta \boldsymbol{H}(\boldsymbol{U}^{**})}{\Delta z} = \mathbf{0}$$



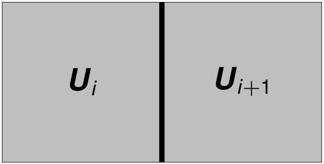
“1-D” Godunov Method (x-sweep)

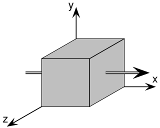
$$\boldsymbol{U}_i^* = \boldsymbol{U}_i^n - \frac{\Delta t}{\Delta x}\left[\boldsymbol{F}_{i+\frac{1}{2}}(\boldsymbol{U}^n) - \boldsymbol{F}_{i-\frac{1}{2}}(\boldsymbol{U}^n)\right]$$

$\boldsymbol{F}_{i-\frac{1}{2}}(\boldsymbol{U})$ $\boldsymbol{F}_{i+\frac{1}{2}}(\boldsymbol{U})$



$\boldsymbol{F}_{i+\frac{1}{2}}(\boldsymbol{U})$



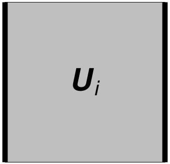


“1-D” Godunov Method (x-sweep)

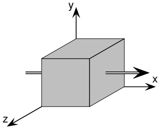
$$\boldsymbol{U}_i^* = \boldsymbol{U}_i^n - \frac{\Delta t}{\Delta x}\left[\boldsymbol{F}_{i+\frac{1}{2}}(\boldsymbol{U}^n) - \boldsymbol{F}_{i-\frac{1}{2}}(\boldsymbol{U}^n)\right]$$

$\boldsymbol{F}_{i-\frac{1}{2}}(\boldsymbol{U})$ $\boldsymbol{F}_{i+\frac{1}{2}}(\boldsymbol{U})$

$$\boldsymbol{F}_{i+\frac{1}{2}}(\boldsymbol{U}) = f(\boldsymbol{q}_S)$$



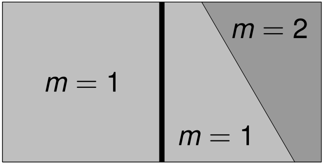
$$\boldsymbol{q}_S = Riemann(\boldsymbol{q}_L, \boldsymbol{q}_R)$$



$$\boldsymbol{q} = \left\{\rho \;\; u \;\; v \;\; w \;\; \sigma_{xx} \;\; \sigma_{yx} \;\; \sigma_{zx}\right\}^{\mathrm{T}}$$

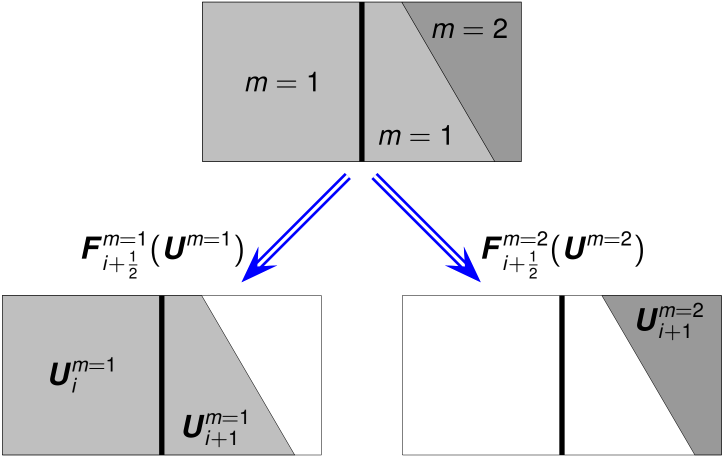
Independent Fields

$$\boldsymbol{U}_i^{m,n+1} = \boldsymbol{U}_i^{m,n} - \frac{\Delta t}{\Delta x}\left[\boldsymbol{F}_{i+\frac{1}{2}}^{m}(\boldsymbol{U}^{m,n}) - \boldsymbol{F}_{i-\frac{1}{2}}^{m}(\boldsymbol{U}^{m,n})\right]$$



Independent Fields

$$\boldsymbol{U}_i^{m,n+1} = \boldsymbol{U}_i^{m,n} - \frac{\Delta t}{\Delta x}\left[\boldsymbol{F}^m_{i+\frac{1}{2}}(\boldsymbol{U}^{m,n}) - \boldsymbol{F}^m_{i-\frac{1}{2}}(\boldsymbol{U}^{m,n})\right]$$



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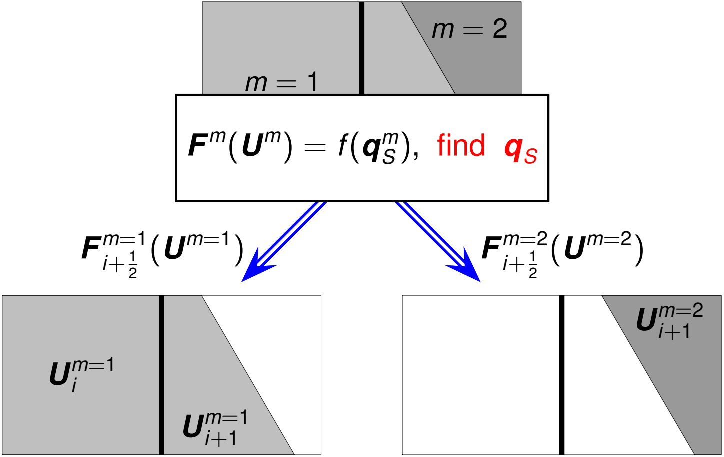
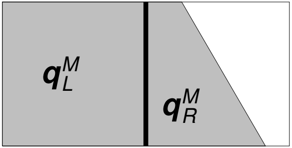
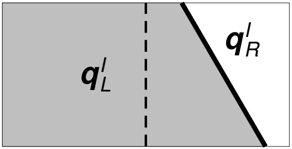


Find Face Values



Material Solution

$$\boldsymbol{q}_S^M = Riemann_E(\boldsymbol{q}_L^M, \boldsymbol{q}_R^M)$$



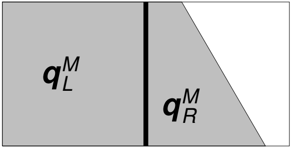
Interface Solution

$$\boldsymbol{q}_S^I = Riemann_L(\boldsymbol{q}_L^I, \boldsymbol{q}_R^I)$$

Interpolation Scheme

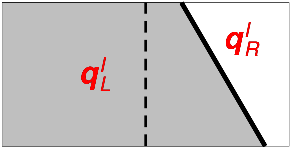
$$\boldsymbol{q}_S = f(\boldsymbol{q}_S^M, \boldsymbol{q}_S^I)$$

Find Face Values



Material Solution

$$\boldsymbol{q}_S^M = Riemann_E(\boldsymbol{q}_L^M, \boldsymbol{q}_R^M)$$



Interface Solution

$$\boldsymbol{q}_S^I = Riemann_L(\boldsymbol{q}_L^I, \boldsymbol{q}_R^I)$$

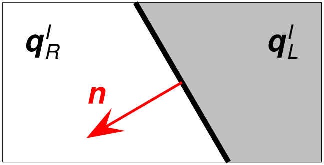
Interpolation Scheme

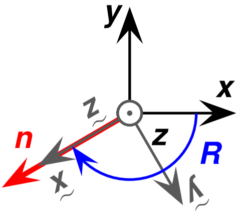
$$\boldsymbol{q}_S = f(\boldsymbol{q}_S^M, \boldsymbol{q}_S^I)$$

Use Interface Coordinate System

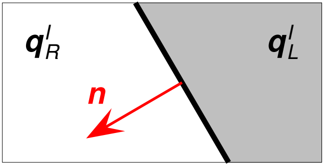
Motivation
XEM
- Overview
- Equations
- Discretization
- Godunov
- Independent Fields
- Find Face Values
- **Find Int. Solution**

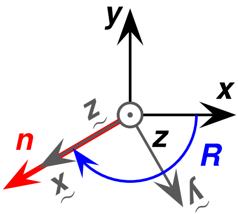
Friction Algorithm
Numerical Results
The End





Use Interface Coordinate System



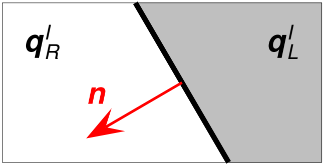


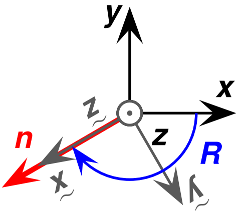
Rotate Stresses and Velocities

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{R}^{\mathrm{T}} \boldsymbol{\sigma} \boldsymbol{R} \qquad \tilde{\boldsymbol{u}} = \boldsymbol{R}^{\mathrm{T}} \boldsymbol{u}$$

$$\tilde{\boldsymbol{q}}^{l}_{L,R} = \left\{ \rho \ \tilde{u} \ \tilde{v} \ \tilde{w} \ \tilde{\sigma}_{xx} \ \tilde{\sigma}_{yy} \ \tilde{\sigma}_{zz} \ \tilde{\sigma}_{xy} \ \tilde{\sigma}_{yz} \ \tilde{\sigma}_{zx} \right\}^{\mathrm{T}}$$

Use Interface Coordinate System





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$$\tilde{\sigma} = \boldsymbol{R}^{\mathrm{T}} \boldsymbol{\sigma} \boldsymbol{R} \qquad \tilde{\boldsymbol{u}} = \boldsymbol{R}^{\mathrm{T}} \boldsymbol{u}$$

$$\tilde{\boldsymbol{q}}^l_{L,R} = \left\{ \rho \;\; \tilde{u} \;\; \tilde{v} \;\; \tilde{w} \;\; \tilde{\sigma}_{xx} \;\; \tilde{\sigma}_{yy} \;\; \tilde{\sigma}_{zz} \;\; \tilde{\sigma}_{xy} \;\; \tilde{\sigma}_{yz} \;\; \tilde{\sigma}_{zx} \right\}^{\mathrm{T}}$$

Find Lagrangian Solution (bonded materials)

$$\tilde{\boldsymbol{q}}^l_S = Riemann_L(\tilde{\boldsymbol{q}}^l_L, \tilde{\boldsymbol{q}}^l_R)$$

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Allow Frictionless Slip

Apply Frictionless Contact to Tangential Components

$$\tilde{v}_S^I = \tilde{v}_L^I - \frac{\tilde{\sigma}_{xy_L}^I}{\rho_L^I c_{T_L}^I} \qquad \tilde{w}_S^I = \tilde{w}_L^I - \frac{\tilde{\sigma}_{xz_L}^I}{\rho_L^I c_{T_L}^I} \qquad \tilde{\sigma}_{xy_S}^I = \tilde{\sigma}_{xz_S}^I = 0$$

Allow Frictionless Slip

Apply Frictionless Contact to Tangential Components

$$\tilde{v}^I_S = \tilde{v}^I_L - \frac{\tilde{\sigma}^I_{xy_L}}{\rho^I_L c^I_{T_L}} \qquad \tilde{w}^I_S = \tilde{w}^I_L - \frac{\tilde{\sigma}^I_{xz_L}}{\rho^I_L c^I_{T_L}} \qquad \tilde{\sigma}^I_{xy_S} = \tilde{\sigma}^I_{xz_S} = 0$$

Update $\tilde{\sigma}_{yy}$ and $\tilde{\sigma}_{zz}$ (1-D Strain Condition)

$$\Delta\tilde{\sigma}^I_{xx} = \tilde{\sigma}^I_{xx_S} - \tilde{\sigma}^I_{xx_L} \qquad \tilde{\sigma}^I_{yy_S} = \tilde{\sigma}^I_{yy_L} + \frac{3K - 2G}{3K + 4G}\Delta\tilde{\sigma}^I_{xx}$$

$$\tilde{\sigma}^I_{zz_S} = \tilde{\sigma}^I_{zz_L} + \frac{3K - 2G}{3K + 4G}\Delta\tilde{\sigma}^I_{xx}$$

Allow Frictionless Slip

Apply Frictionless Contact to Tangential Components

$$\tilde{v}^I_S = \tilde{v}^I_L - \frac{\tilde{\sigma}^I_{xy_L}}{\rho^I_L c^I_{T_L}} \qquad \tilde{w}^I_S = \tilde{w}^I_L - \frac{\tilde{\sigma}^I_{xz_L}}{\rho^I_L c^I_{T_L}} \qquad \tilde{\sigma}^I_{xy_S} = \tilde{\sigma}^I_{xz_S} = 0$$

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$$\Delta\tilde{\sigma}^I_{xx} = \tilde{\sigma}^I_{xx_S} - \tilde{\sigma}^I_{xx_L} \qquad \tilde{\sigma}^I_{yy_S} = \tilde{\sigma}^I_{yy_L} + \frac{3K - 2G}{3K + 4G}\Delta\tilde{\sigma}^I_{xx}$$

$$\tilde{\sigma}^I_{zz_S} = \tilde{\sigma}^I_{zz_L} + \frac{3K - 2G}{3K + 4G}\Delta\tilde{\sigma}^I_{xx}$$

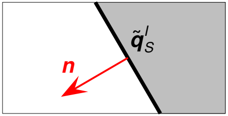
Rotate Stresses and Velocities Back

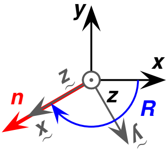
$$\boldsymbol{\sigma} = \boldsymbol{R}\,\widetilde{\boldsymbol{\sigma}}\,\boldsymbol{R}^{\mathrm{T}} \qquad \boldsymbol{u} = \boldsymbol{R}\,\widetilde{\boldsymbol{u}}$$

$$\boldsymbol{q}^I_S = \left\{\rho \;\; u \;\; v \;\; w \;\; \sigma_{xx} \;\; \sigma_{yy} \;\; \sigma_{zz} \;\; \sigma_{xy} \;\; \sigma_{yz} \;\; \sigma_{zx}\right\}^{\mathrm{T}}$$

Friction Algorithm
(current work)

Introduce Coulomb Friction (μ)



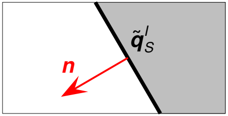


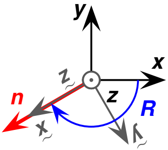
$$\tilde{\boldsymbol{q}}^I_S = \left\{ \rho \quad \tilde{u} \quad \tilde{v} \quad \tilde{w} \quad \tilde{\sigma}_{xx} \quad \tilde{\sigma}_{yy} \quad \tilde{\sigma}_{zz} \quad \tilde{\sigma}_{xy} \quad \tilde{\sigma}_{yz} \quad \tilde{\sigma}_{zx} \right\}^{\mathrm{T}}$$

Values Available by XEM

$$\tilde{\sigma}^I_{xx_S} \qquad \tilde{\sigma}^{bonded}_{xy_S} \qquad \tilde{\sigma}^{bonded}_{xz_S} \qquad \tilde{v}^{slip}_S \qquad \tilde{w}^{slip}_S$$

Introduce Coulomb Friction (μ)



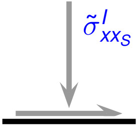


$$\tilde{\boldsymbol{q}}^I_S = \{ \rho \ \tilde{u} \ \tilde{v} \ \tilde{w} \ \tilde{\sigma}_{xx} \ \tilde{\sigma}_{yy} \ \tilde{\sigma}_{zz} \ \tilde{\sigma}_{xy} \ \tilde{\sigma}_{yz} \ \tilde{\sigma}_{zx} \}^{\mathrm{T}}$$

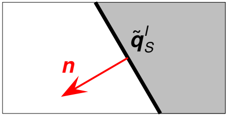
Values Available by XEM

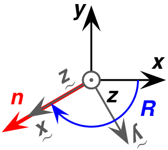
$\tilde{\sigma}^I_{xx_S}$ $\tilde{\sigma}^{bonded}_{xy_S}$ $\tilde{\sigma}^{bonded}_{xz_S}$ $\tilde{v}^{slip}_S$ $\tilde{w}^{slip}_S$

Calculate Maximum Tangential Frictional Stress



Introduce Coulomb Friction (μ)



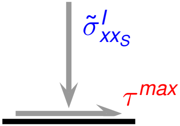


$$\tilde{\boldsymbol{q}}^l_S = \left\{ \rho \;\; \tilde{u} \;\; \tilde{v} \;\; \tilde{w} \;\; \tilde{\sigma}_{xx} \;\; \tilde{\sigma}_{yy} \;\; \tilde{\sigma}_{zz} \;\; \tilde{\sigma}_{xy} \;\; \tilde{\sigma}_{yz} \;\; \tilde{\sigma}_{zx} \right\}^{\mathrm{T}}$$

Values Available by XEM

$\tilde{\sigma}^l_{xx_S}$ $\quad$ $\tilde{\sigma}^{bonded}_{xy_S}$ $\quad$ $\tilde{\sigma}^{bonded}_{xz_S}$ $\quad$ $\tilde{v}^{slip}_S$ $\quad$ $\tilde{w}^{slip}_S$

Calculate Maximum Tangential Frictional Stress



$$\tau^{max} = -min(0, \mu\, \tilde{\sigma}^l_{xx})$$

Calculate Interface Solution

Calculate Allowable Tangential Frictional Stresses

$$\tau_{xy}^{allow} = min(\tau^{max}, |\tilde{\sigma}_{xy_S}^{bonded}|) \qquad \tau_{xz}^{allow} = min(\tau^{max}, |\tilde{\sigma}_{xz_S}^{bonded}|)$$

$$\tilde{\sigma}_{xy_S}^{I} = sign(\tilde{\sigma}_{xy_S}^{bonded})\, \tau_{xy}^{allow} \qquad \tilde{\sigma}_{xz_S}^{I} = sign(\tilde{\sigma}_{xz_S}^{bonded})\, \tau_{xz}^{allow}$$

Calculate Interface Solution

Calculate Allowable Tangential Frictional Stresses

$$\tau_{xy}^{allow} = min(\tau^{max}, |\tilde{\sigma}_{xy_S}^{bonded}|) \qquad \tau_{xz}^{allow} = min(\tau^{max}, |\tilde{\sigma}_{xz_S}^{bonded}|)$$

$$\tilde{\sigma}_{xy_S}^{I} = sign(\tilde{\sigma}_{xy_S}^{bonded})\,\tau_{xy}^{allow} \qquad \tilde{\sigma}_{xz_S}^{I} = sign(\tilde{\sigma}_{xz_S}^{bonded})\,\tau_{xz}^{allow}$$

Update Tangential Velocities

$$\tilde{v}_S^I = \tilde{v}_S^{slip} + \frac{\tilde{\sigma}_{xy_S}^I}{\rho_L^I c_{T_L}^I} \qquad \tilde{w}_S^I = \tilde{w}_S^{slip} + \frac{\tilde{\sigma}_{xz_S}^I}{\rho_L^I c_{T_L}^I}$$

Calculate Interface Solution

Calculate Allowable Tangential Frictional Stresses

$$\tau_{xy}^{allow} = min(\tau^{max}, |\tilde{\sigma}_{xy_S}^{bonded}|) \qquad \tau_{xz}^{allow} = min(\tau^{max}, |\tilde{\sigma}_{xz_S}^{bonded}|)$$

$$\tilde{\sigma}_{xy_S}^{I} = sign(\tilde{\sigma}_{xy_S}^{bonded})\, \tau_{xy}^{allow} \qquad \tilde{\sigma}_{xz_S}^{I} = sign(\tilde{\sigma}_{xz_S}^{bonded})\, \tau_{xz}^{allow}$$

Update Tangential Velocities

$$\tilde{v}_S^{I} = \tilde{v}_S^{slip} + \frac{\tilde{\sigma}_{xy_S}^{I}}{\rho_L^I c_{T_L}^I} \qquad \tilde{w}_S^{I} = \tilde{w}_S^{slip} + \frac{\tilde{\sigma}_{xz_S}^{I}}{\rho_L^I c_{T_L}^I}$$

Contact with Friction Solution

$$\boldsymbol{\sigma} = \boldsymbol{R}\,\tilde{\boldsymbol{\sigma}}\,\boldsymbol{R}^{\mathrm{T}} \qquad \boldsymbol{u} = \boldsymbol{R}\,\tilde{\boldsymbol{u}}$$

$$\boldsymbol{q}_S^I = \left\{\rho \;\; u \;\; v \;\; w \;\; \sigma_{xx} \;\; \sigma_{yy} \;\; \sigma_{zz} \;\; \sigma_{xy} \;\; \sigma_{yz} \;\; \sigma_{zx}\right\}^{\mathrm{T}}$$

Sanity Check

Updated Tangential Velocity

$$\tilde{v}_S^I = \tilde{v}_S^{slip} + \frac{\tilde{\sigma}_{xy_S}^I}{\rho_L^I c_{T_L}^I}$$

Sanity Check

Updated Tangential Velocity

$$\tilde{v}_S^I = \tilde{v}_S^{slip} + \frac{\tilde{\sigma}^I_{S}}{\rho_L c_{T_L}} = \tilde{v}_S^{slip}$$

Sanity Check

Updated Tangential Velocity

$$\tilde{v}_S^I = \tilde{v}_S^{slip} + \frac{\tilde{\sigma}_{xy_S}^I}{\rho_L^I c_{T_L}^I} = \tilde{v}_L^I - \frac{\tilde{\sigma}_{xy_L}^I}{\rho_L^I c_{T_L}^I} + \frac{\tilde{\sigma}_{xy_S}^I}{\rho_L^I c_{T_L}^I}$$

Sanity Check

Updated Tangential Velocity

$$\tilde{v}_S^I = \tilde{v}_S^{slip} + \frac{\tilde{\sigma}_{xy_S}^I}{\rho_L^I c_{T_L}^I} = \tilde{v}_L^I - \frac{\tilde{\sigma}_{xy_L}^I}{\rho_L^I c_{T_L}^I} + \frac{\tilde{\sigma}_{xy_S}^I}{\rho_L^I c_{T_L}^I}$$

Impose Fully Bonded Tangential Stress

$$\tilde{\sigma}_{xy_S}^I = \tilde{\sigma}_{xy_S}^{bonded} = \frac{\rho_L^I c_{T_L}^I \, \tilde{\sigma}_{xy_R}^I + \rho_R^I c_{T_R}^I \, \tilde{\sigma}_{xy_L}^I - \rho_L^I c_{T_L}^I \, \rho_R^I c_{T_R}^I \, (\tilde{v}_L^I - \tilde{v}_R^I)}{\rho_L^I c_{T_L}^I + \rho_R^I c_{T_R}^I}$$

Sanity Check

Updated Tangential Velocity

$$\tilde{v}_S^I = \tilde{v}_S^{slip} + \frac{\tilde{\sigma}_{xy_S}^I}{\rho_L^I c_{T_L}^I} = \tilde{v}_L^I - \frac{\tilde{\sigma}_{xy_L}^I}{\rho_L^I c_{T_L}^I} + \frac{\tilde{\sigma}_{xy_S}^I}{\rho_L^I c_{T_L}^I}$$

Impose Fully Bonded Tangential Stress

$$\tilde{\sigma}_{xy_S}^I = \tilde{\sigma}_{xy_S}^{bonded} = \frac{\rho_L^I c_{T_L}^I \, \tilde{\sigma}_{xy_R}^I + \rho_R^I c_{T_R}^I \, \tilde{\sigma}_{xy_L}^I - \rho_L^I c_{T_L}^I \, \rho_R^I c_{T_R}^I \, (\tilde{v}_L^I - \tilde{v}_R^I)}{\rho_L^I c_{T_L}^I + \rho_R^I c_{T_R}^I}$$

Solve for Tangential Velocity

$$\tilde{v}_S^I = \tilde{v}_L^I - \frac{\tilde{\sigma}_{xy_L}^I}{\rho_L^I c_{T_L}^I} + \frac{\rho_L^I c_{T_L}^I \, \tilde{\sigma}_{xy_R}^I + \rho_R^I c_{T_R}^I \, \tilde{\sigma}_{xy_L}^I - \rho_L^I c_{T_L}^I \, \rho_R^I c_{T_R}^I \, (\tilde{v}_L^I - \tilde{v}_R^I)}{\rho_L^I c_{T_L}^I (\rho_L^I c_{T_L}^I + \rho_R^I c_{T_R}^I)}$$

$$= \frac{\rho_L^I c_{T_L}^I \, \tilde{v}_L^I + \rho_R^I c_{T_R}^I \, \tilde{v}_R^I - (\tilde{\sigma}_{xy_L}^I - \tilde{\sigma}_{xy_R}^I)}{\rho_L^I c_{T_L}^I + \rho_R^I c_{T_R}^I}$$

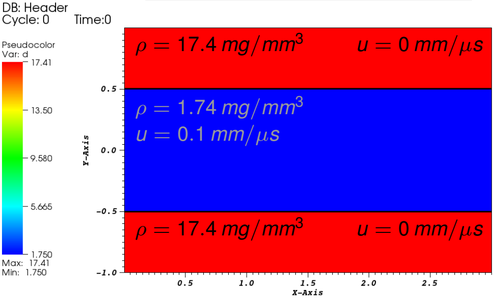
Bonded Velocity Solution

Numerical Results

Sliding Block Under Pressure

B.C.: symmetric & $u = 0$

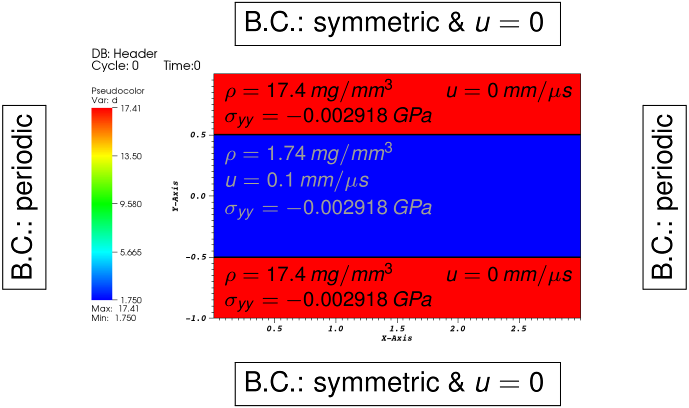
B.C.: periodic



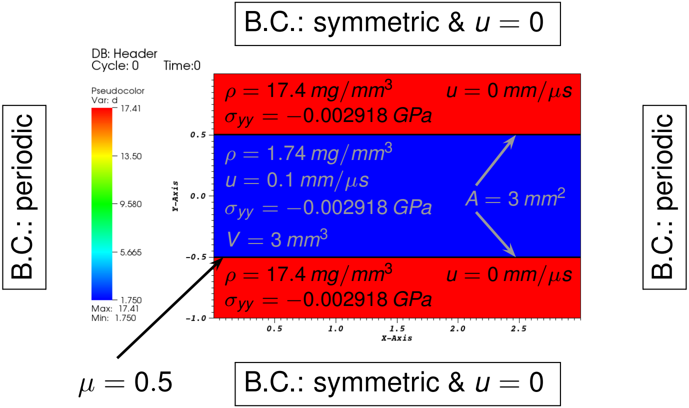
B.C.: periodic

B.C.: symmetric & $u = 0$

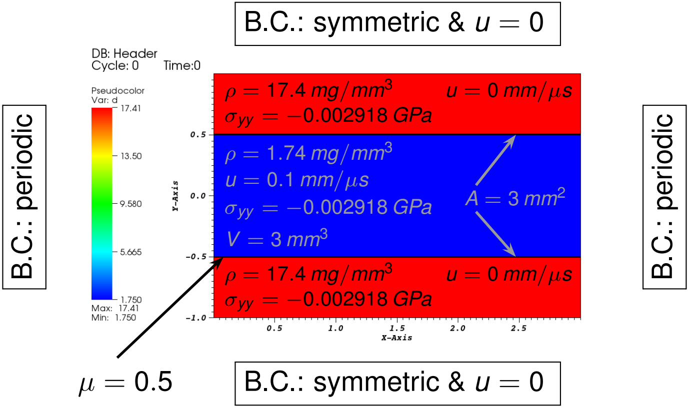
Sliding Block Under Pressure



Sliding Block Under Pressure



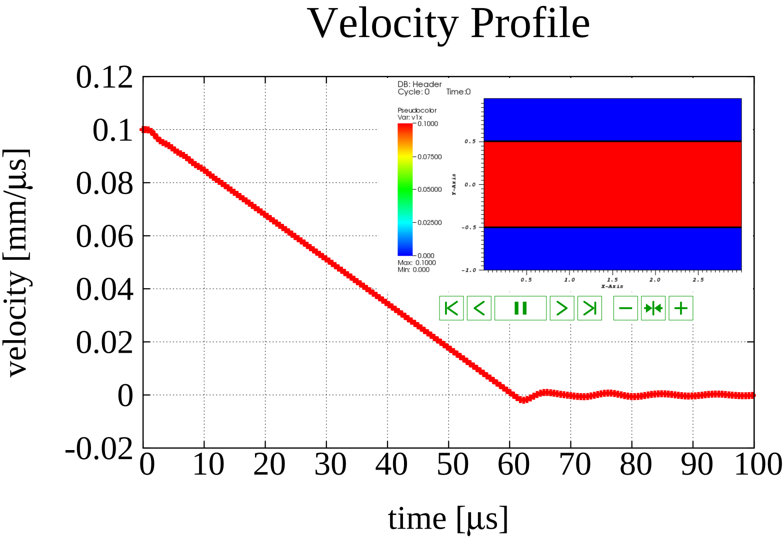
Sliding Block Under Pressure



$$F_x = \mu\, |\sigma_{yy}|\, 2\, A \qquad a = \frac{F_x}{\rho\, V} = 0.0016770\ \frac{mm}{\mu s^2}$$

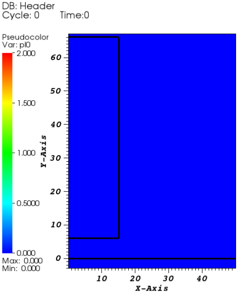
$$t_{stop} = v/a = 59.63\ \mu s$$

Results

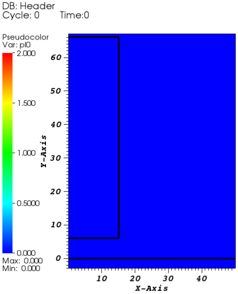

Velocity Profile
velocity [mm/µs]
time [µs]
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0
10
20
30
40
50
60
70
80
90
100
DB: Header
Cycle: 0 Time:0
Pseudocolor
Var: v1x
0.1000
0.07500
0.05000
0.02500
0.000
Max: 0.1000
Min: 0.000
Y-Axis
X-Axis

Taylor Anvil Test

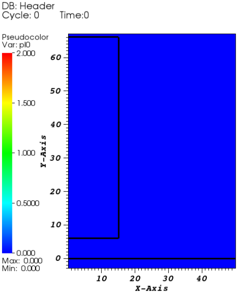
Frictionless slip $\mu = 0$



Slip with Friction $\mu = 0.3$

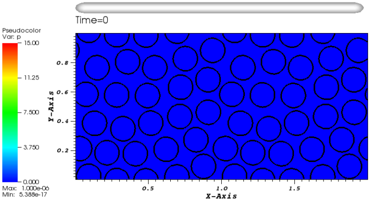


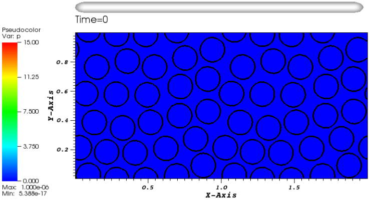
Fully Bonded $\mu = \infty$

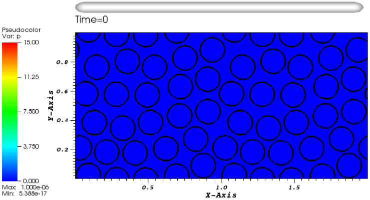


Granular Material Compaction

Fully Bonded $\mu = \infty$

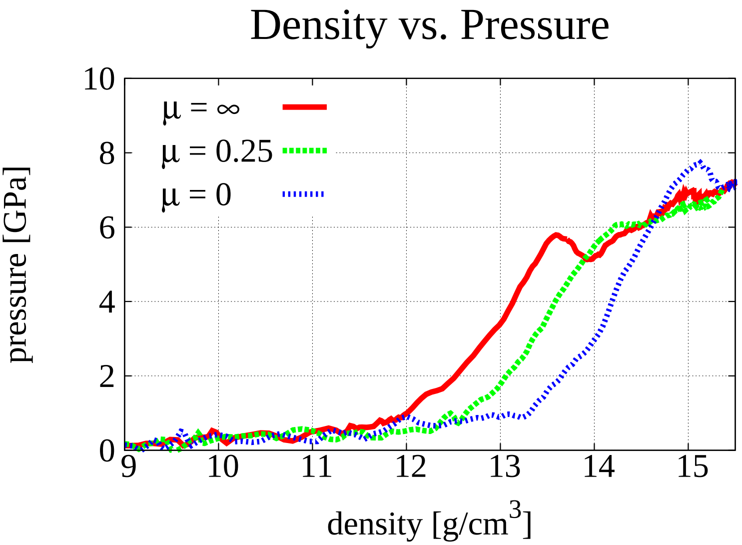






Slip with Friction $\mu = 0.25$ Frictionless slip $\mu = 0$

Compaction Curves



Thank You!